14 Complex Numbers

Formulae

1. Complex numbers:

i. If $z = x + iy$, where $x, y \in R$ and $i = \sqrt{-1}$ is called a complex number whose real part is x and imaginary part is y,

i.e., Re (z) = x and Im (z) = y.

The complex number z is purely real if Im (z) = 0 and purely imaginary if Re(z) = 0.

ii. Integral powers of iota (i):

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{(\text{multiple of } 4)} = 1$

2. Equality of two complex numbers:

The complex numbers $zi = z_1 + ib_1$ and $z_2 = a_2 + ib_2$ are equal iff $a_1 = a_2$ and $b_1 = b_2$

i.e., Re (z_1) = Re (z_2) and Im (z_1) .= Im (z_2)

3. Conjugate of a complex number:

Conjugate of a complex number z = (a + ib) is defined as z = a – ib.

4. Modulus of a complex number:

Modulus of a complex number z = a + ib

denoted by |z| is defined as $|z| = \sqrt{a^2 + b^2}$ or

$$|z| = \sqrt{(\text{Re}(z))^2 + (\text{Im}(z))^2}$$

5. Argument of a complex number:

If $z \neq 0$, the argument (amplitude) θ of z is defined by two equations

$$\cos\theta = \frac{a}{|z|};\ \sin\theta = \frac{b}{|z|}$$

$$\text{So } \arg z = \theta = \tan^{-1}\left(\frac{b}{a}\right),\ 0 \le \theta < 2\pi$$

It is denoted by arg z or amp z.

6. DeMoivre's Theorem:

i. If $n \in Z$ (set of integers), then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

ii. If $n \in Q$ (set of rational numbers), then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

iii. $(\cos\theta + i\sin\theta)^n = \cos n\theta - i\sin n\theta$

iv. $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$

v. $\dfrac{1}{(\cos\theta + i\sin\theta)^1} = \cos\theta - i\sin\theta$

vi. $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$

7. Square root of a Complex numbers:

Let x + iy be a square root of a + ib.

$$x + iy = \sqrt{a + ib}$$

Squaring both sides, we get

$(x + iy)^2 = a + ib$

$\therefore\ x^2 - y^2 + 2xyi = a + ib$

Equating real and imaginary parts, we get

$x^2 - y^2 = a$ and $2xy = b$

Solving these equations, we can find x and y then x + iy will be the required square root of a + ib.

8. Properties of Conjugate:

If $z_1\ z_2,\ z_3$ are complex numbers, then z is the mirror image of z along real axis

i. $\overline{\overline{z}} = z$ ii. $z + \overline{z} = 2\text{Re}(z)$

iii. $z - \overline{z} = 2i\,\text{Im}(z)$

iv. $z = \overline{z} \Leftrightarrow$ z is purely real.

v. $z + \overline{z} = 0 \Leftrightarrow$ is purely imaginary.

vi. $z.\overline{z} = [\text{Re}(z)]^2 + [\text{Im}(z)]^2$

vii. $\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$ viii. $\overline{z_1 - z_2} = \overline{z}_1 - \overline{z}_2$

ix. $\overline{z_1 . z_2} = \overline{z}_1 . \overline{z}_2$

x. $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z}_1}{\overline{z}_2},\ \overline{z}_2 \neq 0$ xi. $\overline{z^n} = (\overline{z})^n$

xii. $z_1\overline{z}_2 + \overline{z}_1 z_2 = 2\text{Re}(\overline{z}_1 z_2) = 2\text{Re}(z_1\overline{z}_2)$

9. Properties of modulus of complex numbers:

If z_1, z_2, z_3 are complex numbers, then

i. $|z| = 0 \Leftrightarrow z = 0$

i.e., Re (z) = Im (z) = 0

ii. $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

iii. $-|z| \le \text{Re}(z) \le |z|;\ -|z| \le \text{Im}(z) \le |z|$

iv. $z\bar{z} = |z|^2$

v. $|z_1 z_2| = |z_1|\,|z_2|$

vi. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|},\ z_2 \ne 0$

vii. $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\text{Re}(z_1 \bar{z}_2)$

viii. $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\text{Re}(z_1 \bar{z}_2)$

ix. $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\,(|z_1|^2 + |z_2|^2)$

x. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$

$= (a^2 + b^2)(|z_1|^2 + |az_2{}^2|)$, where a, b $\in$ R

xi. $|z_1 \pm z_2| \le |z_1| + |z_2|$

xii. $|z_1 \pm z_2| \ge |z_1| - |z_2|$

xiii. $|z^n| = |z|^n$

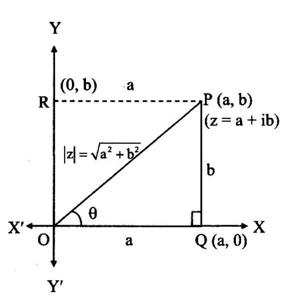
xiv. $|z_1| - |z_2| \le |z_1 + z_2| \le |z_1| + |z_2|$

xv. $|z_1 \pm z_2|^2 = (z_1 \pm z_2)(\bar{z}_1 \pm \bar{z}_2)$

xvi. $z_1 z_2 + z_1 z_2 = 2|z_1||z_2|\cos(\theta_1 - \theta_2)$,

where $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$

10. Geometrical Meaning of Modulus and Argument(Argand Diagram):



i. **Modulus of z** (denoted by |z|) : The length of the line segment OP is called |z|

$\Rightarrow |z| = OP = \sqrt{a^2 + b^2}$

ii. **Argument or Amplitude of** z (denoted by arg (z) or amp (z)) :

The angle θ which OP makes with +ve direction of X-axis in anticlockwise direction is called arg (z).

From the above figure,

$$\cos\theta = \frac{a}{\sqrt{a^2 + b^2}},\ \sin\theta = \frac{b}{\sqrt{a^2 + b^2}},$$

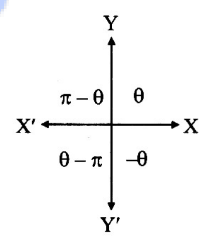
$$\tan\theta = \frac{b}{a} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

iii. **Principal arg (z):** The argument θ which satisfies the inequality $-\pi < \theta \le \pi$ is known as the principal argument of z. This is denoted by Pr. arg (z) or Arg (z).

iv. **Rule to find Arg (z) (Pr value) :**

Let z = a + ib = (a, b) and $\tan^{-1}\left|\frac{b}{a}\right| = \alpha$

Then, arg (z) $= \tan^{-1}\left(\frac{b}{a}\right)$ always gives the principal value. It depends upon the quad, in which the point (a, b) lies.



a. Arg (z) $\tan^{-1}\left|\frac{b}{a}\right|$, when z lies in 1st quadrant.

b. $\pi - \tan^{-1}\left|\frac{b}{a}\right|$, when z lies in 2nd quadrant.

c. Arg (z) $= \tan^{-1}\left|\frac{b}{a}\right|, -\pi$ when z lies in 3 quadrant.

d. Arg (z) $= -\tan^{-1}\left|\frac{b}{a}\right|$ or $2\pi - \tan^{-1}\left|\frac{b}{a}\right|$ when z lies in 4th quadrant.

11. Properties of arg (z):

i. arg (any +ve real no.) = 0

ii. arg (any –ve real no.) = π

iii. $\arg(z - \overline{z}) = \pm\frac{\pi}{2}$

iv. $\arg(z_1.z_2) = \arg(z_1) + \arg(z_2)$

v. $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

vi. $\arg(\overline{z}) = -\arg(z) = \arg\left(\frac{1}{z}\right)$

vii. $\arg(+z) = \pi \pm \arg(z)$ and

$\arg(-z) = \arg z \pm \pi$

viii. $\arg(z) + \arg(\overline{z}) = 0$

12. Polar form of a complex number

The polar form of a complex number $z = x + iy$ is $z = r(\cos\theta + i\sin\theta)$, where

$r = \sqrt{x^2 + y^2} = |z|$ and $x = r\cos\theta$, $y = r\sin\theta$

13. Euler's form or Exponential form:

$e^{i\theta} = \cos\theta + i\sin\theta$

$= \text{cis}\,\theta$

14. If ω is a complex cube root of unity, then

i. $\omega^3 = 1$

ii. $1 + \omega + \omega^2 = 0$

where, $\omega = \frac{-1 + i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1 - i\sqrt{3}}{2}$

Shortcuts

1. If $z = \cos\theta + i\sin\theta$, then

i. $z + \frac{1}{z} = 2\cos\theta$

ii. $z - \frac{1}{z} = 2i\sin\theta$

iii. $z^n + \frac{1}{z^n} = 2\cos n\theta$

iv. If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\alpha$, $z = \cos\gamma + i\sin\gamma$ and $x + y + z — 0$ (given), then

a. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

b. $yz + zx + xy = 0$

c. $x^2 + y^2 + z^2 = 0$

d. $x^3 + y^3 + z^3 = 3xyz$

2. $\sqrt{-a}\sqrt{-b} \neq \sqrt{ab}$ because $\sqrt{-a}.\sqrt{-b} = -\sqrt{ab}$ where $a, b \in R$

3. Square root of $z = a + ib$ is

$$\sqrt{a + ib} = \pm\left[\sqrt{\frac{|z| + a}{2}} + i\sqrt{\frac{|z| - a}{2}}\right], \text{ for } b > 0$$

$$= \pm\left[\sqrt{\frac{|z| + a}{2}} - i\sqrt{\frac{|z| - a}{2}}\right], \text{ for } b < 0$$

4. $\sqrt{a + ib} + \sqrt{a - ib} = \sqrt{2a + 2\sqrt{a^2 + b^2}}$

5. $\sqrt{a + ib} - \sqrt{a - ib} = i\sqrt{2\sqrt{a^2 + b^2} - 2a}$

6. $\left(\frac{-1 + i\sqrt{3}}{2}\right)^n + \left(\frac{-1 - i\sqrt{3}}{2}\right)^n = -1$

i.e., $\omega^n + \omega^{2n} = -1$ if n is a +ve integer other than multiple of 3.

7. $\left(\frac{-1 + i\sqrt{3}}{2}\right)^n + \left(\frac{-1 - i\sqrt{3}}{2}\right)^n = 2$

i.e., $\omega^n + \omega^{2n} = 2$ if n is a +ve integer, which is a multiple of 3.

8. Some results involving complex cube root of unity (ω)

i. $(x^3 \pm 1) = (x \pm 1)(x \pm \omega)(x \pm \omega^2)$

ii. ω and ω^2 are roots of $x^2 + x + 1 = 0$

iii. $a^3 \pm b^3 = (a \pm b)(a \pm b\omega)(a \pm b\omega^2)$

iv. $a^2 + b^2 + c^2 - bc - ca - ab$

$= (a + b\omega + b\omega^2)(a + b\omega^2 + \omega)$

v. $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a + b\omega + \omega^2)(a + b\omega^2 + \omega)$

vi. Cube roots of real number a are

$a^{1/3}$, $a^{1/3}\omega$, $a^{1/3}\omega^2$

vii. $x^2 \pm x + 1 = (x \pm \omega)(x \pm \omega^2)$

viii. $x^2 \pm xy + y^2 = (x \pm y\omega)(x \pm y\omega^2)$

9. If $\omega^{3n} = 1$, $\omega^{3n+1} = \omega$, $\omega^{3n+2} = \omega^2$, then
$\omega^{3n} + \omega^{3n+1} + \omega^{3n+2} = 0$

10. If α, β are non-real cube roots of unity, then
    i. $\alpha + \beta = -1$
    ii. $\alpha\beta = 1$
    iii. $\alpha^3 = \beta^3 - 1$
    iv. $\alpha^2 = \beta$ and $\beta^2 = \alpha$
    v. $\overline{\alpha} = \beta$ and $\overline{\beta} = \alpha$

11. n^{th} root of complex number
$z = r(\cos\theta + i\sin\theta)$, $r > 0$

$$z^{1/n} = r^{1/n}\left[\cos\left(\frac{2m\pi+\theta}{n}\right) + i\sin\left(\frac{2m\pi+\theta}{n}\right)\right]$$

where $m = 0, 1, 2, \ldots, n-1$
    i. Sum of all roots of $z^{1/n}$ is always equal to zero.
    ii. Product of all roots of $z^{1/n} = (-1)^{n-1}$.

12. Area of the triangle with vertices z, ωz and

$z + \omega z$ is $\frac{\sqrt{3}}{4}|z|^2$

13. Area of the triangle whose vertices are z, iz and

$z + iz$ is $\frac{1}{2}|z|^2$

14. If z_1, z_2, z_3 are collinear, then

$$\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = 0$$

i.e., $\frac{z_3 - z_1}{z_2 - z_1}$ is purely real

MULTIPLE CHOICE QUESTIONS

Classical Thinking

14.1 Equality of two complex numbers, Conjugate and Algebra of a complex number

1. A set of complex numbers is denoted by

a) $C = \{a + bi / a, b \in R \text{ and } i^2 = -1\}$

b) $C = (a + bi / a \in R, b \in$ Imaginary number and $i^2 = -1 \}$

c) $C = (a + bi / a, b \in R \text{ and } i^2 = 1\}$

d) $C=\{a + bi/a, b \in R \text{ and } i = -1\}$

2. Let $x, y \in R$, then $x + yi$ is a non-real complex number if

a) $x = 0$ b) $y = 0$

c) $x \neq 0$ d) $y \neq 0$

3. If $z = i - 1$, then $\overline{z} =$

a) $i + 1$ b) $-i - 1$

c) $-i$ d) i

4. Let $x, y \in R$, then $x + yi$ is a purely imaginary number if

a) $x = 0, y \neq 0$

b) $x \neq 0, y = 0$

c) $x \neq 0, y \neq 0$

d) $x = 0, y = 0$

5. a + ib form of the complex number $1 + (2i)(-2 + i)$ is

a) $-4 - 3i$ b) $4 - 3i$

c) $-4 + 3i$ d) $4 + 3i$

6. If $z_1 = 3 + 2i$ and $z_2 = 2 - 3i$, then $z_1 + z_2 =$

a) $7 - i$ b) $7 + i$

c) $5 + i$ d) $5 - i$

7. If $z_1 = 3 + 2i$ and $z_2 = 2 - 3i$, then $\dfrac{z_1}{z_2} =$

a) 0 b) $0 + i$

c) $0 - i$ d) $1 + i$

8. If $z_1 = 1 - 3i$ and $z_2 = 2 + i$, then $\overline{z}_1 + \overline{z}_2 =$

a) $3 - 2i$

b) $2 + 3i$

c) $3 + 2i$

d) $2 - 3i$

9. Multiplicative inverse of the non-zero complex number $x + yi$ $(x, y \in R)$ is

a) $\dfrac{x}{x+y} - \dfrac{y}{x+y}i$

b) $\dfrac{x}{x^2+y^2} - \dfrac{y}{x^2+y^2}i$

c) $-\dfrac{x}{x^2+y^2} + \dfrac{y}{x^2+y^2}i$

d) $\dfrac{x}{x+y} + \dfrac{y}{x+y}i$

10. If z is any complex number, then $\dfrac{z-\overline{z}}{2i}$ is

a) purely real

b) purely imaginary

c) either 0 or purely imaginary

d) none of these

11. The conjugate of a complex number z is $\dfrac{1}{i-1}$. Then, the complex number is

a) $\dfrac{-1}{i+1}$ b) $\dfrac{1}{i-1}$

c) $\dfrac{-1}{i-1}$ d) $\dfrac{1}{i+1}$

12. If $(x + yi)^{1/3} = u + vi$, where $u, v, x, y \in R$, then

a) $\dfrac{x}{u} + \dfrac{y}{v} = 4(u^2 - v^2)$ b) $\dfrac{x}{u} - \dfrac{y}{v} = 4(u^2 - v^2)$

c) $\dfrac{x}{u} + \dfrac{y}{v} = 4(u^2 + v^2)$ d) none of these

13. Additive inverse of $1 - i$ is

a) $0 + 0i$

b) $-1 - i$

c) $-1 + i$

d) $1 - i$

14. $z + \overline{z} \neq 0$, if and only if

a) $Re(z) \neq 0$ b) $Im(z) \neq 0$

c) $z \neq 0$ d) $|z| \neq 0$

15. Complex number $\frac{5-2i}{3-4i}-\frac{5+2i}{3+4i}$ can be written in a + ib form as

a) 0 b) $\left(\frac{28}{25}\right)+i$

c) $0+\left(\frac{28}{25}\right)+i$ d) $\left(\frac{28}{25}\right)$

16. If (3 + i)x + (1 – 2i)y + 7i = 0, then the values of x and y respectively are

a) 1, – 3 b) – 1, 3

c) 2, – 4 d) 4, – 2

17. If (2 – i) + (1 – 3i)y + 2 = 0, then the values of x and y respectively are

a) $-\frac{8}{5},\frac{2}{5}$ b) $\frac{6}{5},-\frac{3}{5}$

c) $-\frac{6}{5},\frac{2}{5}$ d) $-\frac{8}{5},\frac{3}{5}$

18. If $x=1-i\sqrt{3}$, then $x^3 - x^2 + 2x + 4 =$

a) 0 b) 1

c) – 1 d) 2

19. The real values of x and y for which the equation (x + iy) (2 – 3i) = 4 + i is satisfied, are

a) $x=\frac{5}{13}, y=\frac{8}{13}$ b) $x=\frac{8}{13}, y=\frac{5}{13}$

c) $x=\frac{5}{13}, y=\frac{14}{13}$ d) $x=\frac{5}{13}, y=\frac{4}{13}$

20. If z_1 = 1 – i and z_2 = – 2 + 4i, then $\text{Im}\left(\frac{z_1z_2}{z_1}\right)=$

a) 1 b) 2

c) 3 d) 4

14. 2 Modulus, Argument, Power and Square root of a complex number

21. $5 + i^{22} + i^{36} + i^{56} =$

a) – 6 b) 8

c) – 8 d) 6

22. The number $\frac{(1-i)^3}{1-i^3}$ is equal to

a) i b) – i

c) – 1 d) – 2

23. $i^{4k+3} =$

a) i b) i

c) – 1 d) – 2

24. The value of $(1+i)^5 \times (1-i)^5$ is

a) – 8 b) 8i

c) 8 d) 32

25. $\frac{i^6+i^7+i^8+i^9}{-1+i}=$

a) 2i b) 0

c) 1 + i d) 1

26. The modulus of $z = 1+\sqrt{3}i$ is

a) $\sqrt{2}$ b) 2

c) 4 d) $\sqrt{5}$

27. The modulus and argument of $\sqrt{3}+\sqrt{2}\,i$ are

a) $\sqrt{5}, \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ b) $\sqrt{5}, \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

c) $\sqrt{7}, \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ d) $\sqrt{7}, \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$

28. If $i=\sqrt{-1}$, then $1 + i^2 + i^3 - i^6 + i^8$ is equal to

a) 2 – i b) 1

c) 3 d) 2 + i

29. The modulus and amplitude of 3 + 2i are

a) $\sqrt{15}, \tan^{-1}\left(\frac{2}{3}\right)$ b) $\sqrt{13}, \tan^{-1}\left(\frac{2}{3}\right)$

c) $\sqrt{13}, \tan^{-1}\left(\frac{3}{2}\right)$ d) $\sqrt{15}, \tan^{-1}\left(\frac{3}{2}\right)$

30. The square roots of – 2i are

a) 1 + i, – 1 + i b) – 1 – i, – 1 + i

c) 1 – i, 1 + i d) 1 – i, – 1 + i

31. The value of |z – 5|,if z = x + iy is

a) $\sqrt{(x-5)^2+y^2}$ b) $x^2+\sqrt{(y-5)^2}$

c) $\sqrt{(x-y)^2+5^2}$ d) $\sqrt{x^2+(y-5)^2}$

32. If z_1 and z_2 are two complex numbers, then $|z_1 - z_2|$ is

a) $\geq |z_1|-|z_2|$ b) $\leq |z_1|-|z_2|$

c) $\geq |z_1|+|z_2|$ d) $\leq |z_1|-|z_2|$

33. Amp (– i) is

a) $\pi / 2$ b) $-(\pi / 2)$

c) $\pi / 3$ d) $\pi / 4$

34. For z = a + bi, if (a, b) lies in 3rd quadrant, then arg z =

a) $-\pi + \tan^{-1}\left|\frac{b}{a}\right|$ b) $\tan^{-1}\left|\frac{b}{a}\right|$

c) $2\pi + \tan^{-1}\left|\frac{b}{a}\right|$ d) $\frac{\pi}{2} + \tan^{-1}\left|\frac{b}{a}\right|$

35. If z is purely real and Re (z) < 0, then Arg (z) is

a) 0 b) π

c) $-\pi$ d) $\frac{\pi}{2}$

36. Polar form of $z = 4 + 4\sqrt{3}\ i$ is

a) $8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

b) $4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

c) $8\left(\cos\frac{2\pi}{3} + i\sin\frac{5\pi}{3}\right)$

d) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

37. $(\sin\theta + i\cos\theta)^n$ is equal to

a) $\cos n\theta + i\sin n\theta$

b) $\sin n\theta + i\cos n\theta$

c) $\cos n\left(\frac{\pi}{2} - \theta\right) + i\sin n\left(\frac{\pi}{2} - \theta\right)$

d) $\cos n\left(\frac{\pi}{2} - \theta\right) - i\sin n\left(\frac{\pi}{2} - \theta\right)$

38. The value of $\frac{4(\cos 75^0 + i\sin 75^0)}{0.4(\cos 30^0 + i\sin 30^0)}$ is

a) $\frac{\sqrt{2}}{10}(1+i)$ b) $\frac{\sqrt{2}}{10}(1-i)$

c) $\frac{10}{\sqrt{2}}(1-i)$ d) $\frac{10}{\sqrt{2}}(1+i)$

39. The amplitude of the complex number $z = \sin\alpha + i(I - \cos\alpha)$ is

a) $2\sin\frac{\pi}{2}$ b) $\frac{\alpha}{2}$

c) α d) $\cos\frac{\alpha}{2}$

14.4 Fundamental theorem of algebra, Cube roots of unity

40. The roots of equation $x^2 + x + 1 = 0$ are

a) $\frac{-1 \pm i}{2}$ b) $\frac{1 \pm \sqrt{3}i}{2}$

c) $\frac{-1 \pm \sqrt{3}i}{2}$ d) $\frac{-i \pm \sqrt{3}i}{2}$

41. 1, ω, ω^2 are cube roots of

a) 1 b) 2

c) 1 / 2 d) 3

42. If ω is a complex cube root of unity, then $(1 + \omega^2)^3 =$

a) ω b) 1

c) – 1 d) ω

43. If ω is a complex cube root of unity, then the value of $\omega^{99} + \omega^{100} + \omega^{101}$ is

a) 1 b) – 1

c) 3 d) 0

44. If ω is a complex cube root of unity, then $\frac{1}{\omega} + \frac{1}{\omega^2} =$

a) 1 b) – 1

c) $1 / \omega$ d) $-(1 / \omega)$

45. If ω is a complex cube root of unity, then $(1 + \omega - 2\omega^2)^4 + (4 + \omega + 4c\omega^2)^4 =$

a) 0 b) – 81

c) 81 d) – 1

46. If ω is a complex cube root of unity, then $(2 + 5\omega + 2\omega^2)^6 =$

a) 18 b) 0

c) 729 d) 3ω

47. If ω is a complex cube root of unity, then $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 =$

a) ω b) 2ω

c) 1 d) 0

48. If ω is a complex cube root of unity, then $(1 + \omega^2)^4 =$

a) 2ω b) ω

c) 3ω d) 1

Critical Thinking

14.1 Equality of two complex numbers, Conjugate and Algebra of a complex number

1. If $z = \bar{z}$, then
a) z is purely real
b) z is purely imaginary
c) Re(z) = Im(z)
d) z is any complex number

2. a + ib form of the complex number $\frac{1+3i}{2+3i}$ is
a) $\frac{3}{13}+\frac{11}{13}$ b) $\frac{11}{13}+i\frac{3}{13}$
c) $\frac{2}{11}+i\frac{3}{11}$ d) $\frac{2}{11}-i\frac{3}{11}$

3. Which of the following is correct ?
a) $2+3i < 3+4i$ b) $3-4i < 2-3i$
c) $1+i < 1-i$ d) none of these

4. The imaginary part of $\frac{(1+i)^2}{(2-i)}$ is
a) $\frac{1}{5}$ b) $\frac{3}{5}$
c) $\frac{4}{5}$ d) $\frac{2}{5}$

5. The values of x and y satisfying the equation $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ are
a) x = – 1, y = 3 b) x = 3, y = – 1
c) x = 0, y = 1 d) x = 1, y = 0

6. If x = 1 + 2i, the value of $x^3 + 2x^2 - 3x + 5 =$
a) 0 b) – 15
c) 25 d) – 19

7. a + ib form of the complex number $\left(1+\frac{2}{i}\right)\left(1+\frac{3}{i}\right)(2+i)^{-1}$ is
a) – 3– i b) – 3 + i
c) – 5 – i d) – 5 + i

8. If $z=(3\sqrt{7}+4i)^2(3\sqrt{7}-4i)^3$, then Re(z) =
a) $79\times 3\sqrt{7}$ b) $(79)^2(3\sqrt{7})$
c) $-4\,(79)^2$ d) $(79)^2(3\sqrt{7}-4i)$

9. If $(x + iy)(p + iq) - (x^2 + y^2)\, i$, then
a) p = x, q = y b) p = y, q = x
c) $p = x^2, q = y^2$ d) q = – x, p = – y

10. If x = – 3 + 5i, then the value of $x^3 + 6x^2 + 34x + 1$ is equal to
a) 0 b) 1
c) – 1 d) 2

11. If (2 + i)x – (1 + 2i)y = 3i, then the values of x and y respectively are
a) – 1, – 2 b) 1, 2
c) – 1, 2 d) 1, – 2

12. If a, b are real and $(i^4 + 3i)a + (i - 1)b + 5i^3 = 0$, then the values of a and b respectively are
a) $-\frac{5}{4},\frac{3}{4}$ b) $\frac{5}{4},\frac{5}{4}$
c) $-\frac{5}{4},-\frac{5}{4}$ d) $\frac{5}{4},-\frac{3}{4}$

13. If $x=\frac{5+i}{1-i}$, the valueof $x^3 - x^2 + x + 44$ is
a) 8 b) 2
c) 3 d) 5

14. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is
a) $\frac{7-26i}{25}$ b) $\frac{-7-26i}{25}$
c) $\frac{-7+26i}{25}$ d) $\frac{7+26i}{25}$

15. If (x +yi) (3 – 4i) = 5 + 12i, then $\sqrt{x^2+y^2}=$
a) 65 b) $\frac{5}{13}$
c) $\frac{13}{5}$ d) 18

16. If z_1 and z_2 are two complex numbers, then
a) $Re(z_1).Re(z_2)$ b) $Re(z_1).Im(z_2)$
c) $Im(z_1).Re(z_2)$ d) None of these

17. If z = x – iy and $z^{\frac{1}{3}}$ = p + iq, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{p^2+q^2}$ is equal to
a) – 2 b) – 1
c) 2 d) 1

18 If $z = x + iy$, $z^{\frac{1}{3}} = a - ib$ and $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$, then the value of k equals

a) 2 b) 4

c) 6 d) 1

19. If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta}$, then $\frac{1+a}{1-a} =$

a) $\cot\theta$ b) $\cot\frac{\theta}{2}$

c) $i\cot\frac{\theta}{2}$ d) $i\tan\frac{\theta}{2}$

21. The real part of $\frac{1}{1 - \cos\theta + i\sin\theta}$ is equal to

a) 1/4 b) 1/2

c) $\tan \theta/2$ d) $1/1 - \cos\theta$

22. If z is a complex number, then $(\overline{z^{-1}})\,(\overline{z}) =$

a) 1 b) – 1

c) 0 d) i

23. The number of solutions of the equation

$z^2 + \overline{z} = 0$ is

a) 1 b) 2

c) 3 d) 4

24. If $\frac{z-i}{z+i}$ $(z \neq -i)$ is a purely imaginary number,

$z^2 + \overline{z} = 0$ is

a) 0 b) 1

c) 2 d) – 1

25. If$(1 + i)(1 + 2i)(1 + 3i).....(1 + ni) = a + ib$, then $2.5.10....(1 + n^2)$ is equal to

a) $a^2 - b^2$ b) $a^2 + b^2$

c) $\sqrt{a^2 + b^2}$ d) $\sqrt{a^2 - b^2}$

14.2 Modulus, Argument, Power and Square root of a complex number

26. If n is any positive integer, then the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$ equals

a) 1 b) – 1

c) i d) – i

27. The square roots of – 8i are

a $2 - 2i, -2 + 2i$ b) $2 + 2i, -2 + 2i$

c) $2 - 2i, -2 - 2i$ d) $-2 - 2i, 2 + 2i$

28. Square roots of – 48 – 14 i are

a) $\pm(7 + i)$ b) $\pm(7 - i)$

c) $\pm(1 + 7i)$ d) $\pm(1 - 7i)$

29. The square roots of $5 - 2\sqrt{14}\,i$ are

a) $\sqrt{7} + \sqrt{2}i, -\sqrt{7} + \sqrt{2}i$

b) $\sqrt{7} - \sqrt{2}i, -\sqrt{7} + \sqrt{2}i$

c) $\sqrt{7} - \sqrt{2}\,i, -\sqrt{7} - \sqrt{2}i$

d) $-\sqrt{7} - \sqrt{2}\,i, -\sqrt{7} - \sqrt{2}i$

30. If $i = \sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3} =$

a) 1 b) i

c) i^0 d) 0

31. If $z^2 = -24 - 18i$, then $z =$

a) $\pm\sqrt{3}\,(1 + 3i)$ b) $\pm\sqrt{3}\,(3 - i)$

c) $\pm\sqrt{3}\,(1 - 3i)$ d) $\pm\sqrt{3}\,(3 + i)$

32. The square roots of $\frac{35}{4} + 3i$ are

a) $2 + \frac{1}{3}i, -2 - \frac{1}{3}i$ b) $3 + \frac{1}{2}i, -3 - \frac{1}{2}i$

c) $3 - \frac{1}{2}i, -3 + \frac{1}{2}i$ d) $2 - \frac{1}{3}i, -2 + \frac{1}{3}i$

33. If n is an odd integer, then $(1 + i)^{6n} + (1 - i)^{6n}$ is equal to

a) 0 b) 2

c) – 2 d) 1

34. If $p + iq = \sqrt{\frac{a + ib}{c + id}}$, where p, q, a, b, c, d $\in$ R, then $(p^2 + q^2)^2 =$

a) $\sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$ b) $\frac{a^2 + b^2}{c^2 + d^2}$

c) $\left(\frac{a^2 + b^2}{c^2 + d^2}\right)^2$ d) none of these

35. If $z_1 = 5 - 2i$ and $z_2 = 6 + 5i$, then $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| =$

a) $\sqrt{\frac{13}{18}}$ b) $\sqrt{\frac{3}{5}}$

c) $\sqrt{\frac{1}{8}}$ d) $\sqrt{\frac{13}{5}}$

36. The modulus and amplitude of 5 + 12i are

a) $17, \tan^{-1}\left(\frac{12}{7}\right)$ b) $17, \tan^{-1}\left(\frac{5}{12}\right)$

c) $13, \tan^{-1}\left(\frac{5}{12}\right)$ d) $13, \tan^{-1}\left(\frac{12}{5}\right)$

37. $i^{65} + \frac{1}{i^{145}} =$

a) 0 b) 1

c) i d) – i

38. The value of i^{243} is equal to

a) i b) – i

c) – 1 d) 1

39. If $x + 2i + 15i^6 = 7x + i^3(y + 4)$, where $x, y \in R$, then $x + y =$

a) 21 b – 9

c) 9 d) – 21

40. If n is a positive integer, then $\left(\frac{1+i}{1-i}\right)^{4n+1} =$

a) 1 b) – 1

c) i d) – i

41. The value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1 =$

a) – 1 b) – 2

c) – 3 d) – 4

42. If $i^2 = -1$, then $i + i^2 + i^3 + ...$ to 1000 terms is equal to

a) 1 b) – 1

c) i d) 0

43. If $\sum_{k=0}^{100} i^k = x + iy$, then the values of x and y are

a) $x = -1, y = 0$ b) $x = 1, y = 1$

c) $x = 1, y = 0$ d) $x = 0, y = 1$

44. The inequality $|z - 4| < |z - 2|$ represents the region given by

a) Re(z) > 0

b) Re(z) < 0

c) Re(z) > 2

d) Re(z) > 3

45. If $|z_1| = |z_2| .... = |z_n| = 1$, then the value of $|z_1 + z_2 + z_3 + ... + z_n| =$

a) 1

b) $|z_1| + |z_2| + ... + |z_n|$

c) $\left|\frac{1}{z_1} + \frac{1}{z_2} + ... + \frac{1}{z_n}\right|$

d) None of these

46. If $[z] = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then Re(ω) is

a) 0 b) $-\frac{1}{|z+1|^2}$

c) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ d) $\frac{\sqrt{2}}{|z+1|^2}$

47. Let z_1 be a complex number with $|z_1| = 1$ and z_2 be any complex number, then $\left|\frac{z_1 - z_2}{1 - z_1 \bar{z}_2}\right| =$

a) 0 b) 1

c) – 1 d) 2

48. The amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}+i}$ is

a) $\frac{\pi}{6}$ b) $-\frac{\pi}{6}$

c) $\frac{\pi}{3}$ d) $-\frac{\pi}{3}$

49. The modulus and amplitude of $\frac{1+2i}{1-(1+i)^2}$ are

a) $\sqrt{2}$ and $\frac{\pi}{6}$ b) 1 and 0

c) 1 and $\frac{\pi}{3}$ d) 1 and $\frac{\pi}{4}$

50. $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely nnaginary, if $\theta =$

a) $2n\pi \pm \dfrac{\pi}{3}$ b) $n\pi \pm \dfrac{\pi}{4}$

c) $n\pi \pm \dfrac{\pi}{3}$ d) $n\pi \pm \dfrac{\pi}{6}$

14.3 DeMoivre's theorem, Argand diagram and Polar form of a complex number

51. If z = – 1 – i, then arg z is

a) $\dfrac{\pi}{4}$ b) $\dfrac{5\pi}{4}$

c) $-\dfrac{3\pi}{4}$ d) $\dfrac{7\pi}{4}$

52. $\left(\dfrac{1+i}{\sqrt{2}}\right)^8 + \left(\dfrac{1-i}{\sqrt{2}}\right)^8 =$

a) 1 b) 2

c) 4 d) 8

53. Amplitude of $\dfrac{1+i}{1-i}$ is

a) $-\dfrac{\pi}{2}$ b) $\dfrac{\pi}{2}$

c) 0 d) π

54. The amplitude of $\sin\dfrac{\pi}{5} + i\left(1-\cos\dfrac{\pi}{5}\right)$ is

a) $\dfrac{\pi}{5}$ b) $\dfrac{2\pi}{5}$

c) $\dfrac{\pi}{10}$ d) $\dfrac{\pi}{15}$

55. If 0 < amp (z) < π, then amp (z) – amp (–z) =

a) 0 b) 2amp (z)

c) n d) none of these

56. If z = 1 – cos a + i sin α, then amp z =

a) $\dfrac{\alpha}{2}$ b) $-\dfrac{\alpha}{2}$

c) $\dfrac{\pi}{2}+\dfrac{\alpha}{2}$ d) $\dfrac{\pi}{2}-\dfrac{\alpha}{2}$

57. The value of $(-i)^{1/3}$ is

a) $\dfrac{1+\sqrt{3}i}{2}$ b) $\dfrac{1-\sqrt{3}i}{2}$

c) $\dfrac{-\sqrt{3}-i}{2}$ d) $\dfrac{\sqrt{3}-i}{2}$

58. The amplitude of $e^{e^{-i\theta}}$ is equal to

a) $\sin\theta$ b) $-\sin\theta$

c) $e^{\cos\theta}$ d) $e^{\sin\theta}$

59. If $a=\sqrt{2i}$, then which of the following is correct?

a) $a = i + i$ b) $a = 1 - i$

c) $a = -(\sqrt{2})i$ d) $a = -1 - i$

60. $\left(\dfrac{1+\cos\phi+i\sin\phi}{1+\cos\phi-i\sin\phi}\right)^n =$

a) $\cos n\phi - i\sin n\phi$ b) $\cos n\phi + i\sin n\phi$

c) $\sin n\phi + i\cos n\phi$ d) $\sin n\phi - i\cos n\phi$

61. The value of $\left[\dfrac{1-\cos\dfrac{\pi}{10}+i\sin\dfrac{\pi}{10}}{1-\cos\dfrac{\pi}{10}-i\sin\dfrac{\pi}{10}}\right]^{10} =$

a) 0 b) – 1

c) 1 d) 2

62. $\dfrac{(\cos 2\theta - i\sin 2\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2} (\cos 3\theta - i\sin 3\theta)^{-9}} =$

a) cos 49θ – i sin 49θ

b) cos 23θ – i sin 23θ

c) cos 49θ + i sin 49θ

d) cos 21θ + i sin 21θ

63. Which of the following is a fourth root of $\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}$?

a) $\cos\left(\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}\right)$ b) $\cos\left(\dfrac{\pi}{12}+i\sin\dfrac{\pi}{12}\right)$

c) $\cos\left(\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)$ d) $\cos\left(\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)$

14.4 Fundamental theorem of algebra, cube roots of unity

64. The roots of equation $x^2 - (5 + i)x + 18 - i = 0$ are

a) $\frac{3+i\pm(7i+3)}{2}$ b) $\frac{3\pm(7i+3)}{2}$

c) $\frac{5\pm(7i+1)}{2}$ d) $\frac{5+i\pm(7i+1)}{2}$

65. The roots of equation $9x^2 - 12x + 20 = 0$ are

a) $\frac{2}{3}\pm\frac{4i}{3}$ b) $\frac{4}{3}\pm\frac{2i}{3}$

c) $\frac{3}{5}\pm\frac{4i}{5}$ d) $\frac{4}{5}\pm\frac{3i}{5}$

66. The value of $\left(\frac{1+\omega}{\omega^2}\right)^3$ is

a) 1 b) – 1

c) ω d) ω^2

67. If ω is a complex cube root of unity, then

a) 74 b) 68

c) 72 d) 64

68. If ω is a complex cube root of unity, then $(2 - \omega)(2 - \omega^2)(2 - \omega^{10})(2 - \omega^{11})$ is

a) – 47 b) 47

c) 49 d) – 49

69. If α and α are complex cube roots of unity, then $(1 - \alpha)(1 - \beta)(1 - \alpha^2)(1 - \beta^2) =$

a) 3 b) 6

c) 9 d) 12

70. The value of $\frac{1}{1+\omega}+\frac{1}{1+\omega^2}=$

a) 0 b) 1

c) – 1 d) 2

71. Value of $\left(\frac{-1+\sqrt{3}}{2}\right)^{40}+\left(\frac{-1-\sqrt{-3}}{2}\right)^{40}$ is

a) 0 b) 1

c) 2 d) – 1

72. If $i=\sqrt{-1}$, then

$$4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}=$$

a) $1-i\sqrt{3}$ b) $-1+i\sqrt{3}$

c) $i\sqrt{3}$ d) $-i\sqrt{3}$

73. The roots of equation $x^2 - x + 12i = 0$ are

a) $\frac{1+7}{2i}$ b) $\frac{3+5}{2i}$

c) $\frac{1+7}{3i}$ d) $\frac{3+5}{3i}$

74. If α is a complex cube root of unity such that $\alpha^2 + \alpha + 1 = 0$, then α^{31} is

a) α b) α^2

c) 0 d) 1

75. $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 =$

a) 8 b) 16

c) 32 d) 48

76. If 1, ω, ω^2 are the cube roots of unity, then

a) 1 b) – 1

c) i d) 0

77. If ω is a complex cube root of unity, then $(1 - \omega + \omega^2)^3 =$

a) – 6 b) 8

c) 6 d) – 8

78. If ω is a complex cube root of unity, then $(x + y)^3 + (x\omega + y\omega^2)^3 + (x\omega^2 + y\omega)^3 =$

a) $3(x^3 + y^3)$ b) $3(x^3 - y^3)$

c) $4(x^3 + y^3)$ d) $4(x^3 - y^3)$

79. If $z=\frac{\sqrt{3}+i}{2}$, then z^{69} is equal to

a) – i b) i

c) 1 d) – 1

Competitive Thinking

14.1 Equality of two complex numbers, Conjugate and Algebra of a complex number

1. If $\frac{5(-8+6i)}{(1+i)^2} = a + ib$, then (a, b) equals

a) (15, 20) b) (20, 15)
c) (– 15, 20) d) (– 15, – 20)

2. The true statement is

a) $1 - i < 1 + i$ b) $2i + 1 > -2i + 1$
c) $2i > 1$ d) None of these

3. Let z_1, z_2 be two complex numbers such that $z_1 + z_2$ and $z_1 z_2$ both are real, then

a) $z_1 = -z_2$ b) $z_1 = z_2$
c) $z_1 = -z_2$ d) $z_1 = z_2$

4. If $3 - 2yi = 9x - 7i$, where $i^2 = -1$, x and y are real, then

a) x = 0.5, y = 3.5 b) x = 5, y = 3
c) $x = \frac{1}{2}, y = 7$ d) $x = 0, \ y = \frac{3+7i}{2i}$

5. If $(a+ib)(c+id)(e+if)(g+ih) = A + iB$, then $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) =$

a) $A^2 + B^2$ b) $A^2 - B^2$
c) A^2 d) B^2

6. If $\frac{c+i}{c-i}$, where a, b, c are real, then $a^2 + b^2 =$

a) 1 b) – 1
c) c^2 d) $-c^2$

7. If the conjugate of $(x+iy)(1-2i)$ be $1+i$, then

a) $x = \frac{1}{5}$ b) $y = \frac{3}{5}$
c) $x + iy = \frac{1-i}{1-2i}$ d) $x - iy = \frac{1-i}{1+2i}$

8. The conjugate of $\frac{(2+i)^2}{3+i}$ in the form of a + ib is

a) $\frac{13}{2} + i\left(\frac{15}{2}\right)$ b) $\frac{13}{10} + i\left(\frac{-15}{2}\right)$
c) $\frac{13}{10} + i\left(\frac{-9}{10}\right)$ d) $\frac{13}{10} + i\left(\frac{9}{10}\right)$

9. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be real, if $\theta =$

a) $2n\pi$ b) $n\pi + \frac{\pi}{2}$
c) $n\pi$ d) $n\pi - \frac{\pi}{2}$

10. The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is

a) $\frac{1}{3+5\cos\theta}$ b) $\frac{1}{5-3\cos\theta}$
c) $\frac{1}{3-5\cos\theta}$ d) $\frac{1}{5+3\cos\theta}$

11. If $x = 3 + i$, then $x^3 - 3x^2 - 8x + 15 =$

a) 6 b) 10
c) – 18 d) – 15

12. If $z_1 = (4, 5)$ and $z_2 = (-3, 2)$, then $\frac{z_1}{z_2}$ equals

a) $\left(\frac{-23}{12}, \frac{-2}{13}\right)$ b) $\left(\frac{2}{13}, \frac{-23}{13}\right)$
c) $\left(\frac{-2}{13}, \frac{-23}{13}\right)$ d) $\left(\frac{-2}{13}, \frac{23}{13}\right)$

13. If $z = 1 + i$, then the multiplicative inverse of z^2 is (where $i = \sqrt{-1}$)

a) 2i b) 1 – i
c) – i/2 d) i/2

14. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then (x, y) is

a) (3, 1) b) (1, 3)
c) (0, 3) d) (0, 0)

15. If $z_1 = +2i$ and $z_2 = 3 + 5i$, then $\text{Re}\left(\frac{\overline{z}_2 z_1}{z_3}\right)$ is equal to

a) $\frac{-31}{17}$ b) $\frac{17}{22}$
c) $\frac{-17}{31}$ d) $\frac{22}{17}$

16. The complex numbers sin x + i cos 2x and cos x – i sin 2x are conjugate to each other for

a) $x = n\pi$ b) $x = \left(n + \frac{1}{2}\right)\pi$

c) $x = 0$ d) No value of x

17. The real values of x and y for which the equation $(x^4 + 2xi) - (3x^2 + yi) = (3 - 5i) + (1 + 2yi)$ is satisfied, are

a) $x = 2, y = 3$ b) $x = -2, y = \frac{1}{3}$

c) both a) and b) d) none of these

18. For a positive integer n, the expression

$(1-i)^n \left(1 - \frac{1}{i}\right)^n$ equals

a) 0 b) $2i^n$

c) 2^n d) 4^n

19. $\frac{1-2i}{2+i} + \frac{4-i}{3+2i} =$

a) $\frac{24}{13} + \frac{10}{13}i$ b) $\frac{24}{13} - \frac{10}{13}i$

c) $\frac{10}{13} + \frac{24}{13}i$ d) $\frac{10}{13} - \frac{24}{13}i$

20. $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right) =$

a) $\frac{1}{2} + \frac{9}{2}i$ b) $\frac{1}{2} - \frac{9}{2}i$

c) $\frac{1}{4} - \frac{9}{4}i$ d) $\frac{1}{4} + \frac{9}{4}i$

21. If α is a real number such that $z - i\alpha$ is real and $z = \frac{11-3i}{1+i}$, then the value of α is

a) 4 b) – 4

c) – 7 d) 7

22. If $\frac{(1+i)^3}{(1-i)^3} - \frac{(1-i)^3}{(1+i)^3} = x + iy$, then

a) $x = 0, y = -2$ b) $x = -2, y = 0$

c) $x = 1, y = 1$ d) $x = -1, y = 1$

23. For the real parameter t, the locus of the complex number $z = (1 - t^2) + i\sqrt{1+t^2}$ in the complex plane is

a) an ellipse b) a parabola

c) a circle d) a hyperbola

24. If the imaginary part of $\frac{2+i}{ai-1}$ is zero, where a is a real number, then the value of a is equal to

a) 2 b) 2

c) $-\frac{1}{2}$ d) – 2

25. Let z be a complex number such that the imaginary part of z is non zero and $a = z^2 + z + 1$ is real. Then a cannot take the value

a) – 1 b) $\frac{1}{3}$

c) $\frac{1}{2}$ d) $\frac{3}{4}$

26. If $z = \frac{4}{1-i}$, then $\bar{z}$ is (where $\bar{z}$ is complex conjugate of z)

a) $2(1+i)$ b) $1+i$

c) $\frac{2}{1-i}$ d) $\frac{4}{1+i}$

27. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

a) either on the real axis or on a circle passing through the origin

b) on a circle with centre at the origin

c) either on the real axis or on a circle not passing through the origin

d) on the imaginary axis ;

<u>14.2 Modulus, Argument, Power and Square root of a complex number</u>

28. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then the least integral value of m is

a) 2 b) 4

c) 8 d) – 4

29. If $(1-i)^n = 2^n$, then n =

a) 1 b) 0
c) – 1 d) 4

30. $1 + i2 + i4 + i6 + ... + i^{2n}$ is

a) Positive b) Negative
c) Zero d) Cannot be determined

31. $i^2 + i^4 + i^6 + ...$ upto(2n + 1) terms =

a) i b) – i
c) 1 d) – 1

32. If $i^2 = -1$, then the value of $\sum_{n=1}^{200} i^n$ is

a) 50 b) -50
c) 0 d) 100

33. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals

a) i b) i – 1
c) – i d) 0

34. The value of $(1 + i)^6 + (1 - i)^6$ is

a) 0 b 2^7
c) 2^6 d) i^3

35. If $(\sqrt{8} + i)^{50} = 3^{49}(a+ib)$, then $a^2 + b^2$ is

a) 3 b) 8
c) 9 d) $\sqrt{8}$

36. $\sqrt{-8-6i} =$

a) $1 \pm 3i$ b) $\pm(1-3i)$
c) $\pm(1+3i)$ d) $\pm(3-i)$

37. If $(-7-24i)^{\frac{1}{2}} = x - iy$, then $x^2 + y^2 =$

a) 15 b) 25
c) – 25 d) – 15

38. If $\sqrt{a+ib} = x + iy$, then possible value of $\sqrt{a-ib}$ is (a, b, x, y ∈ R)

a) $x^2 + y^2$ b) $\sqrt{x^2+y^2}$
c) $x + iy$ d) $x - iy$

39. If $z = \dfrac{7-i}{3-4i}$, then $z^{14} =$

a) 2^7 b) $2^7 i$
c) $2^{14} i$ d) $-2^7 i$

40. If $\dfrac{2z_1}{3z_2}$ is a purely imaginary number, then

$\left|\dfrac{z_1 - z_2}{z_1 + z_2}\right| =$

a) $\dfrac{3}{2}$ b) 1
c) $\dfrac{2}{3}$ d) $\dfrac{4}{9}$

41. The values of z for which $|z + i| = |z - i|$ are

a) any real number
b) any complex number
c) any natural number
d) any integer

42. Modulus of $\left(\dfrac{3+2i}{3-2i}\right)$ is

a) 1 b) $\dfrac{1}{2}$
c) 2 d) $\sqrt{2}$

43. If z is a complex number such that $\dfrac{z-1}{z+1}$ is purely imaginary, then

a) $|z| = 0$ b) $|z| = 1$
c) $|z| > 1$ d) $|z| < 1$

44. If $\left(\dfrac{1-i}{1+i}\right)^{100} = a + ib$, then

a) a = 2, b = – 1
b) a = 1, b = 0
c) a = 0, b = 1
d) a = – 1, b = 2

45. If z_1 and z_2 are any two complex numbers, then $|z_1 + z_2|^2 + |z_1 - z_2|$ f is equal to

a) $2|z_1|^2 |z_2|^2$ b) $2|z_1|^2 + 2|z_2|^2$
c) $|z_1|^2 - |z_2|^2$ d) $2|z_1| |z_2|$

46. If z is a complex number, then which of the following is not true ?

a) $|z^2| = |z|^2$ b) $|z^2| = |\bar{z}|^2$
c) $z = \bar{z}$ d) $z = \bar{\bar{z}}$

47. If $|z| = 1, (z \neq -1)$ and $z = x + iy$ then $\left(\frac{z-1}{z+1}\right)$ is

a) Purely real　　b) Purely imaginary

c) Zero　　d) Undefined

48. A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta(\alpha, \beta \text{ real})$, if

a) $\alpha^2 - \beta^2 = -1$　　b) $\alpha^2 - \beta^2 = 1$

c) $\alpha^2 + \beta^2 = 1$　　d) $\alpha^2 - \beta^2 = 2$

49. If $|z_1 + z_2| = |z_1 - z_2|$, then the difference in the amplitudes of z_1 and z_2 is

a) $\frac{\pi}{4}$　　b) $\frac{\pi}{3}$

c) $\frac{\pi}{2}$　　d) 0

50. The argument of the complex number $\frac{13-5i}{4-9i}$ is

a) $\frac{\pi}{3}$　　b) $\frac{\pi}{4}$

c) $\frac{\pi}{5}$　　d) $\frac{\pi}{6}$

51. If $z = \frac{-2}{1+\sqrt{3}\,i}$, then the value of arg (z) is

a) π　　b) $\pi/3$

c) $2\pi/3$　　d) $\pi/4$

52. $(1 + i)^{10}$, where $i^2 = -1$, is equal to

a) 32i　　b) 64 + i

c) 24i – 32　　d) 24i

53. $\left|(1+i)\frac{(2+i)}{(3+i)}\right| =$

a) $-\frac{1}{2}$　　b) $\frac{1}{2}$

c) 1　　d) – 1

54. The modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$ is

a) $\sqrt{5}$ units　　b) $\frac{\sqrt{11}}{5}$ units

c) $\frac{\sqrt{5}}{5}$ units　　d) $\frac{\sqrt{12}}{5}$ units

55. The smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$ is

a) 1　　b) 2

c) 3　　d) 4

56. If z is a complex number, then the minimum value of $|z| + z - 1$ is

a) 1　　b) 0

c) $\frac{1}{2}$　　d) none of these

57. If z is any complex number such that $|z + 4| \leq 3$, then the greatest value of $|z + 1|$ is

a) 6　　b) 4

c) 5　　d) 3

58. For any two complex numbers z_1, z_2 and any real numbers a and b;

$|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|2 =$

a) $(a^2 + b^2)(|z_1| + |z_2|)$

b) $(a^2 + b^2)(|z_1|^2 + |z_2|^2)$

c) real and negative

d) none of thes

59. If z_1, z_2, z_3 are complex numbers such that

$$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1,$$

then $|z_1 + z_2 + z_3|$ is

a) equal to 1　　b) less than 1

c) greater than 3　　d) equal to 3

60. The complex number z satisfying the equations

$$\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1 \text{ are}$$

a) 6　　b) $6 \pm 8i$

c) 6 + 8i, 6 + 17 i　　d) – 6

61. If z_1 and z_2 are any two complex numbers, then

$$\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right| \text{ is equal to}$$

a) $|z_1|$　　b) $|z_2|$

c) $|z_1 + z_2|$　　d) $|z_1 + z_2| + |z_1 - z_2|$

62. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

a) 0　　b) 2

c) 7　　d) 17

63. If |z| = max {|z – 2|, |z + 2|}, then

a) $|z+\bar{z}|=1$ b) $z+\bar{z}=2^2$

c) $|z+\bar{z}|=2$ d) none of these

64. If z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{(z_1+z_2)}{(z_1-z_2)}$ may be

a) purely imaginary b) real and positive

c) real and negative d) none of these

65. Let $z_1 = 3 + 4i$ and $z_2 = -1 + 2i$. Then $|z_1 + z_2|^2 - 2(|z_1|^2 + |z_2|^2)$ is equal to

a) $|z_1 - z_2|^2$ b) $-|z_1 - z_2|^2$

c) $|z_1|^2 + |z_2|^2$ d) $|z_1|^2 - z_2|^2$

66. The value of $\left|\frac{1+i\sqrt{3}}{\left(1+\frac{1}{i+1}\right)}\right|$ is

a) 20 b) 9

c) $\frac{5}{4}$ d) $\frac{4}{5}$

67. If $\left(\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)^{50}=3^{25}(x+iy)$, then $x^2+y^2=$

a) – 1 b) 1

c) 0 d) None of these

68. The value of $z|^2 + |z - 3|^2 + |z - i|^2$ is minimum when z equals

a) $2-\frac{2}{3}i$ b) $45+3i$

c) $1+\frac{i}{3}$ d) $1-\frac{i}{3}$

14.3 DeMoivre's theorem, Argand diagram and Polar form of a complex number

69. The complex number $\frac{1+2i}{1-i}$ lies in which quadrant of the complex plane ?

a) First b) Second

c) Third d) Fourth

70. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg(z_1) - \arg(z_2)$ is equal to

a) $-\pi$ b) $-\frac{\pi}{2}$

c) $\frac{\pi}{2}$ d) 0

71. If |z| = 4 and arg $z=\frac{5\pi}{6}$, then $z=$

a) $2\sqrt{3}-2i$ b b) $2\sqrt{3}+2i$

c) $-2\sqrt{3}+2i$ d) $-\sqrt{3}+i$

72. If arg(z) = θ, then arg $(\bar{z})=$

a) θ b) – θ

c) π – θ d) θ – π

73. If $z=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$, then

a) $|z|=1, \arg z=\frac{\pi}{4}$

b) $|z|=1, \arg z=\frac{\pi}{6}$

c) $|z|=\frac{\sqrt{3}}{2}, \arg z=\frac{5\pi}{24}$

d) $|z|=\frac{\sqrt{3}}{2}, \arg z=\tan^{-1}\frac{1}{\sqrt{2}}$

74. If $-1+\sqrt{-3}=re^{i\theta}$, then θ is equal to

a) $\frac{\pi}{3}$ b) $-\frac{\pi}{3}$

c) $\frac{2\pi}{3}$ d) $-\frac{2\pi}{3}$

75. If y = cos θ + i sin θ ,then the value of $y+\frac{1}{y}$ is

a) 2 cos θ b) 2 sin θ

c) 2 cosec θ d) 2 tan θ

76. $(-1+i\sqrt{3})^{20}$ is equal to

a) $2^{20}(-1+i\sqrt{3})^{20}$ b) $2^{20}(1-i\sqrt{3})^{20}$

c) $2^{20}(-1-i\sqrt{3})^{20}$ d) None of these

77. $\dfrac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$ is equal to

a) $\cos\theta - i\sin\theta$ b) $\cos 9\theta - i\sin 9\theta$
c) $\sin\theta - i\cos\theta$ d) $\sin 9\theta - i\cos 9\theta$

78. If $z = \left(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2}\right)^5 + \left(\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\right)^5$, then

a) $Re(z) = 0$
b) $Im(z) = 0$
c) $Re(z) > 0, Im(z) > 0$
d) $Re(z) > 0, Im(z) < 0$

79. $(-\sqrt{3} + i)^{53}$, where $i^2 = -1$, is equal to

a) $2^{53}(\sqrt{3} + 2i)$ b) $2^{52}(\sqrt{3} - i)$
c) $2^{53}\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)$ d) $2^{53}(\sqrt{3} - i)$

80. If $iz^4 + 1 = 0$, then z can take the value

a) $\dfrac{1+i}{\sqrt{2}}$ b) $\cos\dfrac{\pi}{8} + i\sin\dfrac{\pi}{8}$
c) $\dfrac{1}{4i}$ d) i

81. If $(1 + i\sqrt{3})^9 = a + ib$, then b is equal to

a) 1 b) 256
c) 0 d) 2^9

82. The value of $i^{1/3}$ is

a) $\dfrac{\sqrt{3} + i}{2}$ b) $\dfrac{\sqrt{3} - i}{2}$
c) $\dfrac{1 + i\sqrt{3}}{2}$ d) $\dfrac{1 - i\sqrt{3}}{2}$

83. The amplitude of 0 is

a) 0 b) $\pi/2$
c) π d) Not defined

84. $\dfrac{1+7i}{(2-i)^2} =$

a) $\sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$
b) $\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$
c) $\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$
d) $\left(\cos\dfrac{\pi}{4} - i\sin\dfrac{\pi}{4}\right)$

85. Real part of $e^{e^{i\theta}}$ is

a) $e^{\cos\theta}[\cos(\sin\theta)]$
b) $e^{\cos\theta}[\cos(\cos\theta)]$
c) $e^{\sin\theta}[\sin(\cos\theta)]$
d) $e^{\sin\theta}[\sin(\sin\theta)]$

86. If $z = \dfrac{1 + i\sqrt{3}}{\sqrt{3} + i}$, then $(\bar{z})^{100}$ lies in

a) I[st] quadrant b) II[nd] quadrant
c) III[rd] quadrant d) IV[th] quadrant

87. If $x_r = \cos\left(\dfrac{\pi}{2}\right) + i\sin\left(\dfrac{\pi}{2}\right)$, then $x_1 x_2 .....\infty$ is

a) – 3 b) – 2
c) – 1 d) 0

88. If $\dfrac{1}{x} + x = 2\cos\theta$, then $x^n + \dfrac{1}{x^n}$ is equal to

a) $2\cos n\theta$ b) $2\sin n\theta$
c) $\cos n\theta$ d) $\sin n\theta$

89. If $z = \dfrac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$, then arg(z) =

a) 60^0 b) 120^0
c) 240^0 d) 300^0

90. If z_1, z_2, z_3 are three collinear points in argand plane, then $\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$

a) 0 b) – 1
c) 1 d) 2

91. If arg z < 0, then arg(– z) – arg(z) is equal to

a) π b) $-\pi$
c) $-\dfrac{\pi}{2}$ d) $\dfrac{\pi}{2}$

92. If z is a complex number in the argand plane, then the equation $|z-2|+|z+2|=8$ represents

a) parabola b) ellipse

c) hyperbola d) circle

93. If z = x + iy, then area of the triangle whose vertices are z, iz and z + iz is

a) $2|z|^2$ b) $\frac{1}{2}|z|^2$

c) $|z|^2$ d) $\frac{3}{2}|z|^2$

94. If $z = x + iy$ and $|z-2+i| = |z-3-i|$, then locus of z is

a) $2x + 4y - 5 = 0$ b) $2x - 4y - 5 = 0$

c) $x + 2y = 0$ d) $x - 2y + 5 = 0$

95. Let z, ω be complex numbers such that $\bar{z} + i\bar{\omega} = 0$ and $\arg(z\omega) = \pi$. Then arg(z) equals

a) $\frac{3\pi}{4}$ b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$ d) $\frac{5\pi}{4}$

96. If $|z-1| = |z|^2 + 1$, then z lies on

a) an ellipse b) the imaginary axis

c) a circle d) the real axis

97. If $z = x + iy$ and $\omega = \frac{1-iz}{2-i}$, then $|\omega| = 1$ shows that in complex plane

a) z will be at imaginary axis

b) z will be at real axis

c) z will be at unity circle

d) none of these

98. If the amplitude of $z - 2 - 3i$ is $\frac{\pi}{4}$, then the locus of $z = x + yi$ is

a) $x + y - 1 = 0$ b) $x - y - 1 = 0$

c) $x + y + 1 = 0$ d) $x - y + 1 = 0$

99. If z and ω are two non-zero complex numbers such that $|z| = |\omega|$ and $\text{Arg}(z) + \text{Arg}(\omega) = \pi$, then z =

a) $\bar{\omega}$ b) $-\bar{\omega}$

c) ω d) $-\omega$

100. If $z = re^{i\theta}$, then $|e^{iz}| =$

a) $e^{r\sin\theta}$ b) $e^{-r\sin\theta}$

c) $e^{-r\cos\theta}$ d) $e^{r\cos\theta}$

101. If $\left(\frac{1+\cos\theta+i\sin\theta}{i+\sin\theta+i\cos\theta}\right)^4 = \cos n\theta + i\sin n\theta$, then n is equal to

a) 1 b) 2

c) 3 d) 4

102. If $z = (1+i\sqrt{3})$, then $\frac{\text{Re}(z)}{\text{Im}(z)}$ equals

a) 2^{100} b) 2^{50}

c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{3}$

103. The argument of the complex number $\left(\frac{i}{2}-\frac{2}{i}\right)$ is equal to

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$

c) $\frac{\pi}{12}$ d) $\frac{\pi}{2}$

104. If $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x-iy)$, where x, y are real and $i = \sqrt{-1}$, then the order pair (x, y) is given by

a) (0, 3) b) $\left(\frac{1}{2}, \sqrt{3}\right)$

c) (– 3, 0) d) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

105. The modulus of the complex number z such that $|z+3-i| = 1$ and arg z = n is equal to

a) 1 b) 2

c) 9 d) 3

106. If $z = r(\cos\theta + i\sin\theta)$, then the value of $\frac{z}{\bar{z}} + \frac{\bar{z}}{z}$ is

a) cos 2θ b) 2 cos 2θ

c) 2 cos θ d) 2 sin θ

107. If $z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ and

$z_2 = \sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, then $|z_1 z_2|$ is

a) 6 b) $\sqrt{2}$

c) $\sqrt{6}$ d) $\sqrt{3}$

108. Complex number $z = \dfrac{i-1}{\cos(\pi/3) + i\sin(\pi/3)}$ in polar form is :

a) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

b) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

c) $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

d) None of these

109. Let z = cos θ + i sin θ. Then, the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2^0$ is

a) $\dfrac{1}{\sin 2^0}$ b) $\dfrac{1}{3\sin 2^0}$

c) $\dfrac{1}{2\sin 2^0}$ d) $\dfrac{1}{4\sin 2^0}$

110. The value of $\dfrac{\cos 30^0 + i\sin 30^0}{\cos 60^0 - i\sin 60^0}$ is equal to

a) i b) – i

c) $\dfrac{1+\sqrt{3}i}{2}$ d) $\dfrac{1-\sqrt{3}i}{2}$

111. Suppose that z_1, z_2, z_3 are three vertices of an equilateral triangle in the Argand plane. Let $\alpha = \frac{1}{2}(\sqrt{3} + i)$ and β be a non-zero complex number. The points $\alpha z_1 + \beta, \alpha z_2 + \beta, \alpha z_3 + \beta$ will be

a) the vertices of an equilateral triangle

b) the vertices of an isosceles triangle

c) collinear

d) the vertices of a scalene triangle

112. Let z_1 be a fixed point on the circle of radius 1 centered at the origin in the Argand plane and $z_1 \neq \pm 1$. Consider an equilateral triangle inscribed in the circle with z_1 z_2, z_3 as the vertices taken in the counter clockwise direction. Then $z_1 z_2 z_3$ is equal to

a) z_1^2 b) z_1^3

c) z_1^4 d) z_1

14.4 Fundamental theorem of algebra, cube roots of unity

113. If $x + \frac{1}{x} = \sqrt{3}$, then x =

a) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ b) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

c) $\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}$ d) $\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

114. The two numbers such that each one is square of the other are

a) ω, ω^3 b) – i, i

c) – 1, 1 d) ω, ω^2

115. The roots of the equation $x^4 - 1 - 0$ are

a) 1, 1, i, – 1 b) 1, – 1, i, – i

c) $1, -1, \omega, \omega^2$ d) None of these

116. If a + b + c = 0 and 1, ω, ω^2 are three cube roots of unity, then $(a + b\omega + \omega^2)^3 + (a + b\omega^2 + \omega)^3$ is equal to

a) 27 abc b) – 3 abc

c) 3 abc d) – 27 abc

117. If a is an imaginary cube root of unity, then for n ∈ N, the value of $a^{3n+1} + a^{3n+3} + a^{3n+5}$ is

a) – 1 b) 0

c) 1 d) 3

118. If α and π are imaginary cube roots of unity, then the value of $\alpha^4 + \beta^{28} + \dfrac{1}{\alpha\beta}$ is

a) 1 b) – 1

c) 0 d) 2

119. If ω is an imaginary cube root of unity, $(1 + \omega - \omega)^2)^7$ equals

a) 128ω b) – 128ω

c) $128\omega^2$ d) $-128\omega^2$

120. If cube root of 1 is ω, then the value of $(3 + \omega + 3\omega^2)^4$ is

a) 0 b) 16

c) 16ω d) $16\omega^2$

121. The value of $(1 - \omega + \omega^2)(1 - \omega^2 + \omega)^6$, where ω, ω^2 are cube roots of unity, is

a) $128\,\omega$ b) $-128\omega^2$

c) $-128\,\omega$ d) $128\omega^2$

122. If $1, \omega, \omega^2$ are the cube roots of unity, then their product is

a) 0 b) 0)

c) -1 d) 1

123. If $z + z^{-1} = 1$, then $z^{100} + z^{-100}$ is equal to

a) i b) $-i$

c) 1 d) -1

124. The value of

$(1 + 2\omega + \omega^2)^{3n} - (1 + \omega + 2\omega^2)^{3n} =$

a) 0 b) 1

c) ω d) ω^2

125. If ω is a non real cube root of unity, then $(a + b)(a + b\omega)(a + b\omega^2)$ is

a) $a^3 + b^3$ b) $a^3 - b^3$

c) $a^2 + b^2$ d) $a^2 - b^2$

126. The value of $(8)^{1/3}$ is

a) $-1+i\sqrt{3}$ b) $-1-i\sqrt{3}$

c) 2 d) All of these

127. If $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} =$$

a) 0 b) 1

c) ω d) ω^2

128. If $\omega = \dfrac{-1+\sqrt{3}i}{2}$, then $(3 + \omega + 3\omega^2)^4 =$

a) 16 b) -16

c) $16\,\omega$ d) $16\,\omega^2$

129. If $x + \dfrac{1}{x} = 2\cos\theta$, then x is equal to

a) $\cos\theta + i\sin\theta$ b) $\sin\theta - i\cos\theta$

c) $\cos\theta + i\sin\theta$ d) $\sin\theta + i\cos\theta$

130. If ω is an imaginary cube root of unity, then the value of $\sin\left[(\omega^{10} + \omega^{23})\pi - \dfrac{\pi}{4}\right]$ is

a) $-\sqrt{3}/2$ b) $-1/\sqrt{2}$

c) $1/\sqrt{2}$ d) $\sqrt{3}/2$

131. $\left(\dfrac{\sqrt{3}+i}{2}\right)^6 + \left(\dfrac{1-\sqrt{3}}{2}\right)^6$ is equal to

a) -2 b) 0

c) 2 d) 1

132. If ω is a complex root of the equation $z^3 = 1$, then $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\ldots\ldots\right)}$ is equal to

a) -1 b) 0

c) 9 d) i

133. If n is a positive integer not a multiple of 3, then $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$ ...... to 2n factors =

a) 0 b) 1

c) -1 d) 2

134. If ω is a complex cube root of unity, then $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$ .... to 2n factors =

a) 0 b) 1

c) -1 d) 2

135. If α, β, γ are the cube roots of p $(p < 0)$, then for any x, y and z, $\dfrac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha} =$

a) $\dfrac{1}{2}(-1-i\sqrt{3})$ b) $\dfrac{1}{2}(1+i\sqrt{3})$

c) $\dfrac{1}{2}(1-i\sqrt{3})$ d) $\dfrac{1}{2}(-1|i\sqrt{3})$

136. $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)$.... to 2n factors is

a) 2^n b) 2^{2n}

c) 0 d) 1

137. If $\left(\dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n$ is an integer, then n is

a) 1 b) 2

c) 3 d) 4

138. If $(1 + \omega^2)^m = (1 + \omega^4)^m$ and ω is an imaginary cube root of unity, then least positive integral value of m is

a) 6 b) 5

c) 4 d) 3

139. If the cube roots of unity are 1, ω, ω^2, then the roots of the equation $(x - 2)^3 + 27 = 0$ are

a) $-1, -1, -1$

b) $-1, -\omega, -\omega^2$

c) $-1, 2 + 3\omega, 2 + 3\omega)^2$

d) $-1, 2 - 3\omega, 2 - 3\omega^2$

140. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is

a) 2 b) -2

c) 1 d) 0

141. If $2x = -1 + \sqrt{3}\,i$, then the value of $(1 - x^2 + x)^6 - (1 - x + x^2)^6$ is

a) 32 b) -64

c) 64 d) 0

142. Let ω be the imaginary root of $x^n = 1$, then $(5 - \omega)(5 - \omega^2)....(5 - \omega^{n-1})$ is equal to

a) 1 b) $\frac{5^n + 1}{4}$

c) 4^{n-1} d) $\frac{5^n - 1}{4}$

143. If ω is a complex cube root of unity, then for positive integral values of n, the product of $\omega.\omega^2.\omega^3....\omega^n$, will be

a) $\frac{1-i\sqrt{3}}{2}$ b) $-\frac{1-i\sqrt{3}}{2}$

c) 1 d) both (b) and (c)

144. $\frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}} + \frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}$ is equal to

a) -64 b) -32

c) -16 d) $\frac{1}{16}$

145. If $\omega_n = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$, $i^2 = -1$, then $(x + y\omega_3 + z\omega_3^2)(x + y\omega_3^2 + z\omega_3)$ is equal to

a) 0

b) $x^2 + y^2 + z^2$

c) $x^2 + y^2 + z^2 - yz - zx - xy$

d) $x^2 + y^2 + z^2 + yz + zx + xy$

146. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^2+\frac{1}{z^3}\right)^2 + ... + \left(z^6+\frac{1}{z^6}\right)^2$$

is

a) 18 b) 54

c) 6 d) 12

147. If $\omega\ (\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A and B are respectively, the numbers

a) 0, 1 b) 1, 0

c) 1, 1 d) -1, 1

148. $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{1000} =$

a) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ b) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$

c) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ d) None of these

149. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

a) -2 b) -1

c) 1 d) 2

150. 1, ω and ω^2 are the cube roots of unity, then $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)$.... upto 8 terms is

a) 2^6 b) 2^{10}

c) 2^7 d) 2^8

151. In the Argarid plane, the distinct roots of $1 + z + z^3 + z^4 = 0$ (z is a complex number) represent vertices of

a) a square

b) an equilateral triangle

c) a rhombus

d) a rectangle

152. Let α, β denote the cube roots of unity other than 1 and $\alpha \neq \beta$. Let $S, = \sum_{n=0}^{302}(-1)^n\left(\frac{\alpha}{\beta}\right)^n$.

Then, the value of S is

a) either -2ω or $-2\omega^2$

b) either -2ω or $2\omega^2$

c) either 2ω or $-2\omega^2$

d) either 2ω or $2\omega^2$

Evaluation Test

1. If $\frac{|z-2|}{|z-3|}=2$ represents a circle, then its radius is equal to

a) 1 b) $\frac{1}{3}$

c) $\frac{3}{4}$ d) $\frac{2}{3}$

2. The real part of $(1-i)^{-i}$ is

a) $e^{\frac{\pi}{4}}\cos\left(\frac{1}{2}\log 2\right)$ b) $-e^{\frac{\pi}{4}}\sin\left(\frac{1}{2}\log 2\right)$

c) $e^{\frac{\pi}{4}}\cos\left(\frac{1}{2}\log 2\right)$ d) $e^{\frac{\pi}{4}}\sin\left(\frac{1}{2}\log 2\right)$

3. If $z=i\log(2-\sqrt{3})$, then $\cos z=$

a) i b) 2i

c) 1 d) 2

4. If $\omega\neq 1$ is a cube root of unity, then the sum of the series $S = 1 + 2\omega + 3\omega^2 + ... + 3n\omega^{3n-1}$ is

a) $\frac{3n}{\omega-1}$ b) $3n(\omega-1)$

c) $\frac{\omega-1}{3n}$ d) 0

5. If $z = 3 - 4i$, then $z^4 - 3z^3 + 3z^2 + 99z - 95$ is equal to

a) 5 b) 6

c) – 5 d) – 4

6. If for the complex numbers z_1 and z_2

$|1-\bar{z}_1 z_2|^2 - |z_1 - z_2|^2$

$= k(1-|z_1|^2)(1-|z_2|^2)$, then k is equal to

a) 1 b) – 1

c) 2 d) 4

7. If a and b are real numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then

a) $a=b=2+\sqrt{3}$

b) $a=b=2-\sqrt{3}$

c) $a=2-\sqrt{3}, b=2+\sqrt{3}$

d) $a=2+\sqrt{3}, b=2-\sqrt{3}$

8. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta).....(\cos n\theta + i\sin n\theta) = 1$, then the value of θ is

a) $4m\pi$ b) $\frac{2m\pi}{n(n+1)}$

c) $\frac{4m\pi}{n(n+1)}$ d) $\frac{m\pi}{n(n+1)}$

9. If $z_1 = a+ib$ and $z_2 = c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $\mathrm{Re}(z_1\overline{z_2})=0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies

a) $|w_1| = 1$ b) $|w_2| = 1$

c) $\mathrm{Re}(w_1\overline{w_2})=0$ d) All the above

10. If $i=\sqrt{-1}$, then

$4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

a) $1-i\sqrt{3}$ b) $-1+i\sqrt{3}$

c) $i\sqrt{3}$ d) $-i\sqrt{3}$

❍❍❍❍

Classical Thinking

1. (A)	2. (D)	3. (B)	4. (A)	5. (A)	6. (D)	7. (B)	8. (C)	9. (B)	10. (A)
11. (A)	12. (A)	13. (C)	14. (A)	15. (C)	16. (B)	17. (C)	18. (A)	19. (C)	20. (D)
21. (D)	22. (D)	23. (B)	24. (D)	25. (B)	26. (B)	27. (A)	28. (A)	29. (B)	30. (D)
31. (A)	32. (A)	33. (B)	34. (A)	35. (B)	36. (A)	37. (C)	38. (D)	39. (B)	40. (C)
41. (A)	42. (C)	43. (D)	44. (B)	45. (B)	46. (C)	47. (D)	48. (B)		

Critical Thinking

1. (A)	2. (B)	3. (D)	4. (C)	5. (B)	6. (B)	7. (A)	8. (B)	9. (B)	10. (B)
11. (A)	12. (B)	13. (D)	14. (B)	15. (C)	16. (D)	17. (A)	18. (B)	19. (B)	20. (C)
21. (B)	22. (A)	23. (D)	24. (B)	25. (B)	26. (C)	27. (A)	28. (D)	29. (B)	30. (D)
31. (C)	32. (B)	33. (A)	34. (B)	35. (D)	36. (D)	37. (A)	38. (B)	39. (C)	40. (C)
41. (B)	42. (D)	43. (C)	44. (D)	45. (C)	46. (A)	47. (B)	48. (A)	49. (B)	50. (C)
51. (C)	52. (B)	53. (B)	54. (C)	55. (C)	56. (D)	57. (C)	58. (B)	59. (A)	60. (B)
61. (B)	62. (A)	63. (B)	64. (D)	65. (A)	66. (B)	67. (D)	68. (C)	69. (C)	70. (B)
71. (D)	72. (C)	73. (A)	74. (A)	75. (C)	76. (D)	77. (D)	78. (A)	79. (A)	

Competitive Thinking

1. (A)	2. (D)	3. (B)	4. (A)	5. (A)	6. (A)	7. (C)	8. (C)	9. (C)	10. (D)
11. (D)	12. (C)	13. (C)	14. (D)	15. (D)	16. (D)	17. (C)	18. (C)	19. (D)	20. (D)
21. (C)	22. (A)	23. (D)	24. (C)	25. (D)	26. (D)	27. (A)	28. (B)	29. (B)	30. (D)
31. (D)	32. (C)	33. (B)	34. (A)	35. (C)	36. (B)	37. (B)	38. (D)	39. (D)	40. (B)
41. (A)	42. (A)	43. (B)	44. (B)	45. (B)	46. (C)	47. (B)	48. (C)	49. (C)	50. (B)
51. (C)	52. (A)	53. (C)	54. (C)	55. (B)	56. (A)	57. (A)	58. (B)	59. (A)	60. (C)
61. (D)	62. (B)	63. (C)	64. (A)	65. (B)	66. (D)	67. (B)	68. (C)	69. (B)	70. (D)
71. (C)	72. (B)	73. (B)	74. (C)	75. (A)	76. (D)	77. (D)	78. (B)	79. (C)	80. (B)
81. (C)	82. (A)	83. (D)	84. (A)	85. (A)	86. (C)	87. (C)	88. (A)	89. (C)	90. (A)
91. (A)	92. (B)	93. (B)	94. (A)	95. (A)	96. (B)	97. (B)	98. (D)	99. (B)	100. (B)
101. (D)	102. (C)	103. (D)	104. (D)	105. (D)	106. (B)	107. (C)	108. (A)	109. (D)	110. (A)
111. (A)	112. (B)	113. (D)	114. (D)	115. (B)	116. (A)	117. (B)	118. (C)	119. (D)	120. (C)
121. (C)	122. (D)	123. (D)	124. (A)	125. (A)	126. (D)	127. (A)	128. (C)	129. (C)	130. (C)
131. (A)	132. (A)	133. (C)	134. (B)	135. (A)	136. (B)	137. (C)	138. (D)	139. (D)	140. (A)
141. (D)	142. (D)	143. (D)	144. (A)	145. (C)	146. (D)	147. (C)	148. (C)	149. (C)	150. (A)
151. (B)	152. (A)								

Answers to Evaluation Test

1. (D)	2. (A)	3. (D)	4.(A)	5. (A)	6. (A)	7. (B)	8.(C)	9. (D)	10.(C)

Classical Thinking

1. (A)	2. (D)	3. (B)	4. (A)	5. (A)	6. (D)	7. (B)	8. (C)	9. (B)	10. (A)
11. (A)	12. (A)	13. (C)	14. (A)	15. (C)	16. (B)	17. (C)	18. (A)	19. (C)	20. (D)
21. (D)	22. (D)	23. (B)	24. (D)	25. (B)	26. (B)	27. (A)	28. (A)	29. (B)	30. (D)
31. (A)	32. (A)	33. (B)	34. (A)	35. (B)	36. (A)	37. (C)	38. (D)	39. (B)	40. (C)
41. (A)	42. (C)	43. (D)	44. (B)	45. (B)	46. (C)	47. (D)	48. (B)		

Critical Thinking

1. (A)	2. (B)	3. (D)	4. (C)	5. (B)	6. (B)	7. (A)	8. (B)	9. (B)	10. (B)
11. (A)	12. (B)	13. (D)	14. (B)	15. (C)	16. (D)	17. (A)	18. (B)	19. (B)	20. (C)
21. (B)	22. (A)	23. (D)	24. (B)	25. (B)	26. (C)	27. (A)	28. (D)	29. (B)	30. (D)
31. (C)	32. (B)	33. (A)	34. (B)	35. (D)	36. (D)	37. (A)	38. (B)	39. (C)	40. (C)
41. (B)	42. (D)	43. (C)	44. (D)	45. (C)	46. (A)	47. (B)	48. (A)	49. (B)	50. (C)
51. (C)	52. (B)	53. (B)	54. (C)	55. (C)	56. (D)	57. (C)	58. (B)	59. (A)	60. (B)
61. (B)	62. (A)	63. (B)	64. (D)	65. (A)	66. (B)	67. (D)	68. (C)	69. (C)	70. (B)
71. (D)	72. (C)	73. (A)	74. (A)	75. (C)	76. (D)	77. (D)	78. (A)	79. (A)	

Competitive Thinking

1. (A)	2. (D)	3. (B)	4. (A)	5. (A)	6. (A)	7. (C)	8. (C)	9. (C)	10. (D)
11. (D)	12. (C)	13. (C)	14. (D)	15. (D)	16. (D)	17. (C)	18. (C)	19. (D)	20. (D)
21. (C)	22. (A)	23. (D)	24. (C)	25. (D)	26. (D)	27. (A)	28. (B)	29. (B)	30. (D)
31. (D)	32. (C)	33. (B)	34. (A)	35. (C)	36. (B)	37. (B)	38. (D)	39. (D)	40. (B)
41. (A)	42. (A)	43. (B)	44. (B)	45. (B)	46. (C)	47. (B)	48. (C)	49. (C)	50. (B)
51. (C)	52. (A)	53. (C)	54. (C)	55. (B)	56. (A)	57. (A)	58. (B)	59. (A)	60. (C)
61. (D)	62. (B)	63. (C)	64. (A)	65. (B)	66. (D)	67. (B)	68. (C)	69. (B)	70. (D)
71. (C)	72. (B)	73. (B)	74. (C)	75. (A)	76. (D)	77. (D)	78. (B)	79. (C)	80. (B)
81. (C)	82. (A)	83. (D)	84. (A)	85. (A)	86. (C)	87. (C)	88. (A)	89. (C)	90. (A)
91. (A)	92. (B)	93. (B)	94. (A)	95. (A)	96. (B)	97. (B)	98. (D)	99. (B)	100. (B)
101. (D)	102. (C)	103. (D)	104. (D)	105. (D)	106. (B)	107. (C)	108. (A)	109. (D)	110. (A)
111. (A)	112. (B)	113. (D)	114. (D)	115. (B)	116. (A)	117. (B)	118. (C)	119. (D)	120. (C)
121. (C)	122. (D)	123. (D)	124. (A)	125. (A)	126. (D)	127. (A)	128. (C)	129. (C)	130. (C)
131. (A)	132. (A)	133. (C)	134. (B)	135. (A)	136. (B)	137. (C)	138. (D)	139. (D)	140. (A)
141. (D)	142. (D)	143. (D)	144. (A)	145. (C)	146. (D)	147. (C)	148. (C)	149. (C)	150. (A)
151. (B)	152. (A)								

Classical Thinking

5. $(1 + 2i)(-2 + i) = -2 + i - 4i + 2i^2$
$= -2 - 3i - 2$
$= -3i - 4$

6. $z_1 + z_2 = 3 + 2i + 2 - 3i = 5 - i$

7. $\dfrac{z_1}{z_2} = \dfrac{3+2i}{2-3i} = \dfrac{3+2i}{2-3i} \times \dfrac{(2+3i)}{(2+3i)} = \dfrac{13i}{13} = 0 + i$

8. $\bar{z}_1 = 1 + 3i$ and $\bar{z}_2 = 2 - i$
$\therefore \bar{z}_1 + \bar{z}_2 = (1 + 3i) + (2 - i)$
$= 3 + 2i$

9. $(x + yi)^{-1} = \dfrac{1}{x+yi} = \dfrac{x-yi}{(x+yi)(x-yi)} = \dfrac{x-yi}{x^2+y^2}$

10. $\dfrac{z-\bar{z}}{2i} = \dfrac{x+yi-(x-yi)}{2i} = y \Rightarrow$ purely real

11. $\bar{z}=\frac{1}{i-1}\Rightarrow z=\left(\overline{\frac{1}{i-1}}\right)=\frac{1}{-i-1}=-\frac{1}{i+1}$

12. $(x+yi)^{1/3}=u+vi$
$\Rightarrow (u+vi)^3=x+yi$
$\Rightarrow u^3-3uv^2+i\,(3u^2v-v^3)=x+y\,i$
$\Rightarrow u^3-3uv^2=x$ and $3u^2v-v^3=y$
$\Rightarrow \frac{x}{u}=u^2-3v^2$ and $\frac{y}{v}=3u^2-v^2$
$\Rightarrow \frac{x}{u}+\frac{y}{v}=4\,(u^2-v^2)$

13. If $z=x+iy$ is the additive inverse of $1-i$, then $(x+iy)+(1-i)=0$
$\Rightarrow x+1=0, y-1=0\Rightarrow x=-1, y=1$
$\therefore$ The additive inverse of $1-i$ is $z=-1+i$.

15. $\frac{5-2i}{3-4i}-\frac{5+2i}{3+4i}=\frac{28i}{25}=0+\left(\frac{28}{25}\right)i$

16. $3x+ix+y-2yi=0-7i$
$\therefore$ $(3x+y)+(x-2y)i=0-7i$
$\therefore$ $3x+y=0$ and $x-2y=-7$
By solving, we get $x=-1$ and $y=3$

17. $2x-ix+y-3iy+2=0$
$\therefore$ $2x+y-(x+3y)\,i=-2$
$\therefore$ $2x+y=-2$ and $x+3y=0$
By solving, we get
$x=\frac{-6}{5}$ and $y=\frac{2}{5}$

18. $x=1-i\sqrt{3}$
$\therefore$ $(x-1)^2=\left(-i\sqrt{3}\right)^2\Rightarrow x^2-2x+4=0$
$\therefore$ $x^3-x^2+2x+4=(x^2-2x+4)(x+1)$
$=(0)\,(x+1)$
$=0$

19. $x+iy=\frac{4+i}{2-3i}=\frac{(4+i)(2+3i)}{13}=\frac{5}{13}+\frac{14}{13}i$
$\Rightarrow x=\frac{5}{13}, y=\frac{14}{13}$

20. $\frac{z_1z_2}{z_1}=\frac{(1-i)(-2+4i)}{1-i}=-2+4i$
$\Rightarrow \mathrm{Im}\left(\frac{z_1z_2}{z_1}\right)=4$

21. $5+i^{22}+i^{36}+i^{56}=5+(i^2)^{11}+(i^2)^{18}+(i^2)^{28}=6$

22. $\frac{(1-i)^3}{1-i^3}=\frac{(1-i)(1-i)^2}{(1-i)(1+i+i^2)}=-2$

23. $i^{4k+3}=i^{4k}\cdot i^3=1.(-i)=-i$

24. $(1+i)^5(1-i)^5=(1-i^2)^5=2^5=32$

25. Since, $i^n+i^{n+1}+i^{n+2}+i^{n+3}=0$
$\therefore$ $\frac{i^6+i^7+i^8+i^9}{-1+i}=0$

26. $|z|=|1+\sqrt{3}\,i|=\sqrt{(1)^2+\left(\sqrt{3}\right)^2}=\sqrt{1+3}=2$

27. $|z|=\sqrt{a^2+b^2}=\sqrt{3+2}=\sqrt{5}$
Let θ be the argument of z.
$\therefore$ $\tan\theta=\left|\frac{b}{a}\right|=\left|\frac{\sqrt{2}}{\sqrt{3}}\right|=\sqrt{\frac{2}{3}}$
$\Rightarrow \theta=\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$

28. $1+i^2+i^3-i^6+i^8=1-1-i+1+1$
$=2-i$

29. $|z|=\sqrt{a^2+b^2}=\sqrt{13}$
Let θ be the argument of z.
$\therefore$ $\tan\theta=\left|\frac{b}{a}\right|=\left|\frac{2}{3}\right|=\frac{2}{3}$
$\Rightarrow \theta=\tan^{-1}\left(\frac{2}{3}\right)$

30. Let $x+iy=\sqrt{-2i}$
$\Rightarrow x^2-y^2+2xyi=-2i$
$\therefore$ $x^2-y^2=0$ and $2xy=-2$
Solving these equations, we get
$x=1, y=-1$ and $x=-1, y=1$
$\therefore$ Square roots are $1-i, -1+i$.

31. $z=x+iy$, then $|z-5|=|x+iy-5|=|x-5+iy|$
$=\sqrt{(x-5)^2+y^2}$

33. Since $-i=0+(-1)i$, it is represented by $(0,-1)$ which lies on negative Y-axis.
$\therefore$ $\mathrm{amp}\,(-i)=-\frac{\pi}{2}$

36. $z=4+4\sqrt{3}\,i$
$|z|=\sqrt{4^2+(4\sqrt{3})^2}=8$
$\therefore$ Also, $a=4$ and $b=4\sqrt{3}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

Polar form of z = $|z|$ $(\cos\theta + i\sin\theta)$

$= 8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

37. $(\sin\theta + i\cos\theta)^n$

$= \left[\cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)\right]^n$

$= \cos n\left(\frac{\pi}{2}-\theta\right) + i\sin n\left(\frac{\pi}{2}-\theta\right)$

38. $\frac{4(\cos 75^\circ + i\sin 75^\circ)}{0.4(\cos 30^\circ + i\sin 30^\circ)}$

$= 10(\cos 75^\circ + i\sin 75^\circ)(\cos 30^\circ - i\sin 30^\circ)$

$= 10(\cos 45^\circ + i\sin 45^\circ) = \frac{10}{\sqrt{2}}(1+i)$

39. $z = \sin\alpha + i(1-\cos\alpha)$

$\Rightarrow \text{amp}(z) = \tan^{-1}\left(\frac{1-\cos\alpha}{\sin\alpha}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\alpha}{2}\right)\right) = \frac{\alpha}{2}$

40. Here, a = 1, b = 1 and c = 1

a = 1, b = 1 and c = 1

$\therefore \quad x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$

42. $(1+\omega^2)^3 = (1+\omega+\omega^2-\omega)^3 = (0-\omega)^3 = -1$

43. $\omega^{99} + \omega^{100} + \omega^{101} = \omega^{99}[1+\omega+\omega^2] = 0$

44. $\frac{1}{\omega} + \frac{1}{\omega^2} = \frac{\omega+\omega^2}{\omega^3} = \frac{-1}{1} = -1$

45. $(1+\omega-2\omega^2)^4 + (4+\omega+4\omega^2)^4$

$= (-3\omega^2)^4 + [4(-\omega)+\omega]^4$

$= 81\omega^8 + (-3\omega)^4$

$= 81(\omega^3)^2.\omega^2 + 81\omega^4$

$= 81\omega^2 + 81\omega$

$= -81$ ....$[\because 1+\omega+\omega^2 = 0]$

46. $(2+5\omega+2\omega^2)^6 = [2(1+\omega^2)+5\omega]^6$

$= [2(-\omega)+5\omega]^6$

$= [-2\omega+5\omega]^6$

$= (3\omega)^6$

$= 3^6.\omega^6$

$= 729$

47. $(1+\omega-\omega^2)^3 - (1-\omega+\omega^2)^3$

$= (-2\omega^2)^3 - (-2\omega)^3$

$= -8\omega^6 + 8\omega^3$

$= -8 + 8 = 0$

48. $(1+\omega^2)^4 = (-\omega)^4 = \omega^3.\omega = \omega$

Critical Thinking

2. $\frac{1+3i}{2+3i} = \frac{(1+3i)(2-3i)}{(2+3i)(2-3i)} = \frac{2+3i+9}{4+9}$

$= \frac{11}{13} + \frac{3}{13}i$

4. $\frac{(1+i)^2}{2-i} = \frac{(2i)(2+i)}{(2-i)(2+i)} = -\frac{2}{5} + i\frac{4}{5}$

5. $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

$\Rightarrow (4+2i)x - 6i - 2 + (9-7i)y + 3i - 1 = 10i$

Equating real and imaginary parts, we get

$4x + 9y - 3 = 0$ and $2x - 7y - 13 = 0$

Solving both equations, we get

$x = 3, y = -1$

6. $x = 1 + 2i$

$\therefore \quad (x-1)^2 = 4i^2$

$\Rightarrow x^2 - 2x + 1 = -4$

$\Rightarrow x^2 - 2x + 5 = 0$

$\therefore \quad x^3 + 2x^2 - 3x + 5 = (x^2-2x+5)(x+4) + (-15)$

$= 0(x+4) - 15$

$= -15$

7. $\left(1+\frac{2}{i}\right)\left(1+\frac{3}{i}\right)(2+i)^{-1}$

$= \frac{(i+2)(i+3)}{i^2(2+i)} = \frac{5i+5}{-(2+i)} \times \frac{2-i}{2-i} = -3-i$

8. $z = (3\sqrt{7}+4i)^2(3\sqrt{7}-4i)^3$

$= \{(3\sqrt{7}+4i)(3\sqrt{7}-4i)\}^2(3\sqrt{7}-4i)$

$= (63+16)^2(3\sqrt{7}-4i)$

$= (79)^2(3\sqrt{7}-4i)$

$\therefore \quad \text{Re}(z) = (79)^2(3\sqrt{7})$

9. $(x+yi)(p+qi)=(x^2+y^2)i$

$\Rightarrow px-qy=0$ and $qx+py=x^2+y^2$

$\Rightarrow px=qy$ and $qx+py=x^2+y^2$

$\Rightarrow q=x,\ p=y$

10. $x=-3+5i$

$\Rightarrow (x+3)^2=25i^2$

$\Rightarrow x^2+6x+34=0$

$\therefore$ $x^3+6x^2+34x+1$

$=x(x^2+6x+34)+1$

$=x(0)+1$

$=1$

11. $(2+i)x-(1+2i)y=3i$

$\Rightarrow 2x-y+(x-2y)i=0+3i$

$\Rightarrow 2x-y=0$ and $x-2y=3$

By solving, we get

$x=-1$ and $y=-2$

12. $(1+3i)a+(i-1)b+5(-i)=0$

$\therefore$ $(a-b)+(3a+b)i=0+5i$

$\therefore$ $a-b=0$ and $3a+b=5$

By solving, we get $a=\dfrac{5}{4}$ and $b=\dfrac{5}{4}$

13. $x=\dfrac{5+i}{1-i}=\dfrac{(5+i)(1+i)}{(1-i)(1+i)}=2+3i$

$\therefore$ $x-2=3i\Rightarrow (x-2)^2=(3i)^2$

$\therefore$ $x^2-4x+4=9i^2$

$\Rightarrow x^2-4x+13=0$

$\therefore$ $x^3-x^2+x+44=(x^2-4x+13)(x+3)+5$

$=0(x+3)+5$

$=5$

14. $\dfrac{2+5i}{4-3i}=\dfrac{(2+5i)(4+3i)}{25}$

$=\dfrac{-7+26i}{25}$

$\therefore$ conjugate of $\left(\dfrac{-7+26i}{25}\right)=\dfrac{-7-26i}{25}$

15. $(3x+4y)+i(-4x+3y)=5+12i$

$\therefore$ $3x+4y=5$ and $-4x+3y=12$

By solving, we get

$x=-\dfrac{33}{25}$ and $y=\dfrac{56}{25}$

$\therefore$ $\sqrt{x^2+y^2}=\dfrac{13}{5}$

16. If z_1 and z_2 are two complex numbers, then

$\text{Re}(z_1z_2)=\text{Re}(z_1)\text{Re}(z_2)-\text{Im}(z_1)\text{Im}(z_2)$

17. $z=x-iy$ and $z^{\frac{1}{3}}=p+iq$

$\Rightarrow z=x-iy$ and $z=(p+iq)^3$

$\Rightarrow x-iy=(p+iq)^3$

$=(p^3-3pq^2)+i(3p^2q-q^3)$

$\Rightarrow x=p^3-3pq^2$ and $y=q^3-3p^2q$

$\Rightarrow \dfrac{x}{p}+\dfrac{y}{q}=(p^2+q^2)-3(p^2+q^2)$

$=-2(p^2+q^2)$

18. $z=x+iy,\ z^{\frac{1}{3}}=a-ib$

$\Rightarrow z=x+iy$ and $z=(a-ib)^3$

$\Rightarrow x+iy=(a-ib)^3$

$=(a^3-3ab^2)+i(b^3-3a^2b)$

$\Rightarrow x=a^3-3ab^2$ and $y=b^3-3a^2b$

$\Rightarrow \dfrac{x}{a}-\dfrac{y}{b}=a^2-3b^2-b^2+3a^2$

$=4(a^2-b^2)$

$\Rightarrow k=4$

19. $x+iy=\dfrac{3}{2+\cos\theta+i\sin\theta}$

$=\dfrac{3(2+\cos\theta-i\sin\theta)}{(2+\cos\theta)^2+\sin^2\theta}$

$=\dfrac{6+3\cos\theta-3i\sin\theta}{4+\cos^2\theta+4\cos\theta+\sin^2\theta}$

$=\left[\dfrac{6+3\cos\theta}{5+4\cos\theta}\right]+i\left[\dfrac{-3\sin\theta}{5+4\cos\theta}\right]$

$\Rightarrow x=\dfrac{3(2+\cos\theta)}{5+4\cos\theta},\ y=\dfrac{-3\sin\theta}{5+4\cos\theta}$

$\therefore$ x^2+y^2

$=\dfrac{9}{(5+4\cos\theta)^2}[4+\cos^2\theta+4\cos\theta+\sin^2\theta]$

$=\dfrac{9}{5+4\cos\theta}$

$=4\left[\dfrac{6+3\cos\theta}{5+4\cos\theta}\right]-3$

$=4x-3$

20. $\dfrac{1+a}{1-a}=\dfrac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta}$

Rationalising the denominator, we get

$\dfrac{1+a}{1-a}=\dfrac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta}\times\dfrac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)+i\sin\theta}$

$=\frac{(1+\cos\theta)(1-\cos\theta)+(1+\cos\theta)i\sin\theta+(1-\cos\theta)i\sin\theta+i^2\sin^2\theta}{(1-\cos\theta)^2-(i\sin\theta)^2}$

$=\frac{1-\cos^2\theta+i\sin\theta+i\sin\theta\cos\theta+i\sin\theta-i\sin\theta\cos\theta-\sin^2\theta}{1+\cos^2\theta-2\cos\theta+\sin^2\theta}$

$=\frac{1-(\cos^2\theta+\sin^2\theta)+2i\sin\theta}{1+(\cos^2\theta+\sin^2\theta)-2\cos\theta}$

$=\frac{2i\sin\theta}{2(1-\cos\theta)}=\frac{i.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=i\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=i\cot\frac{\theta}{2}$

21. $\frac{1}{1-\cos\theta+i\sin\theta}$

$=\frac{1}{(1-\cos\theta)+i\sin\theta}\times\frac{(1-\cos\theta)-i\sin\theta}{(1-\cos\theta)-i\sin\theta}$

$=\frac{(1-\cos\theta)-i\sin\theta}{(1-\cos\theta)^2+\sin^2\theta}$

$=\frac{(1-\cos\theta)-i\sin\theta}{2(1-\cos\theta)}$

$=\frac{(1-\cos\theta)}{2(1-\cos\theta)}-i\frac{\sin\theta}{2(1-\cos\theta)}$

Therefore, its real part $=\frac{1-\cos\theta}{2(1-\cos\theta)}=\frac{1}{2}$

22. Let $z=x+iy$. Then, $\bar{z}=x-iy$ and $z^{-1}=\frac{1}{x+iy}$

$\Rightarrow(\overline{z^{-1}})=\frac{1}{x-iy}\Rightarrow(\overline{z^{-1}})=\frac{x+iy}{x^2+y^2}$

$\therefore\quad(\overline{z^{-1}})\bar{z}=\frac{x+iy}{x^2+y^2}(x-iy)=1$

23. Let $z=x+iy$, so that $\bar{z}=x-iy$, therefore

$z^2+\bar{z}=0\Leftrightarrow(x^2-y^2+x)+i(2xy-y)=0$

Equating real and imaginary parts, we get

$x^2-y^2+x=0$ .....(i)

and $2xy-y=0$

$\Rightarrow y=0$ or $x=\frac{1}{2}$

If $y=0$, then (i) gives $x^2+x=0$

$\Rightarrow x=0$ or $x=-1$

If $x=\frac{1}{2}$, then (i) gives

$y^2=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}\Rightarrow y=\pm\frac{\sqrt{3}}{2}$

Hence, there are four solutions in all.

24. $\frac{z-i}{z+i}=\frac{x+i(y-1)}{x+i(y+1)}\cdot\frac{x-i(y+1)}{x-i(y+1)}$

$=\frac{(x^2+y^2-1)+i(-2x)}{x^2+(y+1)^2}$

Since, $\frac{z-i}{z+i}$ is a purely imaginary number.

$\therefore\quad x^2+y^2-1=0$

$\Rightarrow x^2+y^2=1$

$\Rightarrow z\bar{z}=1$

25. $(1+i)(1+2i)(1+3i)\ldots..(1+ni)=a+ib$ .....(i)

$\Rightarrow(1-i)(1-2i)(1-3i)\ldots.(1-ni)=a-ib$ .....(ii)

Multiplying (i) and (ii), we get

$2.5\ldots.(1+n^2)=a^2+b^2$

26. $\frac{i^{4n+1}-i^{4n-1}}{2}=\frac{i^{4n}.i^1-i^{4n}.i^{-1}}{2}$

$=\frac{i-i^{-1}}{2}=\frac{1}{2}\left(i-\frac{1}{i}\right)$

$=\frac{1}{2}(i+i)$

$=i$

27. Let $x+iy$ be the square root of $-8i$.

$\therefore\quad(x+iy)=\sqrt{-8i}\Rightarrow x^2-y^2+2xyi=-8i$

$\therefore\quad x^2-y^2=0$ and $2xy=-8$

By solving, we get $x=2$, $y=-2$ and $x=-2$, $y=2$

$\therefore\quad$ the square roots are $2-2i$ and $-2+2i$.

28. $\sqrt{a+ib}=\pm\left[\sqrt{\frac{|z|+a}{2}}-i\sqrt{\frac{|z|-a}{2}}\right]$, for $b<0$

$\therefore\quad\sqrt{-48-14i}=\pm\left[\sqrt{\frac{50-48}{2}}-i\sqrt{\frac{50+48}{2}}\right]$

$=\pm(1-7i)$

29. Let $(x+iy)=\sqrt{5-2\sqrt{14}i}$

$\therefore\quad x^2-y^2+2xyi=5-2\sqrt{14}\,i$

$\therefore\quad x^2-y^2=5$ and $2xy=-2\sqrt{14}$

By solving, we get $x=\sqrt{7}$, $y=-\sqrt{2}$ and $x=-\sqrt{7}$, $y=\sqrt{2}$

$\therefore\quad$ the square roots are $\sqrt{7}-\sqrt{2}i$ and $-\sqrt{7}+\sqrt{2}i$.

31. $\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} - i\sqrt{\frac{|z|-a}{2}}\right]$, for $b < 0$

$\therefore \quad \sqrt{-24-18i} = \pm\left[\sqrt{\frac{30-24}{2}} - i\sqrt{\frac{30+24}{2}}\right]$

$= \pm\left(\sqrt{3} - i\sqrt{27}\right)$

$= \pm\sqrt{3}(1-3i)$

32. $\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + i\frac{\sqrt{|z|-a}}{2}\right]$, for $b > 0$

$\therefore \quad \sqrt{\frac{35}{4}+3i} = \pm\left[\sqrt{\frac{\frac{37}{4}+\frac{35}{4}}{2}} + i\sqrt{\frac{\frac{37}{4}-\frac{35}{4}}{2}}\right]$

$= \pm\left(3+\frac{1}{2}i\right)$

33. $(1+i)^{6n} + (1-i)^{6n} = \{(1+i)^2\}^{3n} + \{(1-i)^2\}^{3n}$

$= (2i)^{3n} + (-2i)^{3n}$

$= 2^{3n}\{i^{3n} + (-i)^{3n}\}$

$= 0 \quad \ldots[\because n \text{ is odd}]$

34. $p + iq = \sqrt{\frac{a+ib}{c+id}}$

$\Rightarrow |p+iq|^2 = \frac{|a+ib|}{|c+id|}$

$\Rightarrow \left(\sqrt{p^2+q^2}\right)^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

$\Rightarrow \left(p^2+q^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$

35. $z_1 + z_2 = 11 + 3i$ and $z_1 - z_2 = -1 - 7i$

$\therefore \quad \frac{z_1+z_2}{z_1-z_2} = \frac{11+3i}{-1-7i} = \frac{-32}{50} + \frac{74}{50}i$

$\therefore \quad \left|\frac{z_1+z_2}{z_1-z_2}\right| = \left|\frac{-32}{50} + \frac{74}{50}i\right| = \sqrt{\left(\frac{-32}{50}\right)^2 + \left(\frac{74}{50}\right)^2}$

$= \sqrt{\frac{13}{5}}$

36. $|z| = \sqrt{a^2+b^2} = \sqrt{(5)^2+(12)^2} = 13$

Let θ be the argument of z.

$\therefore \quad \tan\theta = \left|\frac{b}{a}\right| = \left|\frac{12}{5}\right| \Rightarrow \theta = \tan^{-1}\left(\frac{12}{5}\right)$

37. $i^{65} + \frac{1}{i^{145}} = i^{64}.i + \frac{1}{i^{144}.i}$

$= (i^4)^{16}.i + \frac{1}{(i^4)^{36}.i}$

$= i + \frac{1}{i} = 0$

38. $(i)^{243} = (i^4)^{60}.i^3$

$= -i \quad \ldots[\because i^4 = 1, i^3 = -i]$

39. $(x - 15y) + 2i = 7x - i(y + 4)$

$\Rightarrow x - 15y = 7x$ and $2 = -(y + 4)$

By solving, we get $x = 15$ and $y = -6$

$\therefore \quad x + y = 9$

40. Since, $\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = i$

$\therefore \quad \left(\frac{1+i}{1-i}\right)^{4n+1} = i^{4n+1} = i.i^{4n} = i \quad \ldots[\because i^{4n} = 1]$

41. $\frac{i^{584}(i^8+i^6+i^4+i^2+1)}{i^{574}(i^8+i^6+i^4+i^2+1)} - 1$

$= \frac{i^{584}}{i^{574}} - 1$

$= i^{10} - 1 = -1 - 1 = -2$

42. $i + i^2 + i^3 + \ldots$ upto 1000 terms

$= \frac{i(1-i^{1000})}{1-i} = \frac{i(1-(i^4)^{250})}{1-i} = \frac{i(1-1)}{1-i} = 0$

43. $\sum_{k=0}^{100} i^k = x + iy$

$\Rightarrow 1 + i + i^2 + \ldots + i^{100} = x + iy$

Given series is in G.P.

$\Rightarrow \frac{1.(1-i^{101})}{1-i} = x + iy$

$\Rightarrow \frac{1-i}{1-i} = x + iy$

$\Rightarrow 1 + 0i = x + iy$

Equating real and imaginary parts, we get $x = 1, y = 0$

44. $|z-4| < |z-2|$

$\Rightarrow |z-4|^2 < |z-2|^2$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow 4x > 12$

$\Rightarrow \text{Re}(z) > 3$

45. We have, $|z_k| = 1, k = 1, 2, ..., n$

$\Rightarrow |z_k|^2 = 1$

$\Rightarrow z_k \bar{z}_k = 1$

$\Rightarrow \bar{z}_k = \frac{1}{z_k}$

$\therefore \quad |z_1 + z_2 + ... + z_n| = |\overline{z_1 + z_2 + ... + z_n}|$ ....[$\because |z| = |\bar{z}|$]

$= |\bar{z}_1 + \bar{z}_2 + ... + \bar{z}_n|$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + ... + \frac{1}{z_n}\right|$

46. $|z| = 1$

$\Rightarrow |x + iy| = 1$

$\Rightarrow x^2 + y^2 = 1$ ....(i)

$\omega = \frac{z-1}{z+1}$

$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$

$= \frac{(x^2+y^2-1)}{(x+1)^2+y^2} + \frac{2iy}{(x+1)^2+y^2}$

$= \frac{2iy}{(x+1)^2+y^2}$ ....[From (i)]

$\therefore \quad \text{Re}(\omega) = 0$

47. $\left|\frac{z_1 - z_2}{1 - z_1\bar{z}_2}\right| = \frac{|z_1 - z_2|}{\left|1 - \frac{\bar{z}_2}{\bar{z}_1}\right|}$ ....[$\because z_1\bar{z}_1 = |z_1|^2$]

$= \frac{|z_1 - z_2|}{|\bar{z}_1 - \bar{z}_2|}|\bar{z}_1|$

$= \frac{|z_1 - z_2|}{|z_1 - z_2|}$ ....[$\because |\bar{z}_1| = 1$]

$= \frac{|z_1 - z_2|}{|z_1 - z_2|} = 1$

48. $\text{amp}\left(\frac{1+\sqrt{3}i}{\sqrt{3}+i}\right) = \text{amp}(1+\sqrt{3}i) - \text{amp}(\sqrt{3}+i)$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

49. $\frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-(1-1-2i)} = \frac{1+2i}{1+2i} = 1 + 0i$

Modulus = 1

Amplitude $\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

50. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary, if the real part vanishes, i.e., $\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow 3 - 4\sin^2\theta = 0$ (only if θ be real)

$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} = \sin\left(\pm\frac{\pi}{3}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(\pm\frac{\pi}{3}\right)$

$= n\pi \pm \frac{\pi}{3}$, where n is an integer

51. $z = -1 - i$

$\therefore \quad a = -1$ and $b = -1$

$\arg z = \tan^{-1}\left(\frac{b}{a}\right) - \pi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$

52. $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$

$= \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^8 + \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^8$

$= \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^8 + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^8$

$= \cos\frac{8\pi}{4} + i\sin\frac{8\pi}{4} + \cos\frac{8\pi}{4} - i\sin\frac{8\pi}{4}$

$= \cos 2\pi + \cos 2\pi$

$= 1 + 1$

$= 2$

53. $\frac{1+i}{1-i} = \frac{1+i}{1-i}.\frac{1+i}{1+i} = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

Hence, amplitude is $\frac{\pi}{2}$

54. $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right) = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + 2i\sin^2\frac{\pi}{10}$

$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$

For amplitude, $\tan\theta = \frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\frac{\pi}{10}$

$\Rightarrow \theta = \frac{\pi}{10}$

55. amp(z) – amp (–z)

$= \tan^{-1}\left(\frac{y}{x}\right) - \left(\tan^{-1}\left(\frac{y}{x}\right) - \pi\right)$

$= \pi$

56. amp (z) $= \tan^{-1}\left(\frac{\sin\alpha}{1-\cos\alpha}\right)$

$= \tan^{-1}\left(\cot\frac{\alpha}{2}\right)$

$= \tan^{-1}\left\{\tan\left(\frac{\pi}{2} - \frac{\alpha}{2}\right)\right\}$

$= \frac{\pi}{2} - \frac{\alpha}{2}$

57. Since, $\frac{-\sqrt{3}-i}{2} = -\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\Rightarrow \left(\frac{-\sqrt{3}-i}{2}\right)^3 = -\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3 = -i$

58. Let $z = e^{e^{-i\theta}} = e^{\cos\theta - i\sin\theta} = e^{\cos\theta}e^{-i\sin\theta}$

$= e^{\cos\theta}[\cos(\sin\theta) - i\sin(\sin\theta)]$

$= e^{\cos\theta}\cos(\sin\theta) - ie^{\cos\theta}\sin(\sin\theta)$

$\therefore$ amp (z) $= \tan^{-1}\left[-\frac{e^{\cos\theta}\sin(\sin\theta)}{e^{\cos\theta}\cos(\sin\theta)}\right]$

$= \tan^{-1}[\tan(-\sin\theta)]$

$= -\sin\theta$

59. $a = \sqrt{2i} = \sqrt{2}\, i^{1/2} = \sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/2}$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = 1 + i$

60. L.H.S.

$= \left[\frac{2\cos^2(\phi/2) + 2i\sin(\phi/2)\cos(\phi/2)}{2\cos^2(\phi/2) - 2i\sin(\phi/2)\cos(\phi/2)}\right]^n$

$= \left[\frac{\cos(\phi/2) + i\sin(\phi/2)}{\cos(\phi/2) - i\sin(\phi/2)}\right]^n$

$= \left[\frac{e^{i(\phi/2)}}{e^{-i(\phi/2)}}\right]^n$

$= (e^{i\phi})^n$

$= \cos n\phi + i\sin n\phi$

61. Let $\cos\frac{\pi}{10} - i\sin\frac{\pi}{10} = z$ and

$\cos\frac{\pi}{10} + i\sin\frac{\pi}{10} = \frac{1}{z}$

$\therefore \left(\frac{1-z}{1-\frac{1}{z}}\right)^{10} = \left\{\frac{-(z-1)z}{(z-1)}\right\}^{10}$

$= (-z)^{10}$

$= z^{10} = \left(\cos\frac{\pi}{10} - i\sin\frac{\pi}{10}\right)^{10}$

$= \cos\pi - i\sin\pi = -1$

62. $\frac{(\cos 2\theta - i\sin 2\theta)^4(\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 3\theta - i\sin 3\theta)^{-9}}$

$= \frac{[(\cos\theta + i\sin\theta)^{-2}]^4[(\cos\theta + i\sin\theta)^4]^{-5}}{[(\cos\theta + i\sin\theta)^3]^{-2}[(\cos\theta + i\sin\theta)^{-3}]^{-9}}$

$= \frac{(\cos\theta + i\sin\theta)^{-8}(\cos\theta + i\sin\theta)^{-20}}{(\cos\theta + i\sin\theta)^{-6}(\cos\theta + i\sin\theta)^{27}}$

$= (\cos\theta + i\sin\theta)^{-8-20+6-27}$

$= (\cos\theta + i\sin\theta)^{-49}$

$= \cos 49\theta - i\sin 49\theta$

63. $\frac{1}{2} + i\frac{\sqrt{3}}{2} = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Now, $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{1}{4}} = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{1}{4}}$

$= \left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

64. $x = \frac{5 + i \pm \sqrt{(5+i)^2 - 4(18-i)}}{2(1)}$

$= \frac{5 + i \pm \sqrt{-48 + 14i}}{2}$

$= \frac{5 + i \pm (7i + 1)}{2}$

65. $9x^2 - 12x + 20 = 0$

$\therefore$ a = 9, b = –12, c = 20

$\therefore x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(20)}}{2 \times 9}$

$= \frac{12 \pm 24i}{18} = \frac{2}{3} \pm \frac{4i}{3}$

66. $\left(\frac{1+\omega}{\omega^2}\right)^3 = \left(\frac{-\omega^2}{\omega^2}\right)^3 = (-1)^3 = -1$

67. $[3(1+\omega^2)+5\omega]^6 = (-3\omega+5\omega)^6 = 2^6.\omega^6 = 64$

68. $(2-\omega)(2-\omega^2)(2-\omega)(2-\omega^2)$
$= (2-\omega)^2(2-\omega^2)^2$
$= [(2-\omega)(2-\omega^2)]^2$
$= [4-2(\omega+\omega^2)+\omega^3]^2$
$= [4+2+1]^2$
$= 49$

69. $(1-\alpha)(1-\beta)(1-\alpha^2)(1-\beta^2)$
$= (1-\alpha)(1-\alpha^2)(1-\alpha^2)(1-\alpha)$
$= (1-\alpha)^2(1-\alpha^2)^2 = (-\alpha-2\alpha)(1-2\alpha^2+\alpha)$
$= (-3\alpha)(-\alpha^2-2\alpha^2) = (-3\alpha)(-3\alpha^2) = 9$

70. $\frac{1+\omega^2+1+\omega}{(1+\omega)(1+\omega^2)} = \frac{1}{(-\omega^2)(-\omega)} = \frac{1}{\omega^3} = 1$

71. $\omega^{40}+(\omega^2)^{40} = \omega^{40}+\omega^{80}$
$= (\omega^3)^{13}\omega + (\omega^3)^{26}\omega^2$
$= \omega+\omega^2$
$= -1$

72. $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$
$= 4+5\omega^{334}+3\omega^{365}$
$= 4+5\omega+3\omega^2$
$= 4+5\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)+3\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$
$= i\sqrt{3}$

73. $x = \frac{-(-1)\pm\sqrt{(-1)^2-4i(12i)}}{2i} = \frac{1\pm 7}{2i}$

74. $\alpha^2+\alpha+1=0$
$\therefore$ $(\alpha-1)(\alpha^2+\alpha+1)=0$
$\therefore$ $\alpha^3-1=0, \alpha\neq 1$
$\Rightarrow \alpha^3 = 1$
and consequently $\alpha^{31} = (\alpha^3)^{10}.\ \alpha = 1^{10}\ \alpha = \alpha$

75. $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$
$= (-2\omega)^5+(-2\omega^2)^5$
$= -32\ \omega^3\omega^2 - 32\omega^9\omega$
$= -32\ (\omega^2+\omega)$
$= 32$

76. $\omega^2(1+\omega)^3-(1+\omega^2)\omega = \omega^2(-\omega^2)^3-\omega(-\omega)$
$-\omega^2+\omega^2 = 0$

77. $(1-\omega+\omega^2)^3 = (-2\omega)^3 = -8\ \omega^3 = -8$

78. After solving, we get
$3x^3+3y^3+3x^2y(1+\omega+\omega^2)+3xy^2(1+\omega+\omega^2)$
$= 3(x^3+y^3)+3.0+3.0$
$= 3\ (x^3+y^3)$

79. $z^{69} = \left(\frac{\sqrt{3}+i}{2}\right)^{69} = \left[\frac{1}{i}\left(\frac{-1+\sqrt{3}i}{2}\right)\right]^{69}$
$= \left(\frac{\omega}{i}\right)^{69} = \frac{\omega^{69}}{(i^4)^{17}i} = \frac{1}{i}$
$= -i$

Competitive Thinking

1. $\frac{5(-8+6i)}{(1+i)^2} = a+ib$
$\Rightarrow \frac{-40+30i}{2i} = 15+20i = a+ib$
Equating real and imaginary parts, we get
$a = 15$ and $b = 20$

2. The two complex numbers can be compared only when their real and imaginary parts are equal. In other words, there is no meaning of >, < in complex numbers.

3. Let $z_1 = a+ib$, $z_2 = c+id$, then
z_1+z_2 is real
$\Rightarrow (a+c)+i(b+d)$ is real
$\Rightarrow b+d = 0$
$\Rightarrow d = -b$
z_1z_2 is real
$\Rightarrow (ac-bd)+i(ad+bc)$ is real
$\Rightarrow\ ad+bc = 0$
$\Rightarrow a(-b)+bc = 0 \Rightarrow a = c$
$\therefore$ $z_1 = a+ib = c-id = \bar{z_2}$
....[$\because$ a = c and b = –d]

4. $3-2yi = 9^x-7i$
Equating real and imaginary parts, we get
$9^x = 3 \Rightarrow 3^{2x} = 3^1 \Rightarrow 2x = 1 \Rightarrow x = 0.5$
and $2y = 7 \Rightarrow y = 3.5$

5. $(a+ib)(c+id)(e+if)(g+ih) = A+iB$ ....(i)
$\Rightarrow (a-ib)(c-id)(e-if)(g-ih) = A-iB$ .....(ii)
Multiplying (i) and (ii), we get
$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

6. $\dfrac{c+i}{c-i} = a + ib$ .....(i)

$\therefore$ $\dfrac{c-i}{c+i} = a - ib$ .....(ii)

Multiplying (i) and (ii), we get

$\dfrac{c^2+1}{c^2+1} = a^2 + b^2 \Rightarrow a^2 + b^2 = 1$

7. $\overline{(x+iy)(1-2i)} = 1 + i$

$\Rightarrow x - iy = \dfrac{1+i}{1+2i} \Rightarrow x + iy = \dfrac{1-i}{1-2i}$

8. Let $z = \dfrac{(2+i)^2}{3+i} = \dfrac{3+4i}{3+i} \times \dfrac{3-i}{3-i} = \dfrac{13}{10} + i\dfrac{9}{10}$

Conjugate of $z = \dfrac{13}{10} - i\dfrac{9}{10}$

9. $\dfrac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}$

$= \left(\dfrac{3-4\sin^2\theta}{1+4\sin^2\theta}\right) + i\left(\dfrac{8\sin\theta}{1+4\sin^2\theta}\right)$

Since it is real, therefore Im $(z) = 0$

$\Rightarrow \dfrac{8\sin\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi$, where n=0, 1, 2, 3, ...

10. $\{(1-\cos\theta)+i.2\sin\theta\}^{-1} = \left\{2\sin^2\dfrac{\theta}{2} + i.4\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right\}^{-1}$

$= \left(2\sin\dfrac{\theta}{2}\right)^{-1}\left\{\sin\dfrac{\theta}{2} + i.2\cos\dfrac{\theta}{2}\right\}^{-1}$

$= \left(2\sin\dfrac{\theta}{2}\right)^{-1} \dfrac{1}{\sin\dfrac{\theta}{2} + i.2\cos\dfrac{\theta}{2}} \times \dfrac{\sin\dfrac{\theta}{2} - i.2\cos\dfrac{\theta}{2}}{\sin\dfrac{\theta}{2} - i.2\cos\dfrac{\theta}{2}}$

$= \dfrac{\sin\dfrac{\theta}{2} - i.2\cos\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\left(\sin^2\dfrac{\theta}{2} + 4\cos^2\dfrac{\theta}{2}\right)}$.

Its real part

$= \dfrac{\sin\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\left(1+3\cos^2\dfrac{\theta}{2}\right)} = \dfrac{1}{2\left(1+3\cos^2\dfrac{\theta}{2}\right)}$

$= \dfrac{1}{2\left[1+3\left(\dfrac{1+\cos\theta}{2}\right)\right]} = \dfrac{1}{5+3\cos\theta}$

11. $x = 3 + i$

$\Rightarrow x - 3 = i$

$\Rightarrow x^2 - 6x + 10 = 0$

$\Rightarrow x^3 - 3x^2 - 8x + 15$

$= x(x^2 - 6x + 10) + 3(x^2 - 6x + 10) - 15$

$= x(0) + 3(0) - 15$

$= -15$

12. $\dfrac{z_1}{z_2} = \dfrac{4+5i}{-3+2i} \times \dfrac{-3-2i}{-3-2i}$

$= \dfrac{-12-8i-15i+10}{9-(2i)^2}$

$= \dfrac{-2}{13} - i\left(\dfrac{23}{13}\right)$

$= \left(\dfrac{-2}{13}, \dfrac{-23}{13}\right)$

13. Given, $z = 1 + i$ and $i = \sqrt{-1}$

Squaring on both sides, we get

$z^2 = (1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$

Since, it is a multiplicative identity, therefore multiplicative inverse of

$z^2 = \dfrac{1}{2i} \times \dfrac{i}{i} = \dfrac{i}{2i^2} = -\dfrac{i}{2}$

14. $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$

Applying $R_1 \to R_1 + R_2$, we get

$\begin{vmatrix} 6i+4 & 0 & 0 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$

$\Rightarrow (6i+4)(3i^2+3) = x + iy$

$\Rightarrow (6i+4)(-3+3) = x + iy$

$\Rightarrow x + iy = 0 = 0 + i0$

$\Rightarrow (x, y) = (0, 0)$

15. $z_1 = 1 + 2i$, $z_2 = 3 + 5i$ and $\overline{z}_2 = 3 - 5i$

$\therefore$ $\dfrac{\overline{z}_2 z_1}{z_2} = \dfrac{(3-5i)(1+2i)}{(3+5i)} = \dfrac{13+i}{3+5i}$

$= \dfrac{13+i}{3+5i} \times \dfrac{3-5i}{3-5i}$

$= \dfrac{44-62i}{34}$

$\therefore$ $\text{Re}\left(\dfrac{\overline{z}_2 z_1}{z_2}\right) = \dfrac{44}{34} = \dfrac{22}{17}$

16. $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other, if $\sin x = \cos x$ and $\cos 2x = \sin 2x$

or $\tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, ...$ ......(i)

and $\tan 2x = 1 \Rightarrow 2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, ...$

or $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8} ....$ .....(ii)

There exists no value of x common in (i) and (ii). Therefore there is no value of x for which the given complex numbers are conjugate.

17. Given equation

$(x^4 + 2xi) - (3x^2 + yi) = (3 - 5i) + (1 + 2yi)$

$\Rightarrow (x^4 - 3x^2) + i(2x - 3y) = 4 - 5i$

Equating real and imaginary parts, we get

$x^4 - 3x^2 = 4$ ....(i)

and $2x - 3y = -5$ ....(ii)

From (i) and (ii), we get

$x = \pm 2$ and $y = 3, \frac{1}{3}$

18. $(1-i)^n\left(1-\frac{1}{i}\right)^n = (1-i)^n\left(\frac{i-1}{i}\right)^n$

$= [(1-i)^2]^n\left(\frac{-1}{i}\right)^n$

$= (1 + i^2 - 2i)^n\left(\frac{i^2}{i}\right)^n$

$= (1 - 1 - 2i)^n (i)^n$

$= (-2i)^n (i)^n$

$= (-2i^2)^n$

$= 2^n$

19. $\frac{1-2i}{2+i} + \frac{4-i}{3+2i} = \frac{(1-2i)(3+2i)+(4-i)(2+i)}{(2+i)(3+2i)}$

$= \frac{16-2i}{4+7i} \times \frac{4-7i}{4-7i} = \frac{50-120i}{65} = \frac{10}{13} - \frac{24}{13}i$

20. $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$= \left(\frac{1+2i}{1^2+2^2} + \frac{3-3i}{1^2+1^2}\right)\left(\frac{6-16+12i+8i}{2^2+4^2}\right)$

$= \left(\frac{2+4i+15-15i}{10}\right)\left(\frac{-1+2i}{2}\right)$

$= \frac{(17-11i)(-1+2i)}{20} = \frac{5+45i}{20} = \frac{1}{4} + \frac{9}{4}i$

21. $z = \frac{11-3i}{1+i} \times \frac{1-i}{1-i} = \frac{8-14i}{2} = 4 - 7i$

$\therefore$ $z - i\alpha = 4 - (7 + \alpha)\, i$ which is real, if $\alpha = -7$

22. $\frac{(1+i)^3}{(1-i)^3} - \frac{(1-i)^3}{(1+i)^3} = x + iy$

$\Rightarrow \frac{(1+i^2+2i)^3 - (1+i^2-2i)^3}{(1-i^2)^3} = x + iy$

$\Rightarrow \frac{8i^3 + 8i^3}{8} = x + iy$

$\Rightarrow 2i^3 = x + iy \Rightarrow -2i = x + iy$

$\Rightarrow x = 0, y = -2$

23. Let $z = x + iy$

$\therefore$ $x = 1 - t^2, y = \sqrt{1+t^2}$

On eliminating t, we get $x = 1 - (y^2 - 1)$

$\therefore$ $y^2 = -x + 2$, which is a parabola.

24. Let $z = \frac{2+i}{ai-1} = \frac{(2+i)(ai+1)}{(ai)^2 - 1}$

$= \frac{-1}{a^2+1}\{(2+i)(ai+1)\}$

$\text{Im}(z) = \frac{-1}{1+a^2}(1 + 2a)$

But, imaginary part is zero.

$\therefore$ $1 + 2a = 0 \Rightarrow a = -\frac{1}{2}$

25. Let $z = x + iy$, $y \neq 0$

$a = z^2 + z + 1$

$\Rightarrow (x^2 - y^2 + x + 1) + i\,(2xy + y) = a$

$\Rightarrow x^2 - y^2 + x + 1 = a$ and $2xy + y = 0$

$\Rightarrow x^2 - y^2 + x + 1 = a$ and $y\,(2x + 1) = 0$

$\Rightarrow x^2 - y^2 + x + 1 = a$ and $x = -\frac{1}{2}$ ....[$\because y \neq 0$]

$\Rightarrow y^2 = \frac{3}{4} - a$

Since, $y \neq 0$

$\therefore$ $a \neq \frac{3}{4}$

26. $z = \frac{4}{1-i}$

$\Rightarrow \bar{z} = \frac{4}{1+i}$

27. Let $z = x + iy$

Then, $z^2 = (x^2 - y^2) + i(2xy)$

$\therefore \quad \frac{z^2}{z-1} = \frac{(x^2-y^2)+i(2xy)}{x+iy-1}$

$= \frac{(x^2-y^2)+i(2xy)}{x-1+iy} \times \frac{x-1-iy}{x-1-iy}$

Since, $\frac{z^2}{z-1}$ is real.

$\therefore$ its imaginary part = 0

$\Rightarrow 2xy(x-1) - y(x^2-y^2) = 0$

$\Rightarrow y(x^2 - 2x + y^2) = 0$

$\Rightarrow y = 0$ or $x^2 - 2x + y^2 = 0$

$\therefore$ z lies either on real axis or on a circle passing through origin.

28. $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{2} = \frac{2i}{2} = i$

$\therefore \left(\frac{1+i}{1-i}\right)^m = i^m = 1$ (as given)

So the least value of m = 4 ....$[\because i^4 = 1]$

29. $(1-i)^n = 2^n$ ......(i)

We know that if two complex numbers are equal, their moduli must also be equal, therefore from (i), we have

$|(1-i)^n| = |2^n|$

$\Rightarrow |1-i|^n = |2|^n$ ....$[\because 2^n > 0]$

$\Rightarrow \left[\sqrt{1^2+(-1)^2}\right]^n = 2^n$

$\Rightarrow (\sqrt{2})^n = 2^n$

$\Rightarrow 2^{\frac{n}{2}} = 2^n$

$\Rightarrow \frac{n}{2} = n$

$\Rightarrow n = 0$

Trick : By inspection, $(1-i)^0 = 2^0$

$\Rightarrow 1 = 1$

30. $1 + i^2 + i^4 + \ldots + i^{2n}$

$= 1 - 1 + 1 - 1 + \ldots + (-1)^n$

It depends on n.

Hence, cannot be determined unless n is known.

31. Given expression is

$-1 + 1 - 1 + 1$ .... upto (2n+1) terms

Here, number of terms are odd, so expression has the value -1.

32. $\sum_{n=1}^{200} i^n = i + i^2 + i^3 + \ldots + i^{200}$

$= \frac{i(1-i^{200})}{1-i}$ ....[since G.P.]

$= \frac{i(1-1)}{1-i}$

$= 0$

33. $\sum_{n=1}^{13}(i^n + i^{n+1})$

$= (i + i^2 + i^3 + \ldots + i^{13}) + (i^2 + i^3 + \ldots + i^{14})$

$= \frac{i(1-i^{13})}{1-i} + \frac{i^2(1-i^{13})}{1-i} = i\left(\frac{1-i}{1-i}\right) + \frac{i^2(1-i)}{(1-i)}$

$= i + i^2 = i - 1$

34. $(1+i)^6 + (1-i)^6 = [(1+i)^2]^3 + [(1-i)^2]^3$

$= (2i)^3 + (-2i)^3$

$= (8-8)i^3$

$= 0$

35. $(\sqrt{8} + i)^{50} = 3^{49}(a+ib)$

Taking modulus and squaring on both sides, we get

$(8+1)^{50} = 3^{98}(a^2+b^2)$

$\Rightarrow 9^{50} = 3^{98}(a^2+b^2)$

$\Rightarrow 3^{100} = 3^{98}(a^2+b^2)$

$\Rightarrow (a^2+b^2) = 9$

36. Let $\sqrt{-8-6i} = x + iy$

$\Rightarrow -8 - 6i = (x+iy)^2$

$\Rightarrow x^2 - y^2 = -8$ and $2xy = -6$

By solving, we get

$x = 1, y = -3$ and $x = -1, y = 3$

$\therefore \quad x + iy = \pm(1-3i)$

Trick : Since, $\{\pm(1-3i)\}^2 = -8 - 6i$

37. $\sqrt{-7-24i} = x - iy$

Squaring both sides, $-7 - 24i = x^2 - y^2 - i(2xy)$

Equating real and imaginary parts, we get

$x^2 - y^2 = -7$ and $2xy = 24$

$\therefore \quad x^2 + y^2 = \sqrt{49+576} = \sqrt{625} = 25$

38. $\sqrt{a+ib} = x + yi$

$\Rightarrow \left(\sqrt{a+ib}\right)^2 = (x+yi)^2$

$\Rightarrow a = x^2 - y^2, b = 2xy$

$\therefore \quad \sqrt{a-ib} = \sqrt{x^2-y^2-2xyi}$

$= \sqrt{(x-yi)^2} = x - iy$

39. $z=\frac{7-i}{3-4i}\times\frac{3+4i}{3+4i}=\frac{21+25i+4}{16+9}$

$=\frac{25(1+i)}{25}=(1+i)$

$z^{14}=(1+i)^{14}=\left[(1+i)^2\right]^7=(2i)^7=2^7 i^7=-2^7 i$

40. Let $\frac{2z_1}{3z_2}=iy$ Then, $\frac{z_1}{z_2}=\frac{3}{2}iy$

$\therefore \left|\frac{z_1-z_2}{z_1+z_2}\right|=\left|\frac{\frac{z_1}{z_2}-1}{\frac{z_1}{z_2}+1}\right|=\left|\frac{\frac{3}{2}iy-1}{\frac{3}{2}iy+1}\right|=\left|\frac{1-\frac{3}{2}iy}{1+\frac{3}{2}iy}\right|=1$

....$[\because |z|=|\bar{z}|]$

41. Let $z=x+iy$ ......(i)

Given, $|z+i|=|z-i|$

$\Rightarrow |x+iy+i|=|x+iy-i|$

$\Rightarrow |x+i(y+1)|=|x+i(y-1)|$

$\Rightarrow \sqrt{x^2+(y+1)^2}=\sqrt{x^2+(y-1)^2}$

$\Rightarrow x^2+(y+1)^2=x^2+(y-1)^2$

$\Rightarrow y^2+2y+1=y^2-2y+1\Rightarrow 4y=0\Rightarrow y=0$

Hence, from (i), we get $z=x$, where x is any real number.

42. $\left(\frac{3+2i}{3-2i}\right)=\left(\frac{3+2i}{3-2i}\right)\left(\frac{3+2i}{3+2i}\right)$

$=\frac{9-4+12i}{13}=\frac{5}{13}+i\left(\frac{12}{13}\right)$

Modulus $=\sqrt{\left(\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2}=1$

43. Let $\frac{z-1}{z+1}=iy$, where $y\in R$

This gives

$z=\frac{1+iy}{1-iy}=\frac{1+iy}{1-iy}\times\frac{1+iy}{1+iy}=\frac{(1-y^2)+2iy}{1+y^2}$

$\therefore \quad |z|=\frac{1}{1+y^2}\sqrt{(1-y^2)^2+4y^2}=\frac{1+y^2}{1+y^2}=1$

44. Given, $\left(\frac{1-i}{1+i}\right)^{100}=a+ib$

$\Rightarrow \left[\left(\frac{1-i}{1+i}\right)\times\left(\frac{1-i}{1-i}\right)\right]^{100}=a+ib$

$\Rightarrow a+ib=\left[\frac{(1-i)^2}{2}\right]^{100}=\left[\frac{-2i}{2}\right]^{100}=(-i)^{100}$

$\Rightarrow a+ib=\left[(i)^4\right]^{25}=1+0i,$

Hence, $a=1, b=0$

45. $|z_1+z_2|^2+|z_1-z_2|^2$

$=(x_1+x_2)^2+(y_1+y_2)^2+(x_1-x_2)^2+(y_1-y_2)^2$

$=2(x_1^2)+2(y_1^2)+2(x_2^2)+2(y_2^2)$

$=2|z_1|^2+2|z_2|^2$

46. L.H.S. $=|z^2|=|(x+iy)^2|=|x^2-y^2+2ixy|$

$=\sqrt{(x^2-y^2)^2+(2xy)^2}$

$=\sqrt{(x^2+y^2)^2}$

R.H.S. $=|z|^2=|x+iy|^2=\left(\sqrt{x^2+y^2}\right)^2=x^2+y^2$

Therefore, $|z^2|=|z|^2$

(B) True, (C) False ....$[\because z\neq\bar{z}]$

47. $z=x+iy$

$|z|^2=x^2+y^2=1$ .....(i)

Now, $\left(\frac{z-1}{z+1}\right)=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}$

$=\frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$

$=\frac{2iy}{(x+1)^2+y^2}$ ....[From (i)]

Hence, $\left(\frac{z-1}{z+1}\right)$ is purely imaginary.

48. $\alpha-i\beta=\frac{3-4xi}{3+4xi}$.

Taking modulus and squaring on both sides, $\alpha^2+\beta^2=1$

49. Squaring the given relations implies that

$x_1x_2+y_1y_2=0$

Now, amp z_1 – amp z_2

$=\tan^{-1}\left(\frac{y_1}{x_1}\right)-\tan^{-1}\left(\frac{y_2}{x_2}\right)$

$=\tan^{-1}\left(\frac{\frac{y_1}{x_1}-\frac{y_2}{x_2}}{1+\frac{y_1y_2}{x_1x_2}}\right)=\tan^{-1}\left(\frac{y_1x_2-y_2x_1}{x_1x_2+y_1y_2}\right)$

$=\tan^{-1}\infty=\frac{\pi}{2}$

50. $\arg\left(\frac{13-5i}{4-9i}\right)=\arg(13-5i)-\arg(4-9i)$

$$=-\tan^{-1}\left(\frac{5}{13}\right)+\tan^{-1}\left(\frac{9}{4}\right)$$

$$=\tan^{-1}1=\frac{\pi}{4}$$

51. $z=\frac{-2}{1+\sqrt{3}i}=\frac{-2}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}=\frac{-2+2\sqrt{3}i}{1+3}$

$\Rightarrow z=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$

$\Rightarrow \arg(z)=\tan^{-1}\left(-\frac{\sqrt{3}/2}{1/2}\right)=\frac{2\pi}{3}$

52. $(1+i)^{10}=[(1+i)^2]^5=(1+i^2+2i)^5=(2i)^5=32i$

53. $\left|(1+i)\frac{(2+i)}{(3+i)}\right|=|1+i|\left|\frac{2+i}{3+i}\right|=\frac{\sqrt{2}\times\sqrt{5}}{\sqrt{10}}=1$

54. $\left|\frac{1-i}{3+i}+\frac{4i}{5}\right|=\left|\frac{(1-i)(3-i)}{(3+i)(3-i)}+\frac{4i}{5}\right|$

$$=\left|\frac{3-1-4i}{9-(-1)}+\frac{4i}{5}\right|=\left|\frac{2(1-2i)}{10}+\frac{4i}{5}\right|$$

$$=\left|\frac{1}{5}+\frac{2}{5}i\right|=\sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{2}{5}\right)^2}=\sqrt{\frac{5}{25}}$$

$$=\frac{\sqrt{5}}{5}\text{ units}$$

55. We have, $(1+i)^{2n}=(1-i)^{2n}$

$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n}=1\Rightarrow (i)^{2n}=1$

$\Rightarrow (i)^{2n}=(-1)^2\Rightarrow (i)^{2n}=(i^2)^2$

$\Rightarrow (i)^{2n}=(i)^4\Rightarrow 2n=4$

$\Rightarrow n=2$

56. Since, $|z_1+z_2|\le|z_1|+|z_2|$

$\therefore$ $|z|+|z-1|=|z|+|1-z|\ge|z+(1-z)|=|1|=1$

Hence, minimum value of $|z|+|z-1|$ is 1.

57. $|z+1|=|z+4-3|$

$=|(z+4)+(-3)|$

$\le|z+4|+|-3|$ ....$[\because |z_1+z_2|\le|z_1|+|z_2|]$

$=|z+4|+3\le 3+3=6$ ....$[\because |z+4|\le 3]$

$\therefore$ greatest value of $|z+1|=6$

58. $|(az_1-bz_2)|^2+|(bz_1+az_2)|^2$

$=a^2|z_1|^2+b^2|z_2|^2-2\,\text{Re}(ab)|z_1\bar{z}_2|+b^2|z_1|^2$

$+a^2|z_2|^2+2\,\text{Re}(ab)|z_1\bar{z}_2|$

$=(a^2+b^2)(|z_1|^2+|z_2|^2)$

59. $1=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$

$=\left|\frac{z_1\bar{z}_1}{z_1}+\frac{z_2\bar{z}_2}{z_2}+\frac{z_3\bar{z}_3}{z_3}\right|$

....$[\because |z_1|^2=1=z_1\bar{z}_1,\text{etc.}]$

$=|\bar{z}_1+\bar{z}_2+\bar{z}_3|=|\overline{z_1+z_2+z_3}|$

$=|z_1+z_2+z_3|$ ....$[\because |\bar{z}_1|=|z_1|]$

$\therefore$ $|z_1+z_2+z_3|=1$

60. Let $z=x+iy$,

Now, $\left|\frac{z-12}{z-8i}\right|=\frac{5}{3}\Rightarrow 3|z-12|=5|z-8i|$

$\Rightarrow 3|(x-12)+iy|=5|x+(y-8)i|$

$\Rightarrow 9(x-12)^2+9y^2=25x^2+25(y-8)^2$ ....(i)

and $\left|\frac{z-4}{z-8}\right|=1\Rightarrow |z-4|=|z-8|$

$\Rightarrow |x-4+iy|=|x-8+iy|$

$\Rightarrow (x-4)^2+y^2=(x-8)^2+y^2$

$\Rightarrow x=6$

Putting $x=6$ in (i), we get

$y^2-25y+136=0$

$\therefore$ $y=17, 8$

Hence, $z=6+17i$ or $z=6+8i$

61. $\left[\left|z_1+\sqrt{z_1^2-z_2^2}\right|+\left|z_1-\sqrt{z_1^2-z_2^2}\right|\right]^2$

$=\left|z_1+\sqrt{z_1^2-z_2^2}\right|^2+\left|z_1-\sqrt{z_1^2-z_2^2}\right|^2$

$+2\left|z_1^2-(z_1^2-z_2^2)\right|$

$=2|z_1|^2+2|z_1^2-z_2^2|+2|z_2^2|$

....$\left[\because |z_1+z_2|^2+|z_1-z_2|^2=2\left(|z_1|^2+|z_2|^2\right)\right]$

$=2|z_1|^2+2|z_2|^2+2|z_1^2-z_2^2|$

$=|z_1+z_2|^2+|z_1-z_2|^2+2|z_1+z_2||z_1-z_2|$

$=(|z_1+z_2|+|z_1-z_2|)^2$

Taking square root on both sides, we get

$\left|z_1+\sqrt{z_1^2-z_2^2}\right|+\left|z_1-\sqrt{z_1^2-z_2^2}\right|=|z_1+z_2|+|z_1-z_2|$

62. We have,

$|z_2| = |z_2 - (3+4i) + 3 + 4i|$

$\Rightarrow |z_2| \le |z_2 - (3+4i)| + |3+4i|$

$\Rightarrow |z_2| \le 5 + 5$

....$\left[\because |3+4i| = \sqrt{9+16} = 5\right]$

$\Rightarrow |z_2| \le 10$

$\Rightarrow -|z_2| \ge -10$

$\Rightarrow |z_1| - |z_2| \ge |z_1| - 10$

$\Rightarrow |z_1| - |z_2| \ge 12 - 10$

$\Rightarrow |z_1| - |z_2| \ge 2$

$\Rightarrow |z_1 - z_2| \ge 2$ ....$[\because |z_1 - z_2| \ge |z_1| - |z_2|]$

$\therefore$ minimum value of $|z_1 - z_2| = 2$

63. $|z| = |z-2| \Rightarrow |z|^2 = |z-2|^2$

$\Rightarrow z\bar{z} = (z-2)(\bar{z}-2)$

$\Rightarrow z\bar{z} = z\bar{z} - 2\bar{z} - 2z + 4$

$\Rightarrow z + \bar{z} = 2$ ....(i)

Also, $|z| = |z+2| \Rightarrow |z|^2 = |z+2|^2$

$\Rightarrow z\bar{z} = (z+2)(\bar{z}+2)$

$= z\bar{z} + 2(z + \bar{z}) + 4$

$\Rightarrow z + \bar{z} = -2$ ....(ii)

From (i) and (ii), we get

$|z + \bar{z}| = 2$

64. Let $z_1 = a + ib = (a, b)$ and $z_2 = c - id = (c, -d)$, where $a > 0$ and $d > 0$

Then $|z_1| = |z_2| \Rightarrow a^2 + b^2 = c^2 + d^2$ ....(i)

Now, $\dfrac{z_1 + z_2}{z_1 - z_2} = \dfrac{(a+ib)+(c-id)}{(a+ib)-(c-id)}$

$= \dfrac{[(a+c)+i(b-d)][(a-c)-i(b+d)]}{[(a-c)+i(b+d)][(a-c)-i(b+d)]}$

$= \dfrac{(a^2+b^2)-(c^2+d^2)-2(ad+bc)i}{a^2+c^2-2ac+b^2+d^2+2bd}$

$= \dfrac{-(ad+bc)i}{a^2+b^2-ac+bd}$ ....[From (i)]

$\therefore$ $\dfrac{(z_1+z_2)}{(z_1-z_2)}$ is purely imaginary.

Trick : Assume any two complex numbers satisfying both conditions i.e., $z_1 \neq z_2$ and $|z_1| = |z_2|$

Let $z_1 = 2 + i$, $z_2 = 1 - 2i$

$\therefore$ $\dfrac{z_1+z_2}{z_1-z_2} = \dfrac{3-i}{1+3i} = -i$

Hence the result.

65. $z_1 + z_2 = 2 + 6i$

$\Rightarrow |z_1 + z_2|^2 = (4 + 36) = 40$,

$|z_1|^2 + |z_2|^2 = 25 + 5 = 30$

$\Rightarrow |z_1 + z_2|^2 - 2(|z_1|^2 + |z_2|^2) = 40 - 60 = -20$

$|z_1 - z_2|^2 = (16 + 4) = 20$

$\Rightarrow |z_1 + z_2|^2 - 2(|z_1|^2 + |z_2|^2) = -|z_1 - z_2|^2$

66. $\left|1 + i\sqrt{3}\right| = \sqrt{1+3} = 2$

$1 + \dfrac{1}{i+1} = 1 + \dfrac{i-1}{i^2-1} = 1 + \dfrac{(i-1)}{-2} = \dfrac{3}{2} - \dfrac{i}{2}$

$\therefore$ $\left|1 + \dfrac{1}{i+1}\right| = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(-\dfrac{1}{2}\right)^2} = \sqrt{\dfrac{9}{4} + \dfrac{1}{4}} = \sqrt{\dfrac{10}{4}}$

$\therefore$ $\left|\dfrac{1+i\sqrt{3}}{\left(1+\dfrac{1}{i+1}\right)^2}\right| = \dfrac{2}{\frac{10}{4}} = \dfrac{4}{5}$

67. $\left(\dfrac{3}{2} + i\dfrac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$

Taking modulus on both sides, we get

$\left(\sqrt{\dfrac{9}{4} + \dfrac{3}{4}}\right)^{50} = 3^{25}\sqrt{x^2 + y^2}$

$\Rightarrow \left(\sqrt{3}\right)^{50} = 3^{25}\sqrt{x^2+y^2} \Rightarrow 1 = \sqrt{x^2+y^2}$

$\Rightarrow x^2 + y^2 = 1$

68. $|z|^2 + |z-3|^2 + |z-i|^2$

$= x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-1)^2$

$= 3x^2 + 3y^2 - 6x - 2y + 10$

$= 3\left(x^2 + y^2 - 2x - 2.y.\dfrac{1}{3}\right) + 10$

$= 3\left|z - \left(1 + \dfrac{i}{3}\right)\right|^2 + \dfrac{20}{3}$

$\therefore$ the given expression is minimum, when z equals $1 + \dfrac{i}{3}$.

69. $z = \dfrac{1+2i}{1-i} \Rightarrow z = \dfrac{1+2i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{-1}{2} + i\dfrac{3}{2}$

This complex number will lie in the II quadrant.

70. $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow |z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}\,|z_1\bar{z}_2|$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow 2\text{Re}|z_1\bar{z}_2| = 2|z_1||z_2|$

$\Rightarrow 2|z_1||z_2|\cos(\theta_1 - \theta_2) = 2|z_1||z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$

$\therefore$ arg (z_1) = arg (z_2)

Trick : $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow z_1, z_2$ lies on same straight line.

$\therefore \arg z_1 = \arg z_2 \Rightarrow \arg z_1 - \arg z_2 = 0$

71. Let $z = x + iy$, then $|z| = r = \sqrt{x^2 + y^2} = 4$

and $\theta = \dfrac{5\pi}{6} = 150^0$

$\therefore x = r\cos\theta = 4\cos 150^\circ = -2\sqrt{3}$

and $y = r\sin\theta = 4\sin 150^\circ = \dfrac{4}{2} = 2$

$\therefore z = x + iy = -2\sqrt{3} + 2i$

Trick: Since, arg z = $\dfrac{5\pi}{6}$ = 150^0, here the complex number must lie in second quadrant, so (A) and (B) are rejected. Also $|z| = 4$ which satisfies (C) only.

73. $z = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2}$

$\therefore |z| = \sqrt{\dfrac{3}{4} + \dfrac{1}{4}} = 1$

and $\arg(z) = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$

$= \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

$\Rightarrow \arg(z) = \tan^{-1}\left(\tan\dfrac{\pi}{6}\right) = \dfrac{\pi}{6}$

74. Here, $-1 + \sqrt{-3} = re^{i\theta}$

$\Rightarrow -1 + i\sqrt{3} = re^{i\theta}$

$= r\cos\theta + ir\sin\theta$

Equating real and imaginary parts, we get

$r\cos\theta = -1$ and $r\sin\theta = \sqrt{3}$

Hence, $\tan\theta = -\sqrt{3}$

$\Rightarrow \tan\theta = \tan\dfrac{2\pi}{3}$

Hence, $\theta = \dfrac{2\pi}{3}$

75. $y = \cos\theta + i\sin\theta = e^{i\theta}$,

then $\dfrac{1}{y} = e^{-i\theta} = \cos\theta - i\sin\theta$

$\therefore y + \dfrac{1}{y} = 2\cos\theta$

76. Let $z = -1 + i\sqrt{3}$, $r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\dfrac{\sqrt{3}}{-1}\right) = \dfrac{2\pi}{3}$

$\therefore z = 2\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

$\therefore (z)^{20} = \left[2\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)\right]^{20}$

$= 2^{20}\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)^{20}$

$= 2^{20}\left(-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right)^{20}$

77. $\dfrac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5} = \dfrac{(\cos\theta + i\sin\theta)^4}{i^5\left(\frac{1}{i}\sin\theta + \cos\theta\right)^5}$

$= \dfrac{(\cos\theta + i\sin\theta)^4}{i(\cos\theta - i\sin\theta)^5} = \dfrac{(\cos\theta + i\sin\theta)^4}{i(\cos\theta + i\sin\theta)^{-5}}$

$= \dfrac{1}{i}(\cos\theta + i\sin\theta)^9 = \sin 9\theta - i\cos 9\theta$

78. Given that $z = \left(\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}\right)^5 + \left(\dfrac{\sqrt{3}}{2} - i\dfrac{1}{2}\right)^5$

$= \left[\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\right)\right]^5 + \left[\cos\left(\dfrac{\pi}{6}\right) - i\sin\left(\dfrac{\pi}{6}\right)\right]^5$

$= \cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6} + \cos\dfrac{5\pi}{6} - i\sin\dfrac{5\pi}{6}$

Hence, Im $(z) = 0$

79. $(-\sqrt{3} + i)^{53} = 2^{53}\left(\dfrac{-\sqrt{3}}{2} + \dfrac{i}{2}\right)^{53}$

$= 2^{53}(\cos 150^\circ + i\sin 150^\circ)^{53}$

$= 2^{53}[\cos(150^\circ \times 53) + i\sin(150^\circ \times 53)]$

$= 2^{53}[\cos(22\pi + 30^\circ) + i\sin(22\pi + 30^\circ)]$

$= 2^{53}[\cos 30^\circ + i\sin 30^\circ]$

$= 2^{53}\left[\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}\right]$

80. $iz^4 = -1 \Rightarrow z^4 = \frac{-1}{i} \Rightarrow z^4 = i \Rightarrow z = (i)^{\frac{1}{4}}$

$\Rightarrow z = (0+i)^{\frac{1}{4}} \Rightarrow z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{\frac{1}{4}}$

$\Rightarrow z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$ (using DeMoivre's theorem)

81. $1 + i\sqrt{3} = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = 2e^{\frac{i\pi}{3}}$

$\therefore$ $(1+i\sqrt{3})^9 = (2e^{i\pi/3})^9 = 2^9.e^{i(3\pi)}$

$= 2^9(\cos 3\pi + i\sin 3\pi)$

$= -2^9$

$\therefore$ $a+ib = (1+i\sqrt{3})^9 = -2^9; \therefore b = 0$

82. $i^{1/3} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/3} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2} + \frac{i}{2}$

83. Amplitude of 0 is not defined.

84. $\frac{1+7i}{(2-i)^2} = \frac{(1+7i)}{(3-4i)}\frac{(3+4i)}{(3+4i)} = \frac{-25+25i}{25} = -1+i$

Let $z = x + iy = -1 + i$

$\therefore$ $r\cos\theta = -1$ and $r\sin\theta = 1$

$\therefore$ $\theta = \frac{3\pi}{4}$ and $r = \sqrt{2}$

Thus, $\frac{1+7i}{(2-i)^2} = \sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$

85. $e^{e^{i\theta}} = e^{\cos\theta + i\sin\theta} = e^{\cos\theta}[e^{i\sin\theta}]$

$= e^{\cos\theta}[\cos(\sin\theta) + i\sin(\sin\theta)]$

$\therefore$ Real part of $e^{e^{i\theta}}$ is $e^{\cos\theta}[\cos(\sin\theta)]$

86. $z = \frac{1+i\sqrt{3}}{\sqrt{3}+i} \Rightarrow z = \frac{1+i\sqrt{3}}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i}$

$z = \frac{\sqrt{3}+3i-i+\sqrt{3}}{3+1} = \frac{2(\sqrt{3}+i)}{4}$

$\Rightarrow$ $z = \frac{\sqrt{3}+i}{2} = \left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$

Now $\bar{z} = \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$

$\Rightarrow$ $(\bar{z})^{100} = \left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]^{100}$

$\Rightarrow$ $(\bar{z})^{100} = \cos\frac{50\pi}{3} - i\sin\frac{50\pi}{3}$

$= \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3} = \frac{-1-i\sqrt{3}}{2}$

$(\bar{z})^{100}$ lies in III[rd] quadrant.

87. $x_1 x_2 x_3 \ldots$ upto ∞

$= \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right) \ldots\ldots \infty$

$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \ldots\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \ldots\right)$

$= \cos\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right) + i\sin\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right)$

$= \cos\pi + i\sin\pi = -1$

88. $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$

$\Rightarrow x = \cos\theta \pm i\sin\theta \Rightarrow x^n = \cos n\theta \pm i\sin n\theta$

$\Rightarrow \frac{1}{x} = \frac{1}{\cos\theta \pm i\sin\theta} \Rightarrow \frac{1}{x} = \cos\theta \mp i\sin\theta$

$\Rightarrow \frac{1}{x^n} = \cos n\theta \mp i\sin n\theta$

Thus, $x^n + \frac{1}{x^n} = 2\cos n\theta$

89. $\arg\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right) = \arg(1-i\sqrt{3}) - \arg(1+i\sqrt{3})$

$= -60° - 60° = -120°$ or $240°$

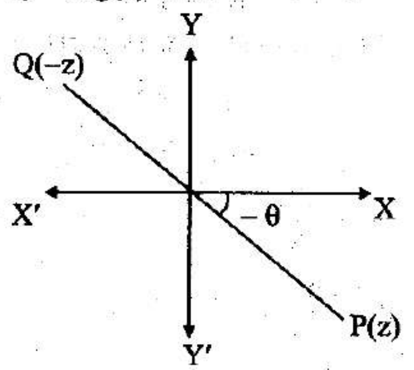
$\left[\begin{array}{l} \because \arg(1-i\sqrt{3}) = -\tan^{-1}\sqrt{3} = -60° \\ \text{and } \arg(1+i\sqrt{3}) = \tan^{-1}\sqrt{3} = 60° \end{array}\right]$

91. Since, $\arg(z) < 0$ i.e., –ve

we choose $\arg(z) = -\theta$, where θ is +ve

$\arg(-z) = \pi - \theta$

$\therefore$ $\arg(-z) - \arg(z) = \pi - \theta + \theta = \pi$



92. $|z-2|+|z+2|=8$

$\Rightarrow \sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=8$

$\sqrt{(x-2)^2+y^2}=8-\sqrt{(x+2)^2+y^2}$

Squaring on both sides, we get

$x^2+y^2+4-4x=64+x^2+y^2+4+4x$
$-16\sqrt{(x+2)^2+y^2}$

$\Rightarrow -8x-64=-16\sqrt{(x+2)^2+y^2}$

$\Rightarrow (x+8)=2\sqrt{(x+2)^2+y^2}$

Again squaring on both sides

$\Rightarrow x^2+64+16x=4(x^2+y^2+4+4x)$

$\Rightarrow 3x^2+4y^2-48=0$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{12}=1$, which is an ellipse.

93. Let $z=x+iy$

$\therefore$ $z+iz=(x-y)+i(x+y)$ and $iz=-y+ix$

If A denotes the area of the triangle formed by

z, z + iz and iz, then $A=\frac{1}{2}\begin{vmatrix} x & y & 1 \\ x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix}$

Applying $R_2 \to R_2-R_1-R_3$, we get

$A=\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{vmatrix}=\frac{1}{2}(x^2+y^2)=\frac{1}{2}|z|^2$

94. $|z-2+i|=|z-3-i|$

$\Rightarrow |(x-2)+i(y+1)|=|(x-3)+i(y-1)|$

$\Rightarrow \sqrt{(x-2)^2+(y+1)^2}=\sqrt{(x-3)^2+(y-1)^2}$

$\Rightarrow x^2+4-4x+y^2+1+2y$
$=x^2+9-6x+y^2+1-2y$

$\Rightarrow 2x+4y-5=0$

95. $\bar{z}+i\bar{\omega}=0$

$\Rightarrow \bar{z}=-i\bar{\omega} \Rightarrow z=i\omega$

$\Rightarrow \omega=\frac{z}{i} \Rightarrow \omega=-iz$

Now, $\arg(z\omega)=\pi$

$\Rightarrow \arg(z(-iz))=\pi \Rightarrow \arg(-iz^2)=\pi$

$\Rightarrow \arg(-i)+2\arg(z)=\pi$

$\Rightarrow -\frac{\pi}{2}+2\arg(z)=\pi \Rightarrow 2\arg(z)=\frac{3\pi}{2}$

$\Rightarrow \arg(z)=\frac{3\pi}{4}$

96. Let $z=x+iy$

We have, $|z^2-1|=|z|^2+1$

$\Rightarrow |(x+iy)^2-1|=|x+iy|^2+1$

$\Rightarrow |(x^2-y^2-1)+2xyi|=\left(\sqrt{x^2+y^2}\right)^2+1$

$\Rightarrow \sqrt{(x^2-y^2-1)^2+(2xy)^2}=x^2+y^2+1$

Squaring on both sides, we get

$x^4+y^4+1-2x^2y^2+2y^2-2x^2+4x^2y^2$
$=x^4+y^4+1+2x^2y^2+2y^2+2x^2$

$\Rightarrow 2x^2y^2=2x^2y^2+4x^2$

$\Rightarrow x=0$

$\therefore$ $z=x+iy=0+iy=iy$

$\therefore$ z lies on imaginary axis.

97. $\omega=\frac{1-iz}{z-i}$, then $|\omega|=1$

$\Rightarrow \left|\frac{1-iz}{z-i}\right|=1$

$\Rightarrow |1-iz|=|z-i|$

$\Rightarrow |1-i(x+iy)|=|x+iy-i|$

$\Rightarrow |(1+y)-ix|=|x+i(y-1)|$

$\Rightarrow \sqrt{x^2+1+y^2+2y}=\sqrt{x^2+y^2+1-2y}$

$\Rightarrow y=0$

$\therefore$ $z=x+iy=x$

$\therefore$ z lies on real axis.

98. Let $z-2-3i=r\text{ cis }\frac{\pi}{4}$

$\Rightarrow x+yi-2-3i=r\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\Rightarrow (x-2)+(y-3)i=\left(r\cos\frac{\pi}{4}\right)+i\left(r\sin\frac{\pi}{4}\right)$

$\Rightarrow x-2=r\cos\frac{\pi}{4}$ and $y-3=r\sin\frac{\pi}{4}$

Dividing, we get $\frac{x-2}{y-3}=\cot\frac{\pi}{4}=1$

$\Rightarrow x-y+1=0$

99. Let $|z|=|\omega|=r$ and $\text{Arg }\omega=\theta$

Then, $\omega=r\text{ cis }\theta$, $\text{Arg }z=\pi-\theta$

$\therefore$ $z=r\text{ cis }(\pi-\theta)$

$=r[\cos(\pi-\theta)+i\sin(\pi-\theta)]$

$=r[-\cos\theta+i\sin\theta]$

$=-r(\cos\theta-i\sin\theta)$

$=-\bar{\omega}$

100. $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$

$\Rightarrow iz = ir(\cos\theta + i\sin\theta) = -r\sin\theta + ir\cos\theta$

$\Rightarrow e^{iz} = e^{(-r\sin\theta + ir\cos\theta)} = e^{-r\sin\theta} e^{ri\cos\theta}$

$\Rightarrow |e^{iz}| = |e^{-r\sin\theta}||e^{ri\cos\theta}|$

$= |e^{-r\sin\theta}|\,|\cos(r\cos\theta) + i\sin(r\cos\theta)|$

$= e^{-r\sin\theta}[\{\cos^2(r\cos\theta) + \sin^2(r\cos\theta)\}]^{\frac{1}{2}}$

$= e^{-r\sin\theta}$ ....[$\because \cos^2\theta + \sin^2\theta = 1$]

101. $\left[\dfrac{1+\cos\theta+i\sin\theta}{i+\sin\theta+i\cos\theta}\right]^4 = \left[\dfrac{(1+\cos\theta)+i\sin\theta}{\sin\theta+i(1+\cos\theta)}\right]^4$

$= \left[\dfrac{2\cos^2\frac{\theta}{2}+i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+i\left(2\cos^2\frac{\theta}{2}\right)}\right]^4$

$= \left[\dfrac{\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}}\right]^4 = \dfrac{1}{i^4}\left[\dfrac{\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}\right]^4$

$= \left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^4 \cdot \left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^{-4}$

$= \left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^4 \cdot \left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^4$

$= \left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^8 = \cos 4\theta + i\sin 4\theta$

Therefore, n = 4

102. Let $z = (1+i\sqrt{3})$

$r = \sqrt{3+1} = 2$ and $r\cos\theta = 1$, $r\sin\theta = \sqrt{3}$

$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$

$z = 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$\Rightarrow z^{100} = \left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^{100}$

$= 2^{100}\left(\cos\frac{100\pi}{3}+i\sin\frac{100\pi}{3}\right)$

$= 2^{100}\left(-\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right) = 2^{100}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)$

$\therefore \dfrac{\text{Re}(z)}{\text{Im}(z)} = \left|\dfrac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}\right| = \dfrac{1}{\sqrt{3}}$

103. $\dfrac{i}{2}-\dfrac{2}{i} = \dfrac{i}{2}-\dfrac{2i}{i^2} = \dfrac{i}{2}+2i = \dfrac{5}{2}i$

So, argument is $\tan^{-1}\left(\dfrac{b}{a}\right) = \tan^{-1}\left(\dfrac{5/2}{0}\right) = \dfrac{\pi}{2}$

104. $\left(\dfrac{3}{2}+i\dfrac{\sqrt{3}}{2}\right)^{50} = \left[\sqrt{3}\left(\dfrac{\sqrt{3}}{2}+i.\dfrac{1}{2}\right)\right]^{50}$

$= 3^{25}\left(\dfrac{\sqrt{3}}{2}+i.\dfrac{1}{2}\right)^{50} = 3^{25}\left(\cos\dfrac{\pi}{6}+i.\sin\dfrac{\pi}{6}\right)^{50}$

$= 3^{25}\left[\cos\left(\dfrac{50\pi}{6}\right)+i.\sin\left(\dfrac{50\pi}{6}\right)\right]$

....[By DeMoivre's theorem]

$\therefore 3^{25}\left[\cos\left(\dfrac{25\pi}{3}\right)+i\sin\left(\dfrac{25\pi}{3}\right)\right] = 3^{25}(x-iy)$

$\Rightarrow x-iy = \cos\left(\dfrac{25\pi}{3}\right)+i\sin\left(\dfrac{25\pi}{3}\right)$

Equating real and imaginary parts, we get

$x = \cos\left(\dfrac{25\pi}{3}\right) = \cos\left(8\pi+\dfrac{\pi}{3}\right)$

$\Rightarrow x = \cos\left(\dfrac{\pi}{3}\right) \Rightarrow x = \dfrac{1}{2}$

and $y = -\sin\left(\dfrac{25\pi}{3}\right) = -\sin\left(8\pi+\dfrac{\pi}{3}\right)$

$\Rightarrow y = -\sin\left(\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$

$\therefore (x, y) = \left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

105. Since, arg (z) = π

$\therefore$ it lies on negative side of X-axis.

Let $z = x$, where $x < 0$

$|z+3-i| = 1$

$\Rightarrow |x+3-i| = 1 \Rightarrow \sqrt{(x+3)^2+1^2} = 1$

$\Rightarrow (x+3)^2+1 = 1 \Rightarrow (x+3)^2 = 0 \Rightarrow x = -3$

$\therefore |z| = 3$

106. $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$\bar{z} = r(\cos\theta - i\sin\theta) = re^{-i\theta}$

$\therefore \dfrac{z}{\bar{z}}+\dfrac{\bar{z}}{z} = \dfrac{re^{i\theta}}{re^{-i\theta}}+\dfrac{re^{-i\theta}}{re^{i\theta}} = e^{2i\theta}+e^{-2i\theta}$

$= (\cos 2\theta + i\sin 2\theta) + (\cos 2\theta - i\sin 2\theta)$

$= 2\cos 2\theta$

107. $z_1z_2=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]\left[\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]$

$=\sqrt{6}e^{i\pi/4}.e^{i\pi/3}=\sqrt{6}e^{i\left(\frac{7\pi}{12}\right)}=\sqrt{6}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$

∴ $|z_1z_2|=\sqrt{6}$

108. $z=\dfrac{i-1}{\cos(\pi/3)+i\sin(\pi/3)}$

$=\dfrac{i-1}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}=\dfrac{2(i-1)}{1+i\sqrt{3}}$

$=\dfrac{2(i-1)}{1+i\sqrt{3}}\times\dfrac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$=\dfrac{2i+2\sqrt{3}-2+2i\sqrt{3}}{1+3}$

$=\dfrac{2\left(-1+i+\sqrt{3}+i\sqrt{3}\right)}{4}$

$=\dfrac{1}{2}\left[\left(\sqrt{3}-1\right)+i\left(\sqrt{3}+1\right)\right]$

∴ $|z|=\sqrt{\frac{1}{4}\left(3+1-2\sqrt{3}+3+1+2\sqrt{3}\right)}=\sqrt{\frac{8}{4}}=\sqrt{2}$

$\theta=\tan^{-1}\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)=\tan^{-1}\left(\dfrac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right)$

$=\tan^{-1}1+\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)=\dfrac{\pi}{4}+\dfrac{\pi}{6}=\dfrac{5\pi}{12}$

∴ the polar form of $z=\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

109. Given, $z=\cos\theta+i\sin\theta=e^{i\theta}$

∴ $\sum_{m=1}^{15}\text{Im}(z^{2m-1})=\sum_{m=1}^{15}\text{Im}(e^{i\theta})^{2m-1}=\sum_{m=1}^{15}\text{Im}\,e^{i(2m-1)\theta}$

$=\sin\theta+\sin3\theta+\sin5\theta+....+\sin29\theta$

$=\dfrac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$

$=\dfrac{\sin(15\theta)\sin(15\theta)}{\sin\theta}$

At $\theta=2°$,

$\sum_{m=1}^{15}\text{Im}(z^{2m-1})=\dfrac{\sin^2 30°}{\sin2°}=\dfrac{1}{4\sin2°}$

110. $\dfrac{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}}{\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)}=\dfrac{e^{i\pi/6}}{e^{i(-\pi/3)}}=e^{i\pi/2}=i$

111. $\dfrac{1}{(\alpha z_1+\beta)-(\alpha z_2+\beta)}+\dfrac{1}{(\alpha z_2+\beta)-(\alpha z_3+\beta)}+\dfrac{1}{(\alpha z_3+\beta)-(\alpha z_1+\beta)}$

$=\dfrac{1}{\alpha(z_1-z_2)}+\dfrac{1}{\alpha(z_2-z_3)}+\dfrac{1}{\alpha(z_3-z_1)}$

$=\dfrac{1}{\alpha}\left[\dfrac{1}{(z_1-z_2)}+\dfrac{1}{(z_2-z_3)}+\dfrac{1}{(z_3-z_1)}\right]=0$

Hence, $\alpha z_1+\beta$, $\alpha z_2+\beta$, $\alpha z_3+\beta$ are vertices of an equilateral triangle.

112. Let $z_1=re^{i\alpha}$, $z_2=r.e^{i\left(\alpha+\frac{2\pi}{3}\right)}$, $z_3=r.e^{i\left(\alpha+\frac{4\pi}{3}\right)}$

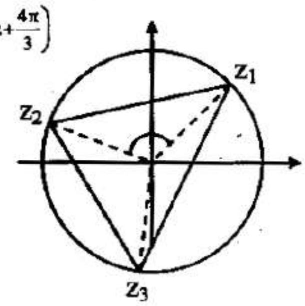
$z_1z_2z_3=r^3e^{i\left(\alpha+\alpha+\frac{2\pi}{3}+\alpha+\frac{4\pi}{3}\right)}$

$=r^3e^{i(3\alpha+2\pi)}$

$=r^3e^{i3\alpha}$

$=(re^{i\alpha})^3$

$=z_1^3$



113. $x^2-\sqrt{3}x+1=0$

$\Rightarrow x=\dfrac{\sqrt{3}\pm\sqrt{3-4}}{2}$

$\Rightarrow x=\dfrac{\sqrt{3}\pm i}{2}=\dfrac{\sqrt{3}}{2}\pm\dfrac{i}{2}$

$\Rightarrow x=\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)$ [Taking +*ve* sign]

114. Since, $(\omega)^2=\omega^2$ and $(\omega^2)^2=\omega^4=\omega\omega^3=\omega$

115. $x^4-1=0$

$\Rightarrow(x^2-1)(x^2+1)=0$

$\Rightarrow x^2=1$ and $x^2=-1$

$\Rightarrow x=\pm1,\pm i$

116. Put $a=1$, $b=1$, $c=-2$, ∵ $a+b+c=0$

∴ $(1+\omega-2\omega^2)^3+(1+\omega^2-2\omega)^3$

$=(-3\omega^2)^3+(-3\omega)^3$

$=-27-27=-54$

Also, option (A) gives the value -54

i.e., $27\times1\times1\times(-2)=-54$

117. Since, α is an imaginary cube root of unity, let it be ω, then

$(\omega)^{3n+1} + (\omega)^{3n+3} + \omega^{3n+5}$

$= \omega + 1 + \omega^5$ ....[$\because \omega^{3n} = 1$ and $\omega^3 = 1$]

$= \omega + 1 + \omega^2 = 0$

118. Since, α and β are complex roots of unity, we may write $\alpha = \omega, \beta = \omega^2$

$\therefore \quad \alpha^4 + \beta^{28} + \dfrac{1}{\alpha\beta} = \omega^4 + (\omega^2)^{28} + \dfrac{1}{\omega.\omega^2}$

$= \omega + \omega^{56} + 1$

$= \omega + \omega^2 + 1$

$= 0$

119. $(1+\omega-\omega^2)^7 = (1+\omega+\omega^2-2\omega^2)^7$

$= (-2\omega^2)^7 = -128\omega^{14}$

$= -128\omega^{12}\omega^2 = -128\omega^2$

120. $(3+\omega+3\omega^2)^4 = (-3\omega+\omega)^4$

....[$\because 1+\omega+\omega^2=0$]

$= (-2\omega)^4$

$= 16\omega^4$

$= 16\omega$

121. $(1-\omega+\omega^2)(1-\omega^2+\omega)^6 = (-2\omega)(-2\omega^2)^6$

$= -128\,\omega$

122. $1.\omega.\omega^2 = \omega^3 = 1$

123. $z + z^{-1} = 1 \Rightarrow z^2 - z + 1 = 0$

$\Rightarrow z = -\omega$ or $-\omega^2$

For $z = -\omega$, $z^{100} + z^{-100} = (-\omega)^{100} + (-\omega)^{-100}$

$= \omega + \dfrac{1}{\omega}$

$= \dfrac{\omega^2+1}{\omega} = -1$

For $z = -\omega^2$, $z^{100} + z^{-100} = (-\omega^2)^{100} + (-\omega^2)^{-100}$

$= \omega^{200} + \dfrac{1}{\omega^{200}}$

$= \omega^2 + \dfrac{1}{\omega^2}$

$= \omega^2 + \omega = -1$

124. $[(1+\omega+\omega^2)+\omega]^{3n} - [(1+\omega+\omega^2)+\omega^2]^{3n}$

$= \omega^{3n} - (\omega^2)^{3n}$

$= (\omega^3)^n - (\omega^3)^{2n}$

$= 1^n - 1^{2n}$

$= 0$

125. $(a+b)(a+b\omega)(a+b\omega^2)$

$= (a+b)(a^2 + ab(\omega+\omega^2) + b^2\omega^3)$

$= (a+b)(a^2 - ab + b^2) = a^3 + b^3$

126. Let $(8)^{1/3} = x \Rightarrow x^3 - 8 = 0$

$\Rightarrow (x-2)(x^2+2x+4) = 0$

$\Rightarrow x = 2, 2\omega, 2\omega^2$ or $x = 2, -1+i\sqrt{3}, -1-i\sqrt{3}$

127. $\Delta = (\omega^{3n}-1) - \omega^n(\omega^{2n}-\omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$= (1-1) + 0 + \omega^{2n}[\omega^n - (\omega^3)^n\omega^n]$

$= 0 + 0 + 0 = 0$

128. $(3+\omega+3\omega^2)^4 = [3(1+\omega^2)+\omega]^4$

$= [3(-\omega)+\omega]^4 = [-2\omega]^4$

$= 16\omega^4 = 16\omega$

129. $x + \dfrac{1}{x} = 2\cos\theta \Rightarrow x^2 - 2x\cos\theta + 1 = 0$

$\Rightarrow x = \dfrac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$\Rightarrow x = \cos\theta \pm i\sin\theta$

130. $\sin\left[(\omega^{10}+\omega^{23})\pi - \dfrac{\pi}{4}\right] = \sin\left[(\omega+\omega^2)\pi - \dfrac{\pi}{4}\right]$

$= \sin\left(-\pi - \dfrac{\pi}{4}\right) = -\sin\left(\pi + \dfrac{\pi}{4}\right) = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

131. $\left(\dfrac{\sqrt{3}+i}{2}\right)^6 + \left(\dfrac{i-\sqrt{3}}{2}\right)^6$

$= \left(\dfrac{-1+\sqrt{3}i}{2i}\right)^6 + \left(\dfrac{-1-\sqrt{3}i}{2i}\right)^6$

$= \dfrac{1}{i^6}[(\omega)^6 + (\omega^2)^6]$

....$\left[\because \omega = \dfrac{-1+\sqrt{3}i}{2}, \omega^2 = \dfrac{-1-\sqrt{3}i}{2}\right]$

$= -[(\omega^3)^2 + (\omega^3)^4] = -(1+1) = -2$

132. $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+.....\right)} = \omega + \omega^{\left(\frac{1/2}{1-3/4}\right)}$

$= \omega + \omega^2 = -1$ ....[$\because 1+\omega+\omega^2 = 0$]

133. Let $n = 3k+1$

$\omega^n + \omega^{2n} = \omega^{3k+1} + \omega^{2(3k+1)}$

$= \omega^{3k}\omega + \omega^{6k}\omega^2$

$= (\omega^3)^k.\,\omega + (\omega^3)^{2k}.\,\omega^2$

$= \omega + \omega^2 = -1$ ....[$\because \omega^3 = 1$]

Hence, $1 + \omega^n + \omega^{2n} = 1 - 1 = 0$

134. $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$....upto 2n factors

$= (-\omega^2)(-\omega)(1+\omega)(1+\omega^2)$ .... upto 2n factors

$= 1.1.1$.... upto n factors $= 1$

135. Since, $p < 0$. Let $p = -q$, where q is positive.

Therefore $p^{\frac{1}{3}} = -q^{\frac{1}{3}}(1)^{\frac{1}{3}}$

Hence $\alpha = -q^{\frac{1}{3}}$, $\beta = -q^{\frac{1}{3}}\omega$ and $\gamma = -q^{\frac{1}{3}}\omega^2$

The given expression $= \dfrac{x + y\omega + z\omega^2}{x\omega + y\omega^2 + z}$

$$= \frac{1}{\omega}\cdot\frac{x\omega + y\omega^2 + z}{x\omega + y\omega^2 + z}$$

$$= \frac{1}{\omega}.1 = \frac{1}{\omega}.\omega^3$$

$$= \omega^2$$

$$= \frac{1}{2}(-1 - i\sqrt{3})$$

136. $(-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2)$....to 2n factors

$= (2^2\omega^3)(2^2\omega^3)$.... to n factors $= (2^2)^n = 2^{2n}$

137. $\dfrac{1+i\sqrt3}{1-i\sqrt3} = \left(\dfrac{1+i\sqrt3}{1-i\sqrt3}\right)\left(\dfrac{1+i\sqrt3}{1+i\sqrt3}\right)$

$$= \frac{-2 + i2\sqrt3}{4}$$

$$= \frac{-1 + i\sqrt3}{2} = \omega$$

$\therefore$ $\left(\dfrac{1+i\sqrt3}{1-i\sqrt3}\right)^n = \omega^n = \omega^3 = 1 \Rightarrow n = 3$

138. We have,

$(1+\omega^2)^m = (1+\omega^4)^m$ ....[$\because \omega^3 = 1$]

$\Rightarrow (1+\omega^2)^m = (1+\omega)^m$

$\Rightarrow (-\omega)^m = (-\omega^2)^m$

$\Rightarrow \left(\dfrac{\omega}{\omega^2}\right)^m = 1$

$\Rightarrow (\omega^2)^m = 1 = (\omega^3)$

Hence, least positive integral value of m is 3.

139. Here, $1^{\frac{1}{3}} = 1, \omega, \omega^2$

$\therefore$ For the equation $(x-2)^3 + 27 = 0$

$\Rightarrow (x-2)^3 = -27 = -3^3$

$\Rightarrow x - 2 = -3(1)^{\frac{1}{3}} = -3(1, \omega, \omega^2)$

$= -3, -3\omega, 3\omega^2$

$\Rightarrow x = -1, 2 - 3\omega, 2 - 3\omega^2$

140. The given expression

$$= \left[\frac{\frac{1}{2} + \frac{\sqrt3}{2}i}{\frac{1}{2} - \frac{\sqrt3}{2}i}\right]^6 + \left[\frac{\frac{1}{2} - \frac{\sqrt3}{2}i}{\frac{1}{2} + \frac{\sqrt3}{2}i}\right]^6$$

$$= \left[\frac{-\left(\frac{-1-\sqrt3 i}{2}\right)}{-\left(\frac{-1+\sqrt3 i}{2}\right)}\right]^6 + \left[\frac{-\left(\frac{-1+\sqrt3 i}{2}\right)}{-\left(\frac{-1-\sqrt3 i}{2}\right)}\right]^6$$

$$= \left(\frac{\omega^2}{\omega}\right)^6 + \left(\frac{\omega}{\omega^2}\right)^6$$

$$= \omega^6 + \frac{1}{\omega^6}$$

$= 1 + 1$ ....[$\because \omega^3 = 1$]

$= 2$

141. Given, $x = \dfrac{-1+\sqrt3 i}{2} \Rightarrow x = \omega$

$\therefore$ $(1 - x^2 + x)^6 - (1 - x + x^2)^6$

$= (1 - \omega^2 + \omega)^6 - (1 - \omega + \omega^2)^6$

$= ((1+\omega) - \omega^2)^6 - ((1+\omega^2) - \omega)^6$

$= (-\omega^2 - \omega^2)^6 - (-\omega - \omega)^6$

$= (-2\omega^2)^6 - (-2\omega)^6$

$= 64\omega^{12} - 64\omega^6$

$= 64(\omega^3)^4 - 64(\omega^3)^2 = 0$ ....[$\because \omega^3 = 1$]

142. If ω is an imaginary (non-real) n^{th} root of unity, then all the nth roots are

$1, \omega, \omega^2, \ldots, \omega^{n-1}$

$\therefore$ $x^n - 1 = (x-1)(x-\omega)(x-\omega^2)\ldots(x-\omega^{n-1})$

Substituting $x = 5$, we get

$5^n - 1 = (5-1)(5-\omega)(5-\omega^2)(5-\omega^2)$ ....$(5-\omega^{n-1})$

$\therefore$ $\dfrac{5^n - 1}{4} = (5-\omega)(5-\omega^2)\ldots\ldots(5-\omega^{n-1})$

143. The product is given by

$\omega.\omega^2.\omega^3\ldots\omega^n = \omega^{1+2+3+\ldots+n} = \omega^{\frac{n(n+1)}{2}}$

On putting $n = 1, 2, 3, \ldots$, we get

$\omega^{\frac{1(1+1)}{2}} = \omega$, $\omega^{\frac{2(2+1)}{2}} = \omega^3 = 1, \ldots, \omega^{\frac{4(5)}{2}} = \omega^{10}$

$= \omega$

Hence, it gives the values 1 and ω only.

144. $2^{15}\left[\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{15}}{(1-i)^{20}}+\frac{\left(\frac{-1}{2}-\frac{i\sqrt{3}}{2}\right)^{15}}{(1+i)^{20}}\right]$

$= 2^{15}\left[\frac{\omega^{15}}{(1-i)^{20}}+\frac{\omega^{30}}{(1+i)^{20}}\right]$

$=2^{15}\left[\frac{1}{(1-i)^{20}}+\frac{1}{(1+i)^{20}}\right]$

$= 2^{15}\left[\frac{(1+i)^{20}+(1-i)^{20}}{(1-i^2)^{20}}\right]$

$=\frac{2^{15}}{2^{20}}[(1+i)^{20}+(1-i)^{20}]$

$= \frac{1}{2^5}[\{(1+i)^2\}^{10}+\{(1-i)^2\}^{10}]$

$= \frac{1}{2^5}[(2i)^{10}+(-2i)^{10}]$

$= \frac{2^{11}.i^{10}}{2^5}$

$=-2^6$ ....$[\because i^{10}=(i^4)^2.\ i^2=i^2=-1]$

$=-64$

145. $\omega_n = \cos\left(\frac{2\pi}{n}\right)+i\sin\left(\frac{2\pi}{n}\right)$

$\Rightarrow \omega_3 = \cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3} = -\frac{1}{2}+\frac{i\sqrt{3}}{2} = \omega$

and $\omega_3^2 = \left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)^2$

$= \left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

$= -\frac{1}{2}-\frac{i\sqrt{3}}{2} = \omega^2$

$\therefore$ $(x+y\omega_3+z\omega_3^2)(x+y\omega_3^2+z\omega_3)$

$=(x+y\omega+z\omega^2)(x+y\omega^2+z\omega)$

$=x^2+y^2+z^2-xy-yz-zx$

146. $z^2+z+1=0 \Rightarrow z=\omega$ or ω^2

Let $z=\omega$

$\therefore$ $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+....+\left(z^6+\frac{1}{z^6}\right)^2$

$=\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+....+\left(\omega^6+\frac{1}{\omega^6}\right)^2$

$=1+1+4+1+1+4=12$

147. $(1+\omega)^7 = A+B\omega \Rightarrow (-\omega^2)^7 = A+B\omega$

$\Rightarrow \omega^{14} = -A-B\omega$

$\Rightarrow \omega^2.\omega^{12} = -A-B\omega \Rightarrow A+B\omega+\omega^2=0$

$\Rightarrow A=1, B=1$ ....$[\because 1+\omega+\omega^2=0]$

148. Since, $\omega = -\frac{1}{2}+\frac{1}{2}i\sqrt{3}$

$\therefore$ $\omega^{1000} = \omega^{999}\omega = (\omega^3)^{333}\omega = \omega = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

149. Roots of the equation $x^2-x+1=0$ are

$\alpha=-\omega, \beta=-\omega^2$

$\therefore$ $\alpha^{2009}+\beta^{2009} = (-\omega)^{2009}+(-\omega^2)^{2009}$

$=-(\omega^2+\omega)=1$

150. $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^3+\omega^6)$

$(1-\omega^4+\omega^8)(1-\omega^5+\omega^{10})(1-\omega^6+\omega^{12})$

$(1-\omega^7+\omega^{14})(1-\omega^8+\omega^{16})$

$=(1-\omega+\omega^2)(1-\omega^2+\omega)(1)(1-\omega+\omega^2)$

$(1-\omega^2+\omega)(1)(1-\omega+\omega^2)(1-\omega^2+\omega)$

$=(1-\omega+\omega^2)^3(1-\omega^2+\omega)^3 = (-2\omega)^3(-2\omega^2)^3$

$=(2^3\omega^6)(2^3\omega^3)=2^6$

151. $1+z+z^3+z^4=0$

$\Rightarrow (1+z)(1+z^3)=0$

$\Rightarrow z=-1,-1,-\omega,-\omega^2$, where ω is a cube root of unity.

$\therefore$ the distinct roots are $(-1,0)$, $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$, $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$.

Distance between each of them is $\sqrt{3}$. So, they form an equilateral triangle.

152. $a=\omega^2, \beta=\omega \Rightarrow \frac{\alpha}{\beta}=\omega$

$S = \sum_{n=0}^{302}(-1)^n(\omega)^n$

$=\omega^0-\omega^1+\omega^2-\omega^3+\omega^4-....+\omega^{302}$

$=\frac{1-(-\omega)^{303}}{1-(-\omega)} = \frac{2}{-\omega^2} = -2\omega$

$\alpha=\omega, \beta=\omega^2 \Rightarrow \frac{\alpha}{\beta}=\frac{1}{\omega}=\omega^2$

$S=(\omega^2)^0-(\omega^2)^1+(\omega^2)^2-....+(\omega^2)^{302}$

$=\frac{1-(-\omega^2)^{303}}{1-(-\omega^2)} = \frac{2}{-\omega} = -2\omega^2$