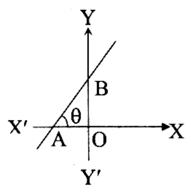
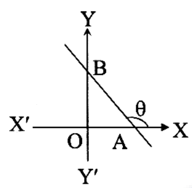
6 Straight Line

Formulae

1. Slope (Gradient) of a line:

The trigonometrical tangent of the angle that a line makes with the positive direction of the X-axis in anticlockwise sense is called the slope or gradient of the line. The slope of a line is generally denoted by m. Thus, $m = \tan\theta$.





i. Slope of line parallel to X - axis is

ii. Slope of line parallel to Y - axis is $m = \tan 90^0 = \infty$.

iii. Slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$

iv. Slope of the line $ax + by + c = 0$, $b \neq 0$ is a $-\frac{a}{b}$

v. If m_1 and m_2 be the slopes of two perpendicular lines, then $m_1 m_2 = -1$.

2. Equation of straight line in standard forms:

i. Slope intercept form: The equation of a line with slope 'm' and making an intercept on Y – axis is $y = mx + c$

ii. Slope - point form: The equation of a line which passes through the point (x_1, y_1) and has slope 'm' is $y - y_1 = m(x - x_1)$

iii. Two point form: The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is $\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$

iv. Double - intercept form: The equation of a line which cuts off intercepts a and b from the X and Y – axis is $\frac{x}{a} + \frac{y}{b} = 1$

v. Normal form: The equation of a straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle a with X – axis is $x \cos\alpha + y \sin\alpha = p$

vi. Parametric form: The equation of the straight line passing through (x_1, y_1) and making an angle θ with the positive direction of X – axis is $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$, where r is the distance of the point (x, y) on the line from the point (x_1, y_1).

3. General equation of a straight line and its transformation in standard forms:

General form of equation of a line is $ax + by + c = 0$, its

i. Slope intercept form: $y = -\frac{a}{b}x - \frac{c}{b}$, slope $m = -\frac{a}{b}$ and intercept on Y-axis is $C = -\frac{c}{b}$

ii. Intercept form : $\frac{x}{-c/a} + \frac{y}{-c/b} = 1$,

x intercept $= \left(-\frac{c}{a}\right)$ and

y intercept $= \left(-\frac{c}{b}\right)$

iii. Normal form : To change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation

by $\sqrt{a^2 + b^2}$ like

$$-\frac{ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}},$$

where $\cos\alpha = -\frac{a}{\sqrt{a^2+b^2}}$,

$\sin\alpha = -\frac{b}{\sqrt{a^2+b^2}}$, $p = \frac{c}{\sqrt{a^2+b^2}}$

4. Two intersecting lines:

i. Angle between two intersecting lines:

a. If θ is the acute angle between the lines with slopes mi and m_2, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

b. If θ is the acute angle between the lines $a_1x + b_1y + c_1 = 0$ and $+ c_2 = 0$, and $a_2x + b_2y + c_2 = 0$, then

$$\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$$

ii. Point of intersection of two lines:

Point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is

$$(x', y') = \left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

5. Concurrent lines:

Three or more lines are said to be concurrent if they meet at a point.

i. If the three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent, then $a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2) = 0$

6. Length of perpendicular:

i. Distance of a point from a line :

The length p of the perpendicular from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by $p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

Length of perpendicular from origin to the line $ax + by + c = 0$ is $\left|\frac{c}{\sqrt{a^2 + b^2}}\right|$

ii. Distance between two parallel lines :

The distance between two parallel lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

Shortcuts

1. Slope of the line equally inclined with the axis is 1 or – 1
2. Slope of two parallel lines are equal.
3. Equation of a line parallel to $ax + by + c = 0$ is $ax + by + \lambda = 0$, where λ, is a constant.
4. Equation of a line perpendicular to $ax + by + c = 0$ is $bx - ay + \lambda, = 0$, where λ, is a constant.
5. If the equation of line be $a \sin\theta + b\cos\theta = c$, then line
   i. Parallel to it is $a \sin\theta + b\cos\theta = d$
   ii. Perpendicular to it is a

   $$\sin\left(\frac{\pi}{2} + \theta\right) + b\cos\left(\frac{\pi}{2} + \theta\right) = d$$

MULTIPLE CHOICE QUESTIONS

Classical Thinking

6.1 Slope of a line, Equation of a line in different forms

1. Slope of a line which cuts intercepts of equal lengths on the axes is

a) – 1 b) 0

c) 2 d) $\sqrt{3}$

2. The equation of the line whose slope is 3 and which cuts off an intercept 3 from the positive X - axis is

A) y = 3x – 9 b) y = 3x + 3

c) y = 3x + 9 d) None of these

3. If the co-ordinates of A and B are (1, 1) and (5, 7), then the equation of the perpendicular bisector of the line segment AB is

a) 2x + 3y = 18 b) 2x – 3y + 18 = 0

c) 2x + 3y – 1 = 0 d) 3x – 2y + 1 = 0

4. The equation of the line passing through the point (x', y') and perpendicular to the line

a) xy' + 2ay + 2ay' – x'y' = 0

b) xy' + 2ay – 2ay' – x'y' = 0

c) xy' + 2ay + 2ay' + x'y' = 0

d) xy' + 2ay – 2ay' + x'y' = 0

5. The equation of the line bisecting the line segment joining the points (a, b) and (a', b') at right angle, is

a) $2(a - a')x + 2(b - b')y = a^2 + b^2 - a'^2 - b'^2$

b) $(a - a')x + (b - b')y = a^2 + b^2 - a'^2 - b'^2$

c) $2(a - b)x + 2(b - b')y = a'^2 + b'^2 - a'^2 - b'^2$

d) None of these

6. The equation of a line joining the origin to the point (– 4, 5) is

a) 5x + 4y = 0 b) 3x + 4y = 2

c) 5x – 4y = 0 d) 4x – 5y = 0

7. The equation of the line which cuts off an intercept 3 units on OX and an intercept – 2 units on OY is

a) $\frac{x}{3} - \frac{y}{2} = 1$ b) $\frac{x}{3} + \frac{y}{2} = 1$

c) $\frac{x}{2} + \frac{y}{3} = 1$ d) $\frac{x}{2} - \frac{y}{3} = 1$

8. The equation of a line through (3, – 4) and perpendicular to the line 3x + 4y = 5 is

a) 4x + 3y = 24 b) y – 4 = x + 3

c) 3y – 4x = 24 d) $y + 4 = \frac{4}{3}(x - 3)$

9. Equation of a line passing through (1, 2) and parallel to the line y = 3x – 1 is

a) y + 2 = x + 1 b) y + 2 = 3 (x + 1)

c) y – 2 = 3 (x – 1) d) – y – 2 = x – 1

10. Equation of a line through the origin and perpendicular to the line joining (a, 0) and (– a, 0) is

a) y = 0 b) x = 0

c) x = – a d) y = – a

11. The equation of a line which bisects the line joining two points (2, – 19) and (6, 1) and perpendicular to the line joining two points (– 1, 3) and (5, – 1), is

a) 3x – 2y = 30 b) 2x – y – 3 = 0

c) 2x + 3y = 20 d) None of these

12. The equation of line whose midpoint $(x_1\ y_1)$ in between the axes, is

a) $\frac{x}{x_1} + \frac{y}{y_1} = 2$ b) $\frac{x}{x_1} + \frac{y}{y_1} = \frac{1}{2}$

c) $\frac{x}{x_1} + \frac{y}{y_1} = 1$ d) None of these

13. The equation of a line passing through (c, d) and parallel to ax + by + c = 0 is

a) a(x + c) + b(y + d) = 0

b) a(x + c) – b(y + d) = 0

c) a(x – c) + b(y – d) = 0

d) a(x – c) – b(y – d) = 0

14. The equation of a line passing through (4, – 6) and making an angle 45° with positive X-axis, is

a) x – y – 10 = 0 b) x – 2y – 16 = 0

c) x – 3y – 22 = 0 d) x – 2y – 10 = 0

15. The equation of a straight line passing through the points (– 5, – 6) and (3, 10) is

a) x – 2y = 4 b) 2x – y + 4 = 0

c) 2x + y = 4 d) x – 2y + 4 = 0

16. The equation of the line which cuts off intercepts $2a \sec\theta$ and $2a \operatorname{cosec}\theta$ on X-axis and Y-axis respectively is

a) $x \sin\theta + y \cos\theta - 2a = 0$

b) $x \cos\theta + y \sin\theta - 2a = 0$

c) $x \sec\theta + y \operatorname{cosec}\theta - 2a = 0$

d) $x \operatorname{cosec}\theta + y \sec\theta - 2a = 0$

17. A straight line makes an angle of 135° with the X-axis and cuts Y-axis at a distance – 5 from the origin. The equation of the line is

a) $2x + y + 5 = 0$ b) $x + 2y + 3 = 0$

c) $x + y + 5 = 0$ d) $x + y + 3 = 0$

18. The equation of line perpendicular to $x + c$ is

a) $y = d$ b) $x = d$

c) $x = 0$ d) None of these

19. If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points a $(2, -3)$ and $(4, -5)$, then $(a, b) =$

a) $(1, 1)$ b) $(-1, 1)$

c) $(1, -1)$ d) $(-1, -1)$

20. The equation of a line perpendicular to line $ax + by + c = 0$ and passing through (a, b) is equal to

a) $bx - ay = 0$ b) $bx + ay - 2ab = 0$

c) $bx + ay = 0$ d) $bx - ay + 2ab = 0$

21. A line passes through $(2, 2)$ and is perpendicular to the line $3x + y = 3$. Its y – intercept is

a) $\frac{1}{3}$ b) $\frac{2}{3}$

c) 1 d) $\frac{4}{3}$

22. The number of straight lines which are equally inclined to both the axes is

a) 4 b) 2

c) 3 d) 1

23. Let P(–l, 0), Q(0, 0) and R$(3, 3\sqrt{3})$ be three points. Then, the equation of the bisector of the angle PQR is

a) $\frac{\sqrt{3}}{2}x + y = 0$ b) $x + \sqrt{3}y = 0$

c) $\sqrt{3}x + y = 0$ d) $x + \frac{\sqrt{3}}{2}y = 0$

24. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other, if

a) $a_1b_2 - b_1a_2 = 0$

b) $a_1a_2 + b_1b_2 = 0$

c) $a_1^2b_2 + b_1^2a_2 = 0$

d) $a_1b_1 + a_2b_2 = 0$

25. The line passing through (1, 0) and $(-2, \sqrt{3})$ makes an angle of _____ with X-axis.

a) 60^0 b) 120^0

c) 150^0 d) 135^0

6.2 Two intersecting lines and family of lines

26. The equation of a line through the intersection of lines $x - 0$ and $y = 0$ and through the point (2, 2) is

a) $y = x - 1$ b) $y = -x$

c) $y = x$ d) $y = -x + 2$

27. For the lines $2x + 5y = 1$ and $2x - 5y = 9$, which of the following statement is true?

a) Lines are parallel

b) Lines are coincident

c) Lines are intersecting

d) Lines are perpendicular

28. The acute angle between the lines $y = 3$ and $y = \sqrt{3}x + 9$ is

a) 30^0 b) 60^0

c) 45^0 d) 90^0

29. The angle between the lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$ is

a) 30^0 b) 60^0

c) 45^0 d) 90^0

30. Equations of the two straight lines passing through the point (3, 2) and making an angle of 45^0 with the line $x - 2y = 3$, are

a) $3x + y + 1 = 0$ and $x + 3y + 9 = 0$

b) $3x - y - 7 = 0$ and $x + 3y - 9 = 0$

c) $x + 3y - 1 = 0$ and $x + 3y - 9 = 0$

d) None of these

31. The value of k for which the lines $7x - 8x + 5 = 0$, $3x - 4y + 5 = 0$ and $4x + 5y + k = 0$ are concurrent is given by

a) – 45 b) 44

c) 54 d) – 54

32. The lines $15x - 18y + 1 = 0$, $12x + 10y - 3 = 0$ and $6x + 66y - 11 = 0$ are

a) Parallel b) Perpendicular

c) Concurrent d) None of these

33. If $u = a_1x + b_1y + c_1 = 0$, $v = a_2x + b_2y + c_2 = 0$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the curve $u + kv = 0$ is

a) same straight line u

b) different straight line

c) not a straight line

d) None of these

34. The angle between the lines whose intercepts on the axes are $(a, -b)$ and $(b, -a)$ respectively, is

a) $\tan^{-1}\frac{a^2+b^2}{ab}$ b) $\tan^{-1}\frac{b^2-a^2}{2}$

c) $\tan^{-1}\frac{b^2-a^2}{2ab}$ d) None of these

35. The angle between the lines $x\cos 30^0 + y\sin 30^0 = 3$ and $x\cos 60^0 + y\sin 60^0 = 5$ is

a) 90^0 b) 30^0

c) 60^0 d) 45^0

36. If the lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$, are concurrent, then

a) $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

B) $m_1(c_2 - c_1) + m_2(c_3 - c_2) + m_3(c_1 - c_3) = 0$

c) $c_1(m_2 - m_3) + c_2(m_3 - m_1) + c_3(m_1 - m_2) = 0$

d) $C_1(m_1 - m_2) + c_2(m_2 - m_3) + C_3(m_3 - m_1) = 0$

37. The angle between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, is

a) $\tan^{-1}\frac{a_1b_2+a_2b_1}{a_1a_2-b_1b_2}$ b) $\cot^{-1}\frac{a_1b_2+b_1b_2}{a_1b_2-a_2b_1}$

c) $\cot^{-1}\frac{a_1b_2-a_2b_2}{a_1a_2+b_1b_2}$ d) $\tan^{-1}\frac{a_1b_1-a_2b_2}{a_1a_2+b_1b_2}$

6.3 Distance of a point from a line

38. If the length of the perpendicular drawn from the origin to the line whose intercepts on the axes are a and b be p, then

a) $a^2 + b^2 = p^2$ b) $a^2 + b^2 = \frac{1}{p^2}$

c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{p^2}$ d) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

39. The length of the perpendicular from the point (b, a) to the line $\frac{x}{a} - \frac{y}{b} = 1$ is

a) $\left|\frac{a^2-ab+b^2}{\sqrt{a^2+b^2}}\right|$ b) $\left|\frac{b^2-ab+a^2}{\sqrt{a^2+b^2}}\right|$

c) $\left|\frac{a^2+ab-b^2}{\sqrt{a^2+b^2}}\right|$ d) $\left|\frac{a^2+ab+b^2}{\sqrt{a^2+b^2}}\right|$

40. The length of perpendicular drawn from origin on the line joining (x', y') and (x'', y'') is

a) $\left|\frac{x'y''+x''y'}{\sqrt{(x''-x')^2+(y''-y')^2}}\right|$

b) $\left|\frac{x'y''-x''y'}{\sqrt{(x''-x')^2+(y''-y')^2}}\right|$

c) $\left|\frac{x'y''+y'y''}{\sqrt{(x''+x')^2+(y''+y')^2}}\right|$

d) $\left|\frac{x'x''+y'y''}{\sqrt{(x''+x')^2+(y''-y')^2}}\right|$

41. The perpendicular distance of the straight line $12x + 5y = 7$ from the origin is

a) $\frac{7}{13}$ b) $\frac{12}{13}$

c) $\frac{5}{13}$ d) $\frac{1}{13}$

42. The length of perpendicular from (3, 1) on line $4x + 3y + 20 = 0$, is

a) 6 b) 7

c) 5 d) 8

43. The distance of the point $(-2, 3)$ from the line $x - y = 5$ is

a) $5\sqrt{2}$ b) $2\sqrt{5}$

c) $3\sqrt{5}$ d) $5\sqrt{3}$

44. Distance between the lines $5x + 3y - 7 = 0$ and $15x + 9y + 14 = 0$ is

a) $\frac{35}{\sqrt{34}}$ b) $\frac{1}{3\sqrt{34}}$

c) $\frac{35}{3\sqrt{34}}$ d) $\frac{35}{2\sqrt{34}}$

45. The position of the point (8, – 9) with respect to the lines 2x + 3y – 4 = 6 and 6x + 9y + 8 = 0 is

a) Point lies on the same side of the lines

b) Point lies on the different sides of the line

c) Point lies on one of the line

d) None of these

46. The length of perpendicular from the point $(a \cos \alpha, a \sin \alpha)$ upon the straight line $y = x \tan \alpha + c, c > 0$ is

a) $c \cos \alpha$ b) $c \sin^2 \alpha$

c) $c \sec^2 \alpha$ d) $c \cos^2 \alpha$

47. The equations $(b - c)x + (c - a) y + (a - b) = 0$ and $(b^3 - c^3)x + (c^3 - a^3) y + a^3 - b^3 = 0$ will represent the same line, if

a) b = c b) c = a

c) a = b d) All the above

Critical Thinking

6.1 Slope of a line, Equation of a line in different forms

1. Equation of the hour hand at 4 O' clock is

a) $x - \sqrt{3}y = 0 =$ b) $\sqrt{3}x - y = 0$

c) $x + \sqrt{3}y = 0$ d) $\sqrt{3}x + y = 0$

2. A straight line through origin bisects the line passing through the given points $(a \cos\alpha, a \sin\alpha)$ and $(a \cos\beta, a \sin\alpha)$, then the lines are

a) Perpendicular

b) Parallel

c) Angle between them is $\frac{\pi}{4}$

d) None of these

3. A line L is perpendicular to the line 5x – y = 1 and the area of the triangle formed by the line L and coordinate axes is 5. The equation of the line L is

a) x + 5y = 5 b) $x + 5y = \pm 5\sqrt{2}$

c) x - 5y = 5 d) $x - 5y = 5\sqrt{2}$

4. A line passes through the point (3, 4) and cuts off intercepts from the co-ordinates axes such that their sum is 14. The equation of the line is

a) 4x – 3y = 24 b) 4x + 3y = 24

c) 3x – 4y = 24 d) 3x + 4y = 24

5. The equation of the line parallel to the line 2x – 3y = 1 and passing through the middle point of the line segment joining the points (1, 3) and (1, – 7), is

a) 2x – 3y + 8 = 0

b) 2x – 3y = 8

c) 2x – 3y + 4 = 0

d) 2x – 3y = 4

6. The intercept cut off from Y-axis is twice that from X-axis by the line and line passes through (1, 2), then its equation is

a) 2x + y = 4 b) 2x + y + 4 = 0

c) 2x – y = 4 d) 2x – y + 4 = 0

7. The equation of the straight line passing through the point $(a \cos^3 \theta, a \sin^3 \theta)$ and .perpendicular to the line $x \sec\theta + y \operatorname{cosec}\theta = a$ is

a) $x \cos \theta - y \sin 6 = a \cos 2\theta$

b) $x \cos \theta + y \sin \theta = a \cos 2\theta$

c) $x \sin \theta + y \cos 0 = 3 \cos 2\theta$

d) None of these

8. Equation of the line passing through the point (– 4, 3) and the portion of the line intercepted between the axes which is divided internally in the ratio 5 : 3 by this point, is

a) 9x + 20y + 96 = 0

b) 20x + 9y + 96 = 0

c) 9x – 20y + 96 = 0

d) None of these

9. A straight line through P (l, 2) is such that its intercept between the axes is bisected at P. Its equation is

a) x + 2y = 5

b) x – y + 1 = 0

c) x + y – 3 = 0

d) 2x + y – 4 = 0

10. The point P(a, b) lies on the straight line 3x + 2y = 13 and the point Q (b, a) lies on the straight line 4x – y = 5, then the equation of line PQ is

a) x – y = 5 b) x + y = 5

c) x + y = – 5 d) x – y = – 5

11. A line AB makes zero intercepts on X - axis and Y - axis and it is perpendicular to another line CD, 3x + 4y + 6 = 0. The equation of line AB is

a) y = 4 b) 4x – 3y + 8 = 0

c) 4x – 3y = 0 d) 4x – 3y + 6 = 0

12. The line passing through $(-1, \pi/2)$ and perpendicular to $\sqrt{3}\sin\theta + 2\cos\theta = \frac{4}{r}$ is

a) $2 = \sqrt{3}\, r\cos\theta - 2r\sin\theta$

b) $5 = -2\sqrt{3}\, r\cos\theta + 4r\sin\theta$

c) $2 = \sqrt{3}\, r\cos\theta + 2r\sin\theta$

d) $5 = 2\sqrt{3}\, r\cos\theta + 4r\sin\theta$

13. The opposite vertices of a square are (1, 2) and (3, 8), then the equation of a diagonal of the square passing through the point (1, 2), is

a) $3x - y - 1 = 0$ b) $3y - x - 1 = 0$

c) $3x + y + 1 = 0$ d) None of these

14. The ends of the base of an isosceles triangle are at (2a, 0) and(0, a). The equation of one side is x = 2a. The equation of the other side is

a) $x + 2y - a = 0$ b) $x + 2y = 2a$

c) $3x + 4y - 4a = 0$ d) $3x - 4y + 4a = 0$

15. The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the value of c will be

a) 4   b) – 4

c) 2   d) – 2

16. The equation of the lines on which the perpendiculars from the origin make 30^0 angle with X-axis and which form a triangle of area $\frac{50}{\sqrt{3}}$ with axes, are

a) $x + \sqrt{3}y \pm 10 = 0$ b) $\sqrt{3}x + y \pm 10 = 0$

c) $x \pm \sqrt{3}y - 10 = 0$ d) None of these

17. The line joining two points A(2, 0), B(3, l) is rotated about A in anti-clockwise direction through an angle of 15^0. The equation of the line in the new position, is

a) $\sqrt{3}x - y - 2\sqrt{3} = 0$ b) $x - 3\sqrt{y} - 2 = 0$

c) $\sqrt{3}x + y - 2\sqrt{3} = 0$ d) $x + \sqrt{3}y - 2 = 0$

18. If the lines $2x + 3ay - 1 = 0$ and $3x + 4y + 1 = 0$ are mutually perpendicular, then the value of a will be

a) $\frac{1}{2}$ b) 2

c) $-\frac{1}{2}$ d) – 2

19. The distance of the line $2x - 3y = 4$ from the point (1, 1) measured parallel to the line

a) $\sqrt{2}$ b) $\frac{5}{\sqrt{2}}$

c) $\frac{1}{\sqrt{2}}$ d) 6

20. The equation of perpendicular bisectors of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y$ — 0 respectively. If the point A is (1, – 2), then the equation of line BC is

a) $23x + 14y - 40 = 0$

b) $14x - 23y + 40 = 0$

c) $23x - 14y + 40 = 0$

d) $14x + 23y - 40 = 0$

21. Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1,- 1) and parallel to PS is

a) $2x - 9y - 7 = 0$

b) $2x - 9y - 11 = 0$

c) $2x + 9y - 11 = 0$

d) $2x + 9y + 7 = 0$

22. The intercept of a line between the coordinate axes is divided by the point (– 5, 4) in the ratio 1 : 2. The equation of the line will be

a) $5x - 8y + 60 = 0$

b) $8x - 5y + 60 = 0$

c) $2x - 5y + 30 = 0$

d) None of these

23. The number of lines that are parallel to $2x + 6y + 1 = 0$ and have an intercept of length 10 between the coordinate axes is

a) 1   b) 2

c) 4   d) Infinitely many

24. A straight line moves so that the sum of the reciprocals of its intercepts on two perpendicular lines is constant, then the line passes through

a) A fixed point   b) A variable point

c) Origin   d) None of these

25. A line through A(– 5, – 4) meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C and D respectively. If

$$\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$$, then the equation of the line is

a) $2x + 3y + 22 = 0$

b) $5x - 4y + 7 = 0$

x) $3x - 2y + 3 = 0$

d) None of these

26. In what direction a line be drawn through the point (1, 2) so that its points of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point

a) 30^0 b) 45^0

c) 60^0 d) 75^0

27. The sides AB, BC, CD and DA of a quadrilateral are $x + 2y = 3$, $x=1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively. The angle between diagonals AC and BD is

a) 45^0 b) 60^0

c) 90^0 d) 30^0

6.2 Two intersecting lines and family of lines

28. The equation of a line passing through the point of intersection of the lines $4x - 3y - 1 = 0$ and $5x - 2y - 3 = 0$ and parallel to the line $2y - 3x + 2 = 0$ is

a) $x - 3y = 1$

b) $3x - 2y = 1$

c) $2x - 3y = 1$

d) $2x - y = 1$

29. The straight line passing through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is

a) $5x - 4y = 0$ b) $5x + 4y = 0$

c) $4x - 5y = 0$ d) $4x + 5y = 0$

30. The equation of the straight line passing through the point of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$ is

a) $5x + 3y + 8 = 0$ b) $3x - 5y + 8 = 0$

c) $5x + 3y + 11 = 0$ d) $3x - 5y + 11 = 0$

31. The equation of straight line passing through the intersection of the lines $x - 2y = 1$ and $x + 3y = 2$ and parallel to $3x + 4y = 0$ is

a) $3x + 4y + 5 = 0$ b) $3x + 4y - 10 = 0$

c) $3x + 4y - 5 = 0$ d) $3x + 4y + 6 = 0$

32. The point of intersection of the lines

$\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{y}{b} + \frac{y}{a} = 1$ lies on the line

a) $x - y = 0$

b) $(x + y)(a + b) = 2ab$

c) $(lx + my)(a + b) = (l + m)ab$

d) All of these

33. If the co-ordinates of the vertices A, B, C of the triangle ABC are (– 4, 2), (12, – 2) and (8, 6) respectively, then $\angle B =$

a) $\tan^{-1}\left(-\frac{6}{7}\right)$ b) $\tan^{-1}\left(\frac{6}{7}\right)$

c) $\tan^{-1}\left(-\frac{7}{6}\right)$ d) $\tan^{-1}\left(\frac{7}{6}\right)$

34. If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, then m =

a) $\frac{1+3\sqrt{2}}{7}$ b) $\frac{1-3\sqrt{2}}{7}$

c) $\frac{1 \pm 2\sqrt{2}}{7}$ d) $\frac{1 \pm 5\sqrt{2}}{7}$

35. The angle between the lines $x\cos\alpha_1 + y\sin\alpha_1 = p_1$ and $x\cos\alpha_2 + y\sin\alpha_2 = p_2$ is

a) $0, 1 + \alpha_2$ b) $\alpha_1 - \alpha_2$

c) $2\alpha_1$ d) $2\alpha_2$

36. If the lines $y = (2 + \sqrt{3})x + 4$ and $y = kx + 6$ are inclined at an angle 60^0 to each other, then the value of k will be

a) 1 b) 2

c) – 1 d) – 2

37. The lines

$(p - q)(q - r)y + (r - p) = 0$

$(q - r)(r - p)y + (p - q) = 0$

$(r - p)x + (p - q)y + (q - r) = 0$ are

a) parallel b) perpendicular

c) concurrent d) none of these

38. Which of the following lines is concurrent with the lines 3x + 4y + 6 = 0 and 6x + 5y + 9 = 0 ?

a) 2x + 3y + 5 = 0

b) 3x + 3y + 5 = 0

c) 1x + 9y + 3 = 0

d) None of these

39. The straight lines x + 2y – 9 = 0, 3x+ 5y – 5 = 0 and ax + by – 1 = 0 are concurrent, if the straight line 35x – 22y +1 = 0 passes through the point

a) (a, b) b) (b, a)

c) (– a, – b) d) (a, – b)

40. If the lines ax + y + 1 = 0, x + by + 1 = 0 and x + y + c = 0 (a, b, c being distinct and different from 1) are concurrent, then

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$$

a) 0 b) 1

c) $\frac{1}{a+b+c}$ d) 3abc

41. If the lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent, then a, b, c are in

a) A. P. b) G. P.

c) H.P. d) None of these

42. The straight lines 4ax + 3by + c = 0, where a + b + c = 0, will be concurrent, if point is

a) (4, 3) b) (1/4, 1/3)

c) (1/2, 1/3) d) None of these

43. The value of λ, for which the lines 3x + 4y = 5, 5x + 4y = 4 and λx + 4y = 6 meet at a point is

a) 2 b) 1

c) 4 d) 3

44. The equation of a line passing through the point of intersection of lines 2x + 3y + 1 = 0 and 3x – 5y – 5 = 0 and making an angle of 45^0 with positive X-axis is

a) 2x – 19y + 23 = 0

b) 19x – 23y + 15 = 0

c) 19x – 19y – 23 = 0

d) 20x – 19y + 23 = 0

45. Which of the following represents the equation of a line passing through point of intersection of lines x + 2y + 5 = 0 and 3x + 4y + 1 = 0 and passing through point (3, 2)?

a) 2x + 3y – 5 = 0 b) 3x + 2y – 13 = 0

c) x + 3y + 13 = 0 d) 3x – 2y – 7 = 0

46. The equation of a line passing through the point of intersection of lines x + 2y + 3 = 0 and 3x + 4y + 7 = 0 and perpendicular to the line x – y + 9 = 0 is

a) x + y + 2 = 0 b) x – y – 2 = 0

c) x + y – 5 = 0 d) x + 2y – 5 = 0

47. Three sides of a triangle are represented by the equation x + y – 6 = 0, 2x + y – 4 = 0 and x + 2y – 5 = 0. The co-ordinate of its orthocentre is

a) (10, 11) b) (2, 3)

c) (– 2, – 3) d) (– 11, – 10)

48. The equations of two equal sides of an isosceles triangle are 7x – y + 3 = 0 and x + y – 3 = 0 and the third side passes through the point (1, – 10). The equation of the third side is

a) 3x – y – 31 = 0 or x + y + 7 = 0

b) 3x – y + 7 = 0 or x + 3y – 31 = 0

c) 3x + y + 7 = 0 or x – 3y – 31 = 0

d) Neither 3x + y + 7 nor x – 3y – 31 = 0

49. For the straight lines given by the equation (2 + k) x + (1 + k)y = 5 + 7k, for different values of k which of the following statements is true

a) Lines are parallel

b) Lines pass through the point (– 2, 9)

c) Lines pass through the point (2, – 9)

d) None of these

50. The diagonals of the parallelogram whose sides are $lx + my + n = 0$, $lx + my + n' = 0$, $mx + y + n = 0$, $mx + ly + n' = 0$ include an angle

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$

c) $\tan^{-1}\left(\frac{l^2-m^2}{l^2+m^2}\right)$ d) $\tan^{-1}\left(\frac{2lm}{l^2+m^2}\right)$

51. The opposite angular points of a square are (3, 4) and (1, –1). Then the co-ordinates of other two points are

a) $D\left(\frac{1}{2},\frac{9}{2}\right), B\left(-\frac{1}{2},\frac{5}{2}\right)$

b) $D\left(\frac{1}{2},\frac{9}{2}\right), B\left(\frac{1}{2},\frac{5}{2}\right)$

c) $D\left(\frac{9}{2},\frac{1}{2}\right), B\left(-\frac{1}{2},\frac{5}{2}\right)$

d) None of these

52. If $a + b + c = 0$ and $p \neq 0$ the lines

$ax + (b + c) y = p$, $bx + (c + a) y = p$ and $cx + (a + b) y = p$

a) do not intersect b) intersect
c) are concurrent d) none of these

53. The equation of straight line passing through point of intersection of the straight lines $3x - y + 2 = 0$ and $5x - 2y + 7 = 0$ and having infinite slope is

a) $x = 2$ b) $x + y = 3$
c) $x = 3$ d) $x = 4$

54. If vertices of a parallelogram are respectively (0, 0), (1, 0), (2, 2) and (1, 2), then angle between diagonals is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$
c) $\frac{3\pi}{2}$ d) $\frac{\pi}{4}$

6.3 Distance of a point from a line

55. The points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$, are

a) (3, 1), (–7, 11) b) (3, 1), (7, 11)
c) (– 3, 1), (– 7, 11) d) (1, 3), (–7, 11)

56. If p and p' be the distances from origin to the lines $x \sec\alpha + y \operatorname{cosec}\alpha = k$ and $x \cos\alpha - y \sin\alpha = k \cos 2\alpha$, then $4p^2 + p'^2 =$

a) k b) 2k
c) k^2 d) $2k^2$

57. The distance between two parallel lines $3x + 4y - 8 = 0$ and $3x + 4y - 3 = 0$ is given by

a) 4 b) 5
c) 3 d) 1

58. The vertex of an equilateral triangle is (2, –l) and the equation of its base is $x + 2y = 1$. The length of its sides is

a) $\frac{4}{\sqrt{15}}$ b) $\frac{2}{\sqrt{15}}$
c) $\frac{4}{3\sqrt{3}}$ d) $\frac{1}{\sqrt{5}}$

59. The product of the perpendiculars drawn from the points $(\pm \sqrt{a^2 - b^2}, 0)$ on the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$, is

a) a^2 b) b^2
c) $a^2 + b^2$ d) $a^2 - b^2$

60. The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$, is

a) 5 : 3 b) 3 : 7
c) 2 : 3 d) None of these

61. If 2p is the length of perpendicular from the origin to the lines $\frac{x}{a} + \frac{y}{b} = 1$, then a^2, $8p^2$, b^2 are in

a) A. P. b) G.P.
c) H.P. d) None of these

62. A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is

a) (1, –1) b) (1, 1)
c) (0, 0) d) (0, 1)

63. Which pair of points lie on the same side of $3x - 8y - 7 = 0$?

a) (0, – 1) and (0, 0)
b) (4, – 3) and (0, 1)
c) (– 3, – 4) and (1, 2)
d) (– 1, – 1) and (3, 7)

64. To which of the following types the straight lines represented by $2x + 3y - 1 = 0$ and $2x + 3y - 5 = 0$ belong

a) Parallel to each other
b) Perpendicular to each other
c) Inclined at 45^0 to each other
d) Coincident pair of straight lines

65. The equations of two lines through (0, a) which are at distance 'a' from the point (2a, 2a) are

a) $y - a = 0$ and $4x - 3y - 3a = 0$
b) $y - a = 0$ and $3x - 4y + 3a = 0$
c) $y - a = 0$ and $4x - 3y + 3a = 0$
d) None of these

66. If the equation $y = mx + c$ and $x \cos\alpha + y \sin\alpha = p$ represents the same straight line, then

a) $p = c\sqrt{1 + m^2}$
b) $c = p\sqrt{1 + m^2}$
c) $cp = \sqrt{1 + m^2}$
d) $p^2 + c^2 + m^2 = 1$

67. The equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point $(3, 2)$ is $\frac{7}{5}$, is

a) $3x - 4y - 6 = 0$ and $4x + 3y + 1 = 0$
b) $3x - 4y + 6 = 0$ and $4x - 3y - 1 = 0$
c) $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$
d) None of these

68. Equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120^0 with the X-axis, is

a) $x\sqrt{3} + y + 8 = 0$ b) $x\sqrt{3} - y = -8$
c) $x\sqrt{3} - y = 8$ d) $x - \sqrt{3}y + 8 = 0$

69. In the equation $y - y_1 = m(x - x_1)$ if m and x_1 are fixed and different lines are drawn for different values of y_1, then

a) The lines will pass through a single point
b) There will be a set of parallel lines
c) There will be one line only
d) None of these

70. The equations of the lines passing through the point $(1, 0)$ and at a distance $\frac{\sqrt{3}}{2}$ from the origin, are

a) $\sqrt{3}x + y - \sqrt{3} = 0, \sqrt{3}x - y - \sqrt{3} = 0$
b) $\sqrt{3}x + y + \sqrt{3} = 0, \sqrt{3}x - y + \sqrt{3} = 0$
c) $x + \sqrt{3}y - \sqrt{3} = 0, x - \sqrt{3}x - \sqrt{3} = 0$
d) None of these

71. $(\sin\theta, \cos\theta)$ and $(3, 2)$ lies on the same side of the line $x + y = 1$, then θ lies between

a) $(0, \pi/2)$ b) $(0, \pi)$
c) $(\pi/4, \pi/2)$ d) $(0, \pi/4)$

72. If we reduce $3x + 3y + 7 = 0$ to the form $x\cos\alpha + y\sin\alpha = p$, then the value of p is

a) $\frac{7}{2\sqrt{3}}$ b) $\frac{7}{3}$
c) $\frac{3\sqrt{7}}{2}$ d) $\frac{7}{3\sqrt{2}}$

Competitive Thinking

6.1 Slope of a line, Equation of a line in different forms

1. The gradient of the line joining the points on the curve $y = x^2 + 2x$ whose abscissa are 1 and 3, is

a) 6 b) 5
c) 4 d) 3

2. The line passing through the points $(3, -4)$ and $(-2, 6)$ and a line passing through $(-3, 6)$ and $(9, -18)$

a) are perpendicular
b) are parallel
c) make an angle 60° with each other
d) none of these

3. The lines $y = 2x$ and $x = -2y$ are

a) parallel
b) perpendicular
c) equally inclined to axes
d) coincident

4. If the line passing through $(4, 3)$ and $(2, k)$ is perpendicular to $y = 2x + 3$, then $k =$

a) -1 b) 1
c) -4 d) 4

5. The equation of a straight line passing through the point $(3, 2)$ and perpendicular to the line $y = x$ is

a) $x - y = 5$ b) $x + y = 5$
c) $x + y = 1$ d) $x - y = 1$

6. The equation of the line perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ and passing through the point at which it cuts X-axis, is

a) $\frac{x}{a} + \frac{y}{b} + \frac{a}{b} = 0$ b) $\frac{x}{b} + \frac{y}{a} + \frac{b}{a}$
c) $\frac{x}{b} + \frac{y}{a} = 0$ d) $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$

7. The equation of a line passing through the point $(1, 2)$ and perpendicular to the line $x + y + 1 = 0$ is

a) $y - x + 1 = 0$ b) $y - x - 1 = 0$
c) $y - x + 2 = 0$ d) $y - x - 2 = 0$

8. The equation of a straight line passing through (– 3, 2) and cutting an intercept equal in magnitude but opposite in sign from the axes is given by

a) $x - y + 5 = 0$ b) $x + y - 5 = 0$
c) $x - y - 5 = 0$ d) $x + y + 5 = 0$

9. Equation of the line passing through (– 1, 1) and perpendicular to the line $2x + 3y + 4 = 0$ is

a) $2(y - 1) = 3(x + 1)$ b) $3(y - 1) = -2(x + 1)$
c) $y - 1 = 2(x + 1)$ d) $3(y - 1) = x + 1$

10. The points A (1, 3) and C (5, 1) are the opposite vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is

a) $2x + y - 8 = 0$ b) $2x - y - 4 = 0$
c) $2x - y + 4 = 0$ d) $2x + y + 7 = 0$

11. A line is such that its segment between the straight lines $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1,5), then its equation is

a) $83x - 35y + 92 = 0$
b) $35x - 83y + 92 = 0$
c) $35x + 35y + 92 = 0$
d) None of these

12. The equation of a line bisecting perpendicularly the segment joining the points (–4, 6) and (8, 8) is

a) $6x + y - 19 = 0$ b) $y = 7$
c) $6x + 2y - 19 = 0$ d) $x + 2y - 7 = 0$

13. A (–1, 1), B(5, 3) are opposite vertices of a square in jry-plane. The equation of the other diagonal not passing through (A, B) of the square is given by

a) $x - 3y + 4 = 0$ b) $2x - y + 3 = 0$
c) $y + 3x - 8 = 0$ d) $x + 2y - 1\ 0$

14. Equations of diagonals of square formed by lines $x = 0$, $y = 0$, $x = 1$ and $y = 1$ are

a) $y = x, y + x = 1$ b) $y = x, x + y = 2$
c) $2y = x,\ y + x = \frac{1}{3}$ d) $y = 2x, y + 2x = 1$

15. The diagonal passing through origin of a quadrilateral formed by $x = 0$, $y = 0$, $x + y = 1$ and $6x + y = 3$ is

a) $3x - 2y = 0$ b) $2x - 3y = 0$
c) $3x + 2y = 0$ d) None of these

16. Equation of the straight line making equal intercepts on the axes and passing through the point (2,4) is

a) $4x - y - 4 = 0$ b) $2x + y - 8 = 0$
c) $x + y - 6 = 0$ d) $x + 2y - 10 = 0$

17. The equations of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is – 1, is

a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

c) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

d) $\frac{x}{2} + \frac{y}{1} = 1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

18. The equation to the line bisecting the join of (3, – 4) and (5, 2) and having its intercepts on the X-axis and the Y-axis in the ratio 2 : 1 is

a) $x + y - 3 = 0$ b) $2x - y = 9$
c) $x + 2y = 2$ d) $2x + y = 7$

19. If the straight line $ax + by + c = 0$ always passes through (1, – 2), then a, b, c are in

a) A.P. b) H.P.
c) G.P. d) None of these

20. The inclination of the straight line passing through the point (– 3, 6) and the midpoint of the line joining the points (4, – 5) and (– 2, 9) is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$
c) $\frac{\pi}{3}$ d) $\frac{3\pi}{4}$

21. For what values of a and b, the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in sign to those cut off by the line $2x - 3y + 6 = 0$ on the axes ?

a) $a = \frac{8}{3}, b = -4$ b) $a = -\frac{8}{3}, b = -4$
c) $a = \frac{8}{3}, b = 4$ d) $a = -\frac{8}{3}, b = 4$

22. The medians AD and BE of a triangle with vertices A(0, b), B(0, 0) and C(a, 0) are perpendicular to each other, if

a) $a = \sqrt{2}b$ b) $a = -\sqrt{2}b$
c) Both (a) and (b) d) None of these

23. If $\left(\frac{3}{2}, \frac{5}{2}\right)$ is the midpoint of line segment intercepted by a line between axes, the equation of the line is

a) $5x + 3y + 15 = 0$ b) $3x + 5y + 15 = 0$

c) $5x + 3y - 15 = 0$ d) $3x + 5y - 15 = 0$

24. The slope of a line that makes an angle of measure 30^0 with Y-axis is

a) $\sqrt{3}$ b) $-\sqrt{3}$

c) $\pm\sqrt{3}$ d) $\pm\frac{1}{\sqrt{3}}$

25. If l, m, n are in arithmetic progression, then the straight line $lx + my + n = 0$ will pass through the point

a) $(-1, 2)$ b) $(l, -2)$

c) $(1, 2)$ d) $(2, 1)$

26. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is

a) $x + y = 7$

b) $3x - 4y + 7 = 0$

c) $4x + 3y = 24$

d) $3x + 4y = 25$

27. If a straight line passes through the points $\left(\frac{-1}{2}, 1\right)$ and (1,2), then its x-intercept is

a) -2 b) -1

c) 2 d) 1

28. The equation of the perpendicular bisector of the line segment joining A(– 2, 3) and B(6, – 5) is

a) $x - y = -1$ b) $x - y = 3$

c) $x + y = 3$ d) $x + y = 1$

29. The slope of the straight line which does not intersect X-axis is equal to

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$

c) $\sqrt{3}$ d) 0

30. If the three points A(l, 6), B(3, – 4) and C(x,y) are collinear, then the equation satisfying by x and y is

a) $5x + y - 11 = 0$ b) $5x + 13y + 5 = 0$

c) $5x - 13y + 5 = 0$ d) $13x - y + 5 = 0$

31. If the line $px - qy = r$ intersects the co-ordinate axes at (a, 0) and (0, b), then the value of a + b is equal to

a) $r\left(\frac{q+p}{pq}\right)$ b) $r\left(\frac{q-p}{pq}\right)$

c) $r\left(\frac{p-q}{pq}\right)$ d) $r\left(\frac{p+q}{p-q}\right)$

32. Equation of the line through (α, β) which is the midpoint of the line intercepted between the coordinate axes is

a) $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ b) $\frac{x}{\alpha} + \frac{y}{\beta} = 2$

c) $\frac{x}{\alpha} - \frac{y}{\beta} = -1$ d) $\frac{x}{\alpha} - \frac{y}{\beta} = -2$

33. The equation of the line which is such that the portion of line segment between the coordinate axes is bisected at (4, – 3) is

a) $3x + 4y = 24$ b) $3x - 4y = 12$

c) $3x - 4y = 24$ d) $4x - 3y = 24$

34. Two lines represented by equations $x + y = 1$ and $x + ky = 0$ are mutually orthogonal if k is

a) 1 b) -1

c) 0 d) None of these

6.2 Two intersecting lines and family of lines

35. The equation of a line passing through the point of intersection of the lines $x + 5y + 7 = 0$, $3x + 2y - 5 = 0$ and perpendicular to the line $7x + 2y - 5 = 0$, is

a) $2x - 7y - 20 = 0$

b) $2x + 7y - 20 = 0$

c) $-2x + 7y - 20 = 0$

d) $2x + 7y + 20 = 0$

36. A line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 1$ is

a) $3x + 4y + 3 = 0$ b) $3x + 4y = 0$

c) $4x - 3y + 3 = 0$ d) $4x - 3y = 3$

37. Equations of lines which passes through the points of intersection of the lines $4x - 3y - 1 = 0$ and $2x - 5y + 3 = 0$ and are equally inclined to the axes are

a) $y \pm x = 0$ b) $y - 1 = \pm 1 (x - 1)$

c) $x - 1 = \pm 2 (y - 1)$ d) $y \pm x = 2$

38. Angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is

a) $2\tan^{-1}\frac{b}{a}$ b) $\tan^{-1}\frac{2ab}{a^2+b^2}$

c) $\tan^{-1}\frac{a^2-b^2}{a^2+b^2}$ d) None of thes

39. The angle between the two lines y – 2x = 9 and x + 2y = – 1, is

a) 60^0 b) 30^0

c) 90^0 d) 45^0

40. If $\frac{1}{ab'}+\frac{1}{ba'}=0$, then lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b'}+\frac{y}{a'}=1$ are

a) Parallel

b) Inclined at 60^0 to each other

c) Perpendicular to each other

d) Inclined at 30^0 to each other

41. Angle between x = 2 and x – 3y = 6 is

a) ∞ b) $\tan^{-1}(3)$

c) $\tan^{-1}\left(\frac{1}{3}\right)$ d) None of these

42. The angle between the lines 2x – y + 3 = 0 and x + 2y + 3 = 0 is

a) 90^0 b) 60^0

c) 45^0 d) 30^0

43. If the lines ax + by + c = 0, bx + cy + a = 0 and cx + ay + b = 0 be concurrent, then

a) $a^3 + b^3 + c^3 + 3abc = 0$

b) $a^3 + b^3 + c^3 - abc = 0$

c) $a^3 + b^3 + c^3 - 3abc = 0$

d) None of these

44. For what value of 'a' the lines x = 3,y = 4 and 4x – 3y + a = 0 are concurrent

a) 0 b) – 1

c) 2 d) 3

45. The lines 2x + y – 1 = 0, ax + 3y – 3 = 0 and 3x + 2y – 2 = 0 are concurrent for

a) All a b) a = 4 only

c) $-1 \le a \le 3$ d) $a > 0$ only

46. If the lines 4x + 3y = 1, x – y = – 5 and 5y + bx = 3 are concurrent, then b equals

a) 1 b) 3

c) 6 d) 0

47. If a and b are two arbitrary constants, then the straight line (a – 2b)x + (a + 3b)y + 3a + 4b = 0 will pass through

a) (– l, – 2) b) (1, 2)

c) (– 2, – 3) d) (2, 3)

48. If a, b, c are in harmonic progression, then straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point, that point is

a) (– 1 – 2) b) (– 1, 2)

c) (l, – 2) d) (l, – l/2)

49. The equation to the sides of a triangle are x – 3y = 0, 4x + 3y = 5, and 3x + y = 0. The line 3x – 4y = 0 passes through

a) The incentre

b) The centroid

c) The circumcentre

d) The orthocentre of the triangle

50. Two consecutive sides of a parallelogram are 4x + 5y = 0 and 7x + 2y = 0. If the equation to one diagonal is llx + 7y = 9, then the equation of the other diagonal is

a) x + 2y = 0 b) 2x + y = 0

c) x – y = 0 d) None of these

51. The triangle formed by the lines x + y – 4 = 0, 3x + y = 4, x + 3y = 4 is

a) isosceles b) equilateral

c) right-angled d) none of these

52. The lines ax+ by + c = 0, where 3a + 2b + 4c = 0 are concurrent at the point

a) (1/2,3/4) b) (1,3)

c) (3,1) d) (3/4,1/2)

53. The parallelism condition for two straight lines one of which is specified by the equation ax + by + c = 0, the other being represented parametrically by $x = \alpha t + \beta$, $y = \gamma t + \delta$ is given by

a) $\alpha\delta - b\alpha = 0, \beta = \delta = c = 0$

b) $a\alpha - b\gamma = 0, \beta = \delta = 0$

c) $a\alpha + b\gamma = 0$

d) $a\gamma = b\alpha = 0$

54. The equation of the line which passes through the point (3, – 2) and are inclined at 60^0 to the line $\sqrt{3}x + y = 1$, is

a) $y + 2 = 0, \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

b) $x - 2 = 0, \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$

c) $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

d) $x - \sqrt{3}y + 2 + 3\sqrt{3} = 0$

55. The point (– 4, 5) is the vertex of a square and one of its diagonals is $7x - y + 8 = 0$. The equation of the other diagonal is

a) $7x - y + 23 = 0$ b) $7y + x = 30$

c) $7y + x = 31$ d) $x - 7y = 30$

56. The line passing through the point of intersection of x + y = 2, x – y = 0 and is parallel to x + 2y = 5 is

a) $x + 2y = 1$ b) $x + 2y = 2$

c) $x + 2y = 4$ d) $x + 2y = 3$

57. The line parallel to the X-axis and passing through the intersection of the lines ax + 2by + 3b = 0 and bx - 2ay – 3a = 0, where (a, b) ≠ (0, 0) is

a) above the X-axis at a distance of 3/2 from it

b) above the X-axis at a distance of 2/3 from it

c) below the X-axis at a distance of 3/2 from it .

d) below the X-axis at a distance of 2/3 from it

58. The angle between the lines
$x \cos\alpha + y \sin\alpha = a$ and $x \sin\beta - y \cos\beta = a$ is

a) $\beta - \alpha$ b) $\pi + \beta - \alpha$

c) $\frac{\pi}{2} + \beta + \alpha$ d) $\frac{\pi}{2} - \beta + \alpha$

59. A line passes through the point of intersection of the lines 3x + y + 1 = 0 and 2x – y + 3 = 0 and makes equal intercepts with axes. The equation of the line is

a) 5x + 5y – 3 = 0 b) x + 5y – 3 = 0

c) 5x – y – 3 = 0 d) 5x + 5y + 3 = 0

60. The length of the straight line x – 3y = 1 intercepted by the hyperbola $x^2 - 4y^2 = 1$, is

a) $\sqrt{10}$ units b) $\frac{6}{5}$ units

c) $\frac{6}{\sqrt{10}}$ units d) $\frac{6}{5}\sqrt{10}$ units

61. The straight lines x + y = 0, 5x + y = 4 and x + 5y = 4 form

a) an isosceles triangle

b) an equilateral triangle

c) a scalene triangle

d) a right angled triangle

6.3 Distance of a point from a line

62. The points on the X-axis whose perpendicular distance from the line $\frac{x}{a} + \frac{y}{b} = 1$ is a, are

a) $\left[\frac{a}{b}(b \pm \sqrt{a^2 + b^2}), 0\right]$

b) $\left[\frac{b}{a}(b \pm \sqrt{a^2 + b^2}), 0\right]$

c) $\left[\frac{a}{b}(a \pm \sqrt{a^2 + b^2}), 0\right]$

d) None of these

63. The distance between the lines 3x + 4y = 9 and 6x + 8y = 15 is

a) $\frac{3}{2}$ b) $\frac{3}{10}$

c) 6 d) None of these

64. The length of the perpendicular drawn from origin upon the straight line $\frac{x}{3} - \frac{y}{4} = 1$ is

a) $2\frac{2}{5}$ b) $3\frac{1}{5}$

c) $4\frac{2}{5}$ d) $3\frac{2}{5}$

65. Two points A and B have co-ordinates (1, 1) and (3, – 2) respectively. The co-ordinates of a point distant $\sqrt{85}$ from B on the line through B perpendicular to AB are

a) (4, 7) b) (7, 4)

c) (5, 7) d) (– 5, – 3)

66. The equation of the base of an equilateral triangle is x + y = 2 and the vertex is (2, 1). The length of the side of the triangle is

a) $\sqrt{3/2}$ b) $\sqrt{2}$

c) $\sqrt{2/3}$ d) None of these

67. Let α be the distance between the lines $-x + y = 2$ and $x - y = 2$ and β be the distance between the lines $4x - 3y = 5$ and $6y - 8x = 1$, then

a) $20\sqrt{2}\beta = 11\alpha$ b) $20\sqrt{2}\alpha = 11\beta$

c) $11\sqrt{2}\beta = 20\alpha$ d) None of these

68. Choose the correct statement which describe the position of .the point (– 6, 2) relative to straight lines $2x + 3y - 4 = 0$ and $6x + 9y + 8 = 0$?

a) Below both the lines

b) Above both the lines

c) In between the lines

d) None of these

69. If the equation of base of an equilateral triangle is $2x - y = 1$ and the vertex is (– 1, 2), then the length of the side of the triangle is

a) $\sqrt{\frac{20}{3}}$ b) $\frac{2}{\sqrt{15}}$

c) $\sqrt{\frac{8}{15}}$ d) $\sqrt{\frac{15}{2}}$

70. The vertices of a triangle are (2, 1), (5, 2) and (4, 4). The lengths of the perpendicular from these vertices on the opposite sides are

a) $\frac{7}{\sqrt{5}}, \frac{7}{\sqrt{13}}, \frac{7}{\sqrt{6}}$ b) $\frac{7}{\sqrt{6}}, \frac{7}{\sqrt{8}}, \frac{7}{\sqrt{10}}$

c) $\frac{7}{\sqrt{5}}, \frac{7}{\sqrt{8}}, \frac{7}{\sqrt{15}}$ d) $\frac{7}{\sqrt{5}}, \frac{7}{\sqrt{13}}, \frac{7}{\sqrt{10}}$

71. The vertices of a triangle OBC are (0, 0), (– 3, – 1) and (– 1, – 3) respectively. Then the equation of line parallel to BC which is at $\frac{1}{2}$ unit distant from origin and cuts OB and OC, is

a) $2x + 2y + \sqrt{2} = 0$ b) $2x - 2y - \sqrt{2} = 0$

c) $2x - 2y + \sqrt{2} = 0$ d) None of these

72. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to K and the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then, the distance between L and K is

73. The equation of one of the lines parallel to $4x - 3y = 5$ and at a unit distance from the point (– 1, – 4) is

a) $3x + 4y - 3 = 0$ b) $3x + 4y + 3 = 0$

c) $4x - 3y + 3 = 0$ d) $4x - 3y - 3 = 0$

74. The length of the perpendicular from the origin on the line $\frac{x\cos\alpha}{b} - \frac{y\cos\alpha}{a} - 1 = 0$ is

75. The number of points on the line $x + y = 4$ which are unit distance apart from the line $2x + 2y = 5$ is

a) 0 b) 1

c) 2 d) Infinity

76. The distance between the parallel lines $y = x + a$, $y = x + b$ is

a) $\frac{|a - b|}{\sqrt{2}}$ b) $|a - b|$

c) $|a + b|$ d) $\frac{|a + b|}{\sqrt{2}}$

77. The equation of straight line equally inclined to the axis and equidistant from the points (1, – 2) and (3, 4) is $ax + by + c = 0$, where

a) a = 1, b = 1, c = 1 b) a = 1, b = 1, c = – 1

c) a = 1, b = 1, c = 2 d) None of these

78. A straight line passes through the points (5, 0) and (0, 3). The length of perpendicular from the point (4, 4) on the line is

a) $\frac{15}{\sqrt{34}}$ b) $\frac{\sqrt{17}}{2}$

c) $\frac{17}{2}$ d) $\sqrt{\frac{17}{2}}$

Evaluation Test

1. If the line $y = 7x - 25$ meets the circle $x^2 + y^2 = 25$ at the points A, B, then the distance between A and B is

a) $\sqrt{10}$ b) 10

c) $5\sqrt{2}$ d) 5

2. If $f(\alpha) = x \cos\alpha + y \sin\alpha - p(\alpha)$, then the lines $f(\alpha) = 0$ and $f(\beta) = 0$ are perpendicular to each other, if

a) $\alpha = \beta$ b) $\alpha + \beta = \frac{\pi}{2}$

c) $|\alpha - \beta| = \frac{\pi}{2}$ d) none of these

3. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is

a) a hyperbola b) a parabola

c) an ellipse d) a straight line

4. If the straight line $ax + by + c = 0$ make a triangle of constant area with coordinate axes, then

a) a, b, c are in G.P. b) a, c, b are in G.P.

c) c, a, b are in G.P. d) none of these

5. Let $0 < \alpha < \frac{\pi}{2}$ be a fixed angle.

If $P = (\cos\theta, \sin\theta)$ and $Q = (\cos(\alpha - \theta), \sin(\theta - \theta))$, then Q is obtained from P by

a) clockwise rotation around origin through an angle α

b) anti-clockwise rotation around origin through an angle α

c) reflection in the line through origin with slope $\tan\alpha$

d) reflection in the line through origin with slope $\frac{\tan\alpha}{2}$

6. Let $P(-1, 0)$, $Q(0, 0)$ and $R(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is

a) $\frac{\sqrt{3}}{2}x + y = 0$ b) $x + \sqrt{3}y = 0$

c) $\sqrt{3}x + y = 0$ d) $x + \frac{\sqrt{3}}{2}y = 0$

7. A square of side V lies above the X-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha (0 < \alpha < \pi/4)$ with the positive direction of X-axis. The equation of its diagonal not passing through the origin is

a) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

b) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

d) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

8. A line L passes through the points (1, 1) and (2, 0) and another line L' passes through $\left(\frac{1}{2}, 0\right)$ and perpendicular to L. Then the area of the triangle formed by the lines L, L' and Y-axis is

a) $\frac{15}{8}$ b) $\frac{25}{4}$

c) $\frac{25}{8}$ d) $\frac{25}{16}$

9. The number of integer values of m, for which the x-co-ordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

a) 2 b) 0

c) 4 d) 1

10. If straight lines $ax + by + p = 0$ and $x \cos\alpha + y \sin\alpha - p = 0$ are inclined at an angle $\frac{\pi}{4}$ and meet the straight line $x \sin\alpha - y \cos\alpha = 0$ at the same point, then the value of $a^2 + b^2$ is equal to

a) 1 b) 2

c) 3 d) 4

11. A line $Ax + y = 1$ passes through the point $A(2, -7)$ meets the line BC whose equation is $3x - 4y + 1 = 0$ at the point B. The equation to the line AC so that AB = AC is

a) $52x + 89y + 519 = 0$

b) $52x + 89y - 519 = 0$

c) $89x + 52y + 519 = 0$

d) $89x + 52y - 519 = 0$

❍❍❍❍

Answer Key

Classical Thinking

1. (A)	2. (A)	3. (A)	4. (B)	5. (A)	6. (A)	7. (A)	8. (D)	9. (C)	10. (B)
11. (A)	12. (A)	13. (C)	14. (A)	15. (B)	16. (B)	17. (C)	18. (A)	19. (D)	20. (A)
21. (D)	22. (B)	23. (C)	24. (B)	25. (C)	26. (C)	27. (C)	28. (B)	29. (B)	30. (B)
31. (A)	32. (C)	33. (A)	34. (C)	35. (B)	36. (A)	37. (B)	38. (D)	39. (B)	40. (B)
41. (A)	42. (B)	43. (A)	44. (C)	45. (A)	46. (A)	47. (D)			

Critical Thinking

1. (C)	2. (A)	3. (B)	4. (B)	5. (B)	6. (A)	7. (A)	8. (C)	9. (D)	10. (B)
11. (C)	12. (A)	13. (A)	14. (D)	15. (B)	16. (B)	17. (A)	18. (C)	19. (A)	20. (D)
21. (D)	22. (B)	23. (B)	24. (A)	25. (A)	26. (D)	27. (C)	28. (B)	29. (B)	30. (A)
31. (C)	32. (D)	33. (D)	34. (D)	35. (B)	36. (C)	37. (C)	38. (B)	39. (A)	40. (B)
41. (A)	42. (B)	43. (B)	44. (C)	45. (B)	46. (A)	47. (D)	48. (C)	49. (B)	50. (B)
51. (C)	52. (A)	53. (C)	54. (D)	55. (A)	56. (C)	57. (D)	58. (B)	59. (B)	60. (B)
61. (C)	62. (C)	63. (D)	64. (A)	65. (C)	66. (B)	67. (C)	68. (A)	69. (B)	70. (A)
71. (D)	72. (D)								

Competitive Thinking

1. (A)	2. (B)	3. (B)	4. (D)	5. (B)	6. (D)	7. (B)	8. (A)	9. (A)	10. (B)
11. (A)	12. (A)	13. (C)	14. (A)	15. (A)	16. (C)	17. (A)	18. (C)	19. (A)	20. (D)
21. (D)	22. (C)	23. (C)	24. (C)	25. (B)	26. (C)	27. (A)	28. (B)	29. (D)	30. (A)
31. (B)	32. (B)	33. (C)	34. (B)	35. (A)	36. (A)	37. (B)	38. (A)	39. (C)	40. (C)
41. (B)	42. (A)	43. (C)	44. (A)	45. (A)	46. (C)	47. (A)	48. (C)	49. (D)	50. (C)
51. (A)	52. (D)	53. (C)	54. (A)	55. (C)	56. (D)	57. (C)	58. (D)	59. (A)	60. (D)
61. (A)	62. (A)	63. (B)	64. (A)	65. (C)	66. (C)	67. (A)	68. (A)	69. (A)	70. (D)
71. (A)	72. (D)	73. (D)	74. (D)	75. (A)	76. (A)	77. (B)	78. (D)		

Answers to Evaluation Test

1. C) 2. C) 3. D) 4.B) 5. D) 6. C) 7. A) 8. D) 9. (A) 10. (B)
11. (A)

Answer Key

Classical Thinking

1. (A)	2. (A)	3. (A)	4. (B)	5. (A)	6. (A)	7. (A)	8. (D)	9. (C)	10. (B)
11. (A)	12. (A)	13. (C)	14. (A)	15. (B)	16. (B)	17. (C)	18. (A)	19. (D)	20. (A)
21. (D)	22. (B)	23. (C)	24. (B)	25. (C)	26. (C)	27. (C)	28. (B)	29. (B)	30. (B)
31. (A)	32. (C)	33. (A)	34. (C)	35. (B)	36. (A)	37. (B)	38. (D)	39. (B)	40. (B)
41. (A)	42. (B)	43. (A)	44. (C)	45. (A)	46. (A)	47. (D)			

Critical Thinking

1. (C)	2. (A)	3. (B)	4. (B)	5. (B)	6. (A)	7. (A)	8. (C)	9. (D)	10. (B)
11. (C)	12. (A)	13. (A)	14. (D)	15. (B)	16. (B)	17. (A)	18. (C)	19. (A)	20. (D)
21. (D)	22. (B)	23. (B)	24. (A)	25. (A)	26. (D)	27. (C)	28. (B)	29. (B)	30. (A)
31. (C)	32. (D)	33. (D)	34. (D)	35. (B)	36. (C)	37. (C)	38. (B)	39. (A)	40. (B)
41. (A)	42. (B)	43. (B)	44. (C)	45. (B)	46. (A)	47. (D)	48. (C)	49. (B)	50. (B)
51. (C)	52. (A)	53. (C)	54. (D)	55. (A)	56. (C)	57. (D)	58. (B)	59. (B)	60. (B)
61. (C)	62. (C)	63. (D)	64. (A)	65. (C)	66. (B)	67. (C)	68. (A)	69. (B)	70. (A)
71. (D)	72. (D)								

Competitive Thinking

1. (A)	2. (B)	3. (B)	4. (D)	5. (B)	6. (D)	7. (B)	8. (A)	9. (A)	10. (B)
11. (A)	12. (A)	13. (C)	14. (A)	15. (A)	16. (C)	17. (A)	18. (C)	19. (A)	20. (D)
21. (D)	22. (C)	23. (C)	24. (C)	25. (B)	26. (C)	27. (A)	28. (B)	29. (D)	30. (A)
31. (B)	32. (B)	33. (C)	34. (B)	35. (A)	36. (A)	37. (B)	38. (A)	39. (C)	40. (C)
41. (B)	42. (A)	43. (C)	44. (A)	45. (A)	46. (C)	47. (A)	48. (C)	49. (D)	50. (C)
51. (A)	52. (D)	53. (C)	54. (A)	55. (C)	56. (D)	57. (C)	58. (D)	59. (A)	60. (D)
61. (A)	62. (A)	63. (B)	64. (A)	65. (C)	66. (C)	67. (A)	68. (A)	69. (A)	70. (D)
71. (A)	72. (D)	73. (D)	74. (D)	75. (A)	76. (A)	77. (B)	78. (D)		

Hints

Classical Thinking

1. The equation of line is $\frac{x}{a}+\frac{y}{a}=1$.

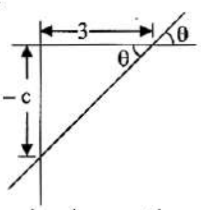
$\Rightarrow x+y-a=0$

$\therefore$ Slope $= -\frac{\text{coefficient of } x}{\text{coefficient of } y} = -1$

2. From the figure, m = tan θ = $\frac{-c}{3}$

$\Rightarrow 3 = \frac{-c}{3}$

$\Rightarrow c = -9$



Hence, the required equation is $y = 3x - 9$

3. Midpoint is (3, 4) and

slope of AB = $\frac{6}{4}$

$\therefore$ Slope of perpendicular $= \frac{-1}{6/4} = \frac{-2}{3}$

$\therefore$ the required equation is

$y-4 = \frac{-2}{3}(x-3)$

$\Rightarrow 2x+3y = 18$

4. Slope of perpendicular $= \frac{-y'}{2a}$

$\therefore$ the required equation is $y - y' = -\frac{y'}{2a}(x-x')$

$\Rightarrow xy' + 2ay - 2ay' - x'y' = 0$

5. $m = \dfrac{-1}{\dfrac{b'-b}{a'-a}} = \dfrac{a'-a}{b-b'}$

Midpoint is $\left(\dfrac{a+a'}{2}, \dfrac{b+b'}{2}\right)$

∴ the required equation is

$y - \left(\dfrac{b+b'}{2}\right) = \dfrac{a'-a}{b-b'}\left[x - \left(\dfrac{a+a'}{2}\right)\right]$

$\Rightarrow 2(b-b')y + 2(a-a')x = b^2 - b'^2 + a^2 - a'^2$

6. $m = \dfrac{5-0}{-4-0} = \dfrac{5}{-4}$

∴ the required equation is $5x + 4y = 0$.

7. Here, intercept on X-axis is 3 and intercept on Y-axis is –2.
So, using double intercept form, the required equation of the line is $\dfrac{x}{3} - \dfrac{y}{2} = 1$.

8. The required equation passing through (3, –4) and having gradient $\dfrac{4}{3}$ is $y + 4 = \dfrac{4}{3}(x-3)$.

9. The required equation which passes through (1, 2) and its gradient m = 3, is $y - 2 = 3(x-1)$.

10. The required equation passing through (0, 0) and having gradient $m = \dfrac{1}{0}$, is $y = \dfrac{1}{0}x$

$\Rightarrow x = 0$

11. Midpoint ≡ (4, –9) and slope $= \dfrac{-1}{\dfrac{3+1}{-1-5}} = \dfrac{3}{2}$

Hence, the required line is $y + 9 = \dfrac{3}{2}(x-4)$

$\Rightarrow 3x - 2y = 30$

12. Intersection point on X-axis is $(2x_1, 0)$ and on Y-axis is $(0, 2y_1)$. Thus, equation of line passing through these points is $\dfrac{x}{x_1} + \dfrac{y}{y_1} = 2$.

13. The required equation which passes through (c, d) and its gradient $-\dfrac{a}{b}$, is

$y - d = -\dfrac{a}{b}(x-c)$

$\Rightarrow a(x-c) + b(y-d) = 0$

14. The required equation is
$y + 6 = \tan 45°(x-4)$
$\Rightarrow x - y - 10 = 0$

15. Equation of a line passing through the given points is $\dfrac{y-(-6)}{-6-10} = \dfrac{x-(-5)}{-5-3}$

$\Rightarrow \dfrac{y+6}{-16} = \dfrac{x+5}{-8} \Rightarrow 2x - y + 4 = 0$

16. Using double intercept form, we get

$\dfrac{x}{2a\sec\theta} + \dfrac{y}{2a\,\mathrm{cosec}\,\theta} = 1$

$\Rightarrow x\cos\theta + y\sin\theta = 2a$

17. Equation of line is $y = mx + c$
$\Rightarrow y = (\tan 135°)x - 5 \Rightarrow y = -x - 5$
$\Rightarrow x + y + 5 = 0$

19. Since, the given line passes through (2, –3) and (4, –5).

∴ $\dfrac{2}{a} - \dfrac{3}{b} = 1$ and $\dfrac{4}{a} - \dfrac{5}{b} = 1$

$\Rightarrow b = -1, a = -1$

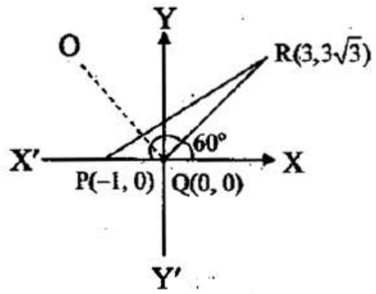
20. Equation of line perpendicular to $ax + by + c = 0$ is $bx - ay + \lambda = 0$ ......(i)
It passes through (a, b).
∴ $ab - ab + \lambda = 0 \Rightarrow \lambda = 0$
Putting $\lambda = 0$ in (i), we get $bx - ay = 0$ which is the required equation.

21. The equation of a line passing through (2, 2) and perpendicular to $3x + y = 3$ is $y - 2 = \dfrac{1}{3}(x-2)$ or $x - 3y + 4 = 0$.

Putting $x = 0$ in this equation, we get $y = \dfrac{4}{3}$

So, y – intercept $= \dfrac{4}{3}$

23.



Slope of QR $= \dfrac{3\sqrt{3}-0}{3-0} = \sqrt{3}$ i.e., $\theta = 60°$
Clearly, ∠PQR = 120°

OQ is the angle bisector of the angle PQR, so line OQ makes 120° with the positive direction of X-axis.

Therefore, equation of the bisector of $\angle PQR$ is $y = \tan 120° x \Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

25. Gradient of the line which passes through $(1, 0)$ and $(-2, \sqrt{3})$ is $m = \frac{\sqrt{3}-0}{-2-1} = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = 150°$

26. The point of intersection is (0, 0)
Thus, the equation of line passing through the points (0, 0) and (2, 2) is $y = x$.

27. Let $L_1 \equiv 2x + 5y - 7 = 0$ and $L_2 \equiv 2x - 5y - 9 = 0$, so that $m_1 = -\frac{2}{5}$, $m_2 = \frac{2}{5}$
Lines are neither parallel nor perpendicular, also not coincident.
Hence, the lines are intersecting.

28. $m_1 = \sqrt{3}$, $m_2 = 0$

$\therefore \tan\theta = \left|\frac{\sqrt{3}-0}{1+0}\right|$

$\Rightarrow \theta = 60° = \sqrt{3}$

29. $\theta = \tan^{-1}\left|\frac{2-\sqrt{3}-2-\sqrt{3}}{1+4-3}\right| = \tan^{-1}(-\sqrt{3})$

$= 120°$

Considering smaller angle $\theta' = 60°$.

30. Slope of given line is $\frac{1}{2}$

Thus, $\tan 45° = \pm\frac{m-\frac{1}{2}}{1+m.\frac{1}{2}} \Rightarrow m = 3$ or $\frac{-1}{3}$

Hence option (B) is correct.

31. The lines are concurrent, if $\begin{vmatrix} 7 & -8 & 5 \\ 3 & -4 & 5 \\ 4 & 5 & k \end{vmatrix} = 0$

$\Rightarrow 7(-4k - 25) + 8(3k - 20) + 5(15 + 16) = 0$

$\Rightarrow k = -45$

32. $3(12x + 10y - 3) - 2(15x - 18y + 1)$
$= 6x + 66y - 11 = 0$
Hence, the lines are concurrent.

33. $u = a_1x + b_1y + c_1 = 0$, $v = a_2x + b_2y + c_2 = 0$

let $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = c$

$\Rightarrow a_2 = \frac{a_1}{c}, b_2 = \frac{b_1}{c}, c_2 = \frac{c_1}{c}$

Given that, $u + kv = 0$

$\Rightarrow a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$

$\Rightarrow a_1x + b_1y + c_1 +$
$k\left(\frac{a_1}{c}\right)x + k\left(\frac{b_1}{c}\right)y + k\left(\frac{c_1}{c}\right) = 0$

$\Rightarrow a_1x\left(1+\frac{k}{c}\right) + b_1y\left(1+\frac{k}{c}\right) + c_1\left(1+\frac{k}{c}\right) = 0$

$\Rightarrow a_1x + b_1y + c_1 = 0 = u$

34. Equation of lines are $\frac{x}{a} - \frac{y}{b} = 1$ and $\frac{x}{b} - \frac{y}{a} = 1$

$\Rightarrow m_1 = \frac{b}{a}$ and $m_2 = \frac{a}{b}$

$\therefore \theta = \tan^{-1}\left|\frac{\frac{b}{a}-\frac{a}{b}}{1+\frac{b}{a}.\frac{a}{b}}\right| = \tan^{-1}\frac{b^2-a^2}{2ab}$

35. $\theta = \tan^{-1}\left|\frac{-\cot 30° + \cot 60°}{1+\cot 30° \cot 60°}\right|$

$= \tan^{-1}\left|\frac{\tan 60° - \tan 30°}{1+\tan 30° \tan 60°}\right| = 30°$

36. $\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$

$\Rightarrow m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

38. $p = \left|\frac{ab}{\sqrt{a^2+b^2}}\right|$

$\Rightarrow \frac{a^2+b^2}{a^2b^2} = \frac{1}{p^2} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

39. Length of perpendicular is

$\left|\frac{\frac{b}{a}-\frac{a}{b}-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(-\frac{1}{b}\right)^2}}\right| = \left|\frac{b^2-a^2-ab}{\sqrt{a^2+b^2}}\right|$

40. Straight line $y - y' = \dfrac{y'' - y'}{x'' - x'}(x - x')$

$\therefore$ Length of perpendicular

$$= \left|\frac{x'(y'' - y') - y'(x'' - x')}{\sqrt{(x'' - x')^2 + (y'' - y')^2}}\right|$$

$$= \left|\frac{x'y'' - y'x''}{\sqrt{(x'' - x')^2 + (y'' - y')^2}}\right|$$

41. Required distance $= \left|\dfrac{-7}{\sqrt{12^2 + 5^2}}\right| = \dfrac{7}{13}$

42. Required length $= \left|\dfrac{4(3) + 3(1) + 20}{5}\right| = 7$

43. Required distance $= \left|\dfrac{-2 - 3 - 5}{\sqrt{1+1}}\right| = \dfrac{10}{\sqrt{2}} = 5\sqrt{2}$

44. Given lines are $5x + 3y - 7 = 0$ .....(i)
and $15x + 9y + 14 = 0$ or

$5x + 3y + \dfrac{14}{3} = 0$ ....(ii)

Lines (i) and (ii) are parallel.

$\therefore$ Required distance $= \left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\dfrac{-7 - \frac{14}{3}}{\sqrt{5^2 + 3^2}}\right|$

$= \left|\dfrac{-35}{3\sqrt{34}}\right| = \dfrac{35}{3\sqrt{34}}$

45. $L_1(8, -9) = 2(8) + 3(-9) - 4 = -15$
$L_2(8, -9) = 6(8) + 9(-9) + 8 = -25$
Hence, point lies on same side of the lines.

46. Here, equation of line is $y = x \tan\alpha + c$, $c > 0$
Length of the perpendicular drawn on line from point $(a \cos\alpha, a \sin\alpha)$ is

$p = \left|\dfrac{-a \sin\alpha + a\cos\alpha \tan\alpha + c}{\sqrt{1 + \tan^2\alpha}}\right| = \dfrac{c}{\sec\alpha} = c\cos\alpha$

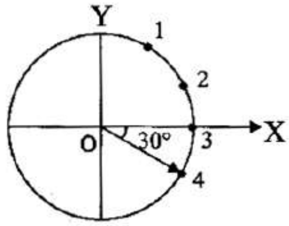
47. The two lines will be identical if there exists some real number k such that
$b^3 - c^3 = k(b - c)$, $c^3 - a^3 = k(c - a)$,
$a^3 - b^3 = k(a - b)$
$\Rightarrow b - c = 0$ or $b^2 + c^2 + bc = k$
$\Rightarrow c - a = 0$ or $c^2 + a^2 + ac = k$
$\Rightarrow a - b = 0$ or $a^2 + b^2 + ab = k$
$\Rightarrow b = c, c = a, a = b$
or $b^2 + c^2 + bc = c^2 + a^2 + ca$
$\Rightarrow b^2 - a^2 = c(a - b)$
$\Rightarrow b = a$ or $a + b + c = 0$

Critical Thinking

1. Since the hour, minute and second hands always pass through origin because one end of these hands is always at origin.
Now, at 4 O' clock, the hour hand makes 30° angle in fourth quadrant.
So, the equation of hour hand is

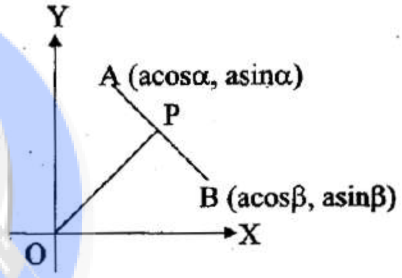
$y = mx \Rightarrow y = -\dfrac{1}{\sqrt{3}}x$

$\Rightarrow x + \sqrt{3}y = 0$



2. Mid point of $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ is

$$P\left(\frac{a(\cos\alpha + \cos\beta)}{2}, \frac{a(\sin\alpha + \sin\beta)}{2}\right)$$



$\therefore$ Slope of line AB is

$\dfrac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha} = \dfrac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha} = m_1$

and slope of OP is $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = m_2$

Now, $m_1 \times m_2 = \dfrac{\sin^2\beta - \sin^2\alpha}{\cos^2\beta - \cos^2\alpha} = -1$

Hence, the lines are perpendicular.

3. A line perpendicular to the line $5x - y = 1$ is given by $x + 5y - \lambda = 0 = L$

In intercept form $\dfrac{x}{\lambda} + \dfrac{y}{\lambda/5} = 1$

So, area of triangle is $\dfrac{1}{2} \times$ (Multiplication of intercepts)

$\Rightarrow \dfrac{1}{2}(\lambda) \times \left(\dfrac{\lambda}{5}\right) = 5$

$\Rightarrow \lambda = \pm 5\sqrt{2}$

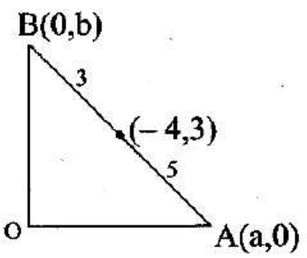
Hence, the equation of required line is $x + 5y = \pm 5\sqrt{2}$

4. Given, $a + b = 14 \Rightarrow a = 14 - b$
Hence, the equation of straight line is
$$\frac{x}{14-b} + \frac{y}{b} = 1$$
Also, it passes through (3, 4)
$$\therefore \quad \frac{3}{14-b} + \frac{4}{b} = 1$$
$\Rightarrow b = 8$ or 7
Therefore, equations are $4x + 3y = 24$ and $x + y = 7$

5. Mid point $\equiv \left(\frac{1+1}{2}, \frac{3-7}{2}\right) = (1, -2)$
Therefore, required line is
$$y + 2 = \frac{2}{3}(x - 1) \Rightarrow 2x - 3y = 8$$

6. Let the intercept be a and 2a, then the equation of line is $\frac{x}{a} + \frac{y}{2a} = 1$, but it also passes through (1, 2), therefore $a = 2$.
Hence, the required equation is $2x + y = 4$

7. $x\cos\theta - y\sin\theta = a(\cos^4\theta - \sin^4\theta) = a\cos 2\theta$

8.


By the section formula, we get $a = -\frac{32}{3}$ and $b = \frac{24}{5}$
Hence, the required equation is given by
$$\frac{x}{-(32/3)} + \frac{y}{(24/5)} = 1$$
$\Rightarrow 9x - 20y + 96 = 0$

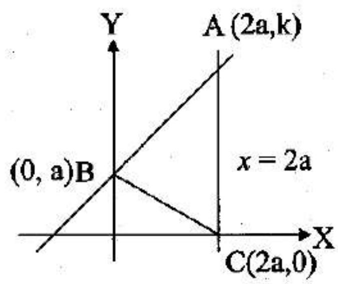
9. Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$
The co-ordinates of the mid point of the intercept AB between the axes are $\left(\frac{a}{2}, \frac{b}{2}\right)$
$$\therefore \quad \frac{a}{2} = 1, \frac{b}{2} = 2 \Rightarrow a = 2, b = 4$$
Hence, the equation of the line is
$$\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow 2x + y = 4$$

10. Point P(a, b) is on $3x + 2y = 13$
So, $3a + 2b = 13$ .....(i)
Point Q(b, a) is on $4x - y = 5$
So, $4b - a = 5$ .....(ii)
By solving (i) and (ii), we get
$a = 3, b = 2$
Now, equation of PQ is
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$\Rightarrow y - 2 = \frac{3-2}{2-3}(x - 3)$$
$\Rightarrow y - 2 = -(x - 3)$
$\Rightarrow x + y = 5$

11. Given, line AB makes 0 intercepts on X–axis and Y – axis so, $(x_1, y_1) = (0, 0)$
Slope of perpendicular $= \frac{4}{3}$
$\therefore$ Equation is $y - 0 = \frac{4}{3}(x - 0)$
$\Rightarrow 4x - 3y = 0$

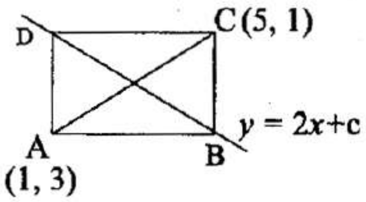
12. Perpendicular to $\sqrt{3}\sin\theta + 2\cos\theta = \frac{4}{r}$ is
$$\sqrt{3}\sin\left(\frac{\pi}{2} + \theta\right) + 2\cos\left(\frac{\pi}{2} + \theta\right) = \frac{k}{r}$$
It is passing through $(-1, \pi/2)$
$$\sqrt{3}\sin\pi + 2\cos\pi = \frac{k}{-1} \Rightarrow k = 2$$
$$\therefore \quad \sqrt{3}\cos\theta - 2\sin\theta = \frac{2}{r}$$
$$\Rightarrow 2 = \sqrt{3}r\cos\theta - 2r\sin\theta$$

13. Slope $= \frac{8-2}{3-1} = 3$
The diagonal is $y - 2 = 3(x - 1)$
$\Rightarrow 3x - y - 1 = 0$

14. Line AB will pass through (0, a) and (2a, k)


But as we are given AB = AC
$$\Rightarrow k = \sqrt{4a^2 + (k-a)^2} \Rightarrow k = \frac{5a}{2}$$
Hence, the required equation is $3x - 4y + 4a = 0$

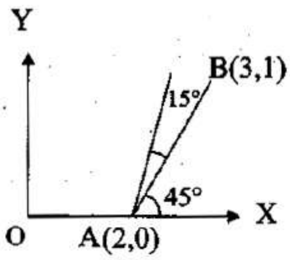
15. Let ABCD be a rectangle.
Given, A (1, 3) and C (5, 1).



Intersecting point of diagonal of a rectangle is same or at midpoint.
So, midpoint of AC is (3, 2).
Also, $y = 2x + c$ passes through (3, 2).
Hence, $c = -4$

16. Let p be the length of the perpendicular from the origin on the given line. Then its equation in normal form is
$x\cos 30° + y\sin 30° = p$ or $\sqrt{3}x + y = 2p$
This meets the coordinate axes at $A\left(\frac{2p}{\sqrt{3}}, 0\right)$
and B(0, 2p)
$\therefore$ Area of $\Delta OAB = \frac{1}{2}\left(\frac{2p}{\sqrt{3}}\right)2p = \frac{2p^2}{\sqrt{3}}$
$\Rightarrow \frac{2p^2}{\sqrt{3}} = \frac{50}{\sqrt{3}} \Rightarrow p = \pm 5$
Hence, the lines are $\sqrt{3}x + y \pm 10 = 0$

17. Here, slope of AB = 1
$\Rightarrow \tan\theta = m_1 = 1$
or $\theta = 45°$
$\therefore$ $\tan(45° + 15°) = \tan 60°$
($\because$ It is rotated anticlockwise so the angle will be $45° + 15° = 60°$)
Thus, slope of new line is $\sqrt{3}$

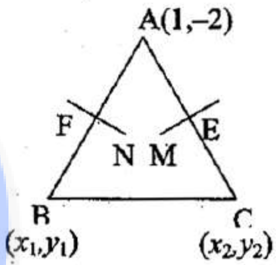


Hence, the equation is $y = \sqrt{3}x + c$, but it still passes through (2,0), $c = -2\sqrt{3}$
Thus, required equation is
$y = \sqrt{3}x - 2\sqrt{3}$

18. $\left(\frac{-2}{3a}\right)\left(\frac{-3}{4}\right) = -1$ or $a = \frac{-1}{2}$

19. The slope of line $x + y = 1$ is -1.
$\therefore$ It makes an angle of 135° with X-axis.
The equation of line passing through (1, 1) and making an angle of 135° is,
$$\frac{x-1}{\cos 135°} = \frac{y-1}{\sin 135°} = r$$
$$\Rightarrow \frac{x-1}{-1/\sqrt{2}} = \frac{y-1}{1/\sqrt{2}} = r$$
$\therefore$ Co-ordinates of any point on this line are $\left(1 - \frac{r}{\sqrt{2}}, 1 + \frac{r}{\sqrt{2}}\right)$
If this point lies on $2x - 3y = 4$, then
$$2\left(1 - \frac{r}{\sqrt{2}}\right) - 3\left(1 + \frac{r}{\sqrt{2}}\right) = 4$$
$\Rightarrow r = \sqrt{2}$

20. Let the equation of perpendicular bisector FN of AB is $x - y + 5 = 0$ ......(i)



The middle point F of AB is $\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$ Which lies on line (i).
$\therefore$ $x_1 - y_1 = -13$ .....(ii)
Also AB is perpendicular to FN. So the product of their slopes is -1.
i.e., $\frac{y_1+2}{x_1-1} \times 1 = -1$ or $x_1 + y_1 = -1$ ....(iii)
On solving (ii) and (iii), we get B(–7, 6)
Similarly, $C\left(\frac{11}{5}, \frac{2}{5}\right)$
Hence, the equation of BC is $14x + 23y - 40 = 0$

21. S = midpoint of QR $= \left(\frac{6+7}{2}, \frac{-1+3}{2}\right)$
$= \left(\frac{13}{2}, 1\right)$
$\therefore$ 'm' of PS $= \frac{2-1}{2 - \frac{13}{2}} = -\frac{2}{9}$
$\therefore$ The required equation is $y + 1 = \frac{-2}{9}(x - 1)$
i.e., $2x + 9y + 7 = 0$

22. Let the co-ordinates of axes are A (a, 0) and B(0, b), but the point (–5, 4) divides the line AB in the ratio of 1 : 2. Therefore, the co-ordinates of axes are $\left(\frac{-15}{2}, 0\right)$ and (0, 12).

Therefore, the equation of line passing through these coordinate axes is given by $8x - 5y + 60 = 0$

23. The equation of any line parallel to $2x + 6y + 7 = 0$ is $2x + 6y + k = 0$.

This meets the axes at $A\left(-\frac{k}{2}, 0\right)$ and $B\left(0, -\frac{k}{6}\right)$

By hypothesis, AB = 10

$\Rightarrow \sqrt{\frac{k^2}{4} + \frac{k^2}{36}} = 10 \Rightarrow \sqrt{\frac{10k^2}{36}} = 10$

$\Rightarrow 10k^2 = 3600 \Rightarrow k = \pm 6\sqrt{10}$

Hence, there are two lines given by $2x + 6y \pm 6\sqrt{10} = 0$

24. Take two perpendicular lines as the coordinate axes. If a, b be the intercepts made by the moving line on the coordinate axes, then the equation of the line is

$\frac{x}{a} + \frac{y}{b} = 1$ .....(i)

Let $\frac{1}{a} + \frac{1}{b} = \frac{1}{k}$

i.e., $\frac{k}{a} + \frac{k}{b} = 1$ .....(ii)

The result (ii) shows that the straight line (i) passes through a fixed point (k, k).

25. The equation of line passing through A(– 5, –4) is $\frac{x+5}{\cos\theta} = \frac{y+4}{\sin\theta}$

Let AB = r_1, AC = r_2, AD = r_3

The co-ordinate of B is

$(r_1 \cos\theta - 5, r_1 \sin\theta - 4)$

which lies on $x + 3y + 2 = 0$

$\therefore$ $r_1 = \frac{15}{\cos\theta + 3\sin\theta}$

Similarly, $\frac{10}{AC} = 2\cos\theta + \sin\theta$ and

$\frac{6}{AD} = \cos\theta - \sin\theta$

Putting in the given relation, we get $(2\cos\theta + 3\sin\theta)^2 = 0$

$\Rightarrow \tan\theta = -\frac{2}{3}$

$\therefore$ The equation of line is $y + 4 = -\frac{2}{3}(x + 5)$

$\Rightarrow 2x + 3y + 22 = 0$

26. Let the required line through the point (1,2) be inclined at an angle θ to the axis of X. Then its equation is $\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = r$ .....(i)

The co-ordinates of a point on the line (i) are $(1 + r\cos\theta, 2 + r\sin\theta)$

If this point is at a distance $\frac{\sqrt{6}}{3}$ form (1, 2), then $r = \frac{\sqrt{6}}{3}$

Therefore, the point is

$\left(1 + \frac{\sqrt{6}}{3}\cos\theta,\ 2 + \frac{\sqrt{6}}{3}\sin\theta\right)$.

But this point lies on the line $x + y = 4$

$\Rightarrow \frac{\sqrt{6}}{3}(\cos\theta + \sin\theta) = 1$ or

$\sin\theta + \cos\theta = \frac{3}{\sqrt{6}}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{\sqrt{3}}{2}$

....(Dividing both sides by $\sqrt{2}$)

$\Rightarrow \sin(\theta + 45^\circ) = \sin 60^\circ$ or $\sin 120^\circ$

$\Rightarrow \theta = 15^\circ$ or 75°

27. The four vertices on solving are A(–3, 3), B(1, 1), C(1, –1) and D(–2, –2).

m_1 = slope of AC = –1,

m_2 = slope of BD = 1

$\therefore$ $m_1 m_2 = -1$

Hence, the angle between diagonals AC and BD is 90°.

28. The point of intersection of the lines is (1, 1). and slope of the line $2y - 3x + 2 = 0$ is $\frac{3}{2}$

Hence, the equation is $y - 1 = \frac{3}{2}(x - 1)$

$\Rightarrow 3x - 2y = 1$

29. From option (B),

$$\begin{vmatrix} 1 & 2 & -10 \\ 2 & 1 & 5 \\ 5 & 4 & 0 \end{vmatrix} = 1(0-20)-2(-25)-10(3)=0$$

Hence, option (B) is the correct answer.

30. The point of intersection of $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ is $(-1, -1)$.
Now the line perpendicular to $3x - 5y + 11 = 0$ is $5x + 3y + k = 0$, but it passes through $(-1, -1)$
$\Rightarrow -5-3+k=0 \Rightarrow k=8$
Hence, required line is $5x + 3y + 8 = 0$.

31. The intersection point of lines $x - 2y = 1$ and $x + 3y = 2$ is $\left(\frac{7}{5}, \frac{1}{5}\right)$ and the slope of required line $= -\frac{3}{4}$

$\therefore$ Equation of required line is

$$y - \frac{1}{5} = \frac{-3}{4}\left(x - \frac{7}{5}\right)$$

$$\Rightarrow \frac{3x}{4} + y = \frac{21}{20} + \frac{1}{5} \Rightarrow 3x + 4y = 5$$

$$\Rightarrow 3x + 4y - 5 = 0$$

32. Intersection point of the line is $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$, which is satisfying all the equations given in options (A), (B) and (C).
Hence, (D) is correct.

33. Here, equation of AB is $x + 4y - 4 = 0$ .....(i)
and equation of BC is $2x + y - 22 = 0$ .....(ii)
Thus angle between (i) and (ii) is given by

$$\tan^{-1} \frac{-\frac{1}{4}+2}{1+\left(-\frac{1}{4}\right)(-2)} = \tan^{-1}\frac{7}{6}$$

34. Let θ be the acute angle which the line $y = mx + 4$ makes with the lines $y = 3x + 1$ and $2y = x + 3$.
Then,

$$\tan\theta = \left|\frac{m-3}{1+3m}\right| \text{ and } \tan\theta = \left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right|$$

$$\Rightarrow \left|\frac{m-3}{1+3m}\right| = \left|\frac{2m-1}{m+2}\right|$$

$$\Rightarrow m^2 - m - 6 = \pm(6m^2 - m - 1)$$

$$\Rightarrow 5m^2 + 5 = 0 \text{ or } 7m^2 - 2m - 7 = 0$$

$$\Rightarrow 7m^2 - 2m - 7 = 0$$

$$\Rightarrow m = \frac{1 \pm 5\sqrt{2}}{7}$$

35. $$\theta = \tan^{-1}\left|\frac{-\cot\alpha_1 + \cot\alpha_2}{1 + \cot\alpha_1\cot\alpha_2}\right|$$

$$= \tan^{-1}\left|\frac{\tan\alpha_1 - \tan\alpha_2}{1 + \tan\alpha_2\tan\alpha_1}\right| = \alpha_1 - \alpha_2$$

36. $$\frac{k-(2+\sqrt{3})}{1+k(2+\sqrt{3})} = \sqrt{3}$$

$$\Rightarrow k - 2 - \sqrt{3} = \sqrt{3} + 2k\sqrt{3} + 3k$$

$$\Rightarrow k = \frac{-2(1+\sqrt{3})}{2(1+\sqrt{3})} = -1$$

37. $$\begin{vmatrix} p-q & q-r & r-p \\ q-r & r-p & p-q \\ r-p & p-q & q-r \end{vmatrix} = \begin{vmatrix} 0 & q-r & r-p \\ 0 & r-p & p-q \\ 0 & p-q & q-r \end{vmatrix} = 0$$

[By $C_1 \to C_1 + C_2 + C_3$]
Hence, the lines are concurrent.

38. Check by options.
From option (A), we get

$$\begin{vmatrix} 3 & 4 & 6 \\ 6 & 5 & 9 \\ 2 & 3 & 5 \end{vmatrix} = 3(25-27) - 4(12) + 6(8) \neq 0$$

From option (B), we get

$$\begin{vmatrix} 3 & 4 & 6 \\ 6 & 5 & 9 \\ 3 & 3 & 5 \end{vmatrix} = 3(25-27) - 4(3) + 6(3) = 0$$

39. The three lines are concurrent, if

$$\begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & b & -1 \end{vmatrix} = 0$$

$$\Rightarrow 35a - 22b + 1 = 0$$

which is true if the line $35x - 22y + 1 = 0$ passes through (a, b).

40. If the given lines are concurrent, then

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

[By $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$\Rightarrow a(b-1)(c-1)-(b-1)(1-a)-(c-1)(1-a)=0$

$\Rightarrow \frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$

[Divide by $(1-a)(1-b)(1-c)$]

$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

41. It is given that the lines $ax+2y+1=0$, $bx+3y+1=0$ and $cx+4y+1=0$ are concurrent, therefore $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix}=0$

$\Rightarrow -a+2b-c=0 \Rightarrow 2b=a+c$

$\Rightarrow$ a, b, c are in A. P.

42. The set of lines is $4ax+3by+c=0$, where $a+b+c=0$.

Eliminating c, we get $4ax+3by-(a+b)=0$

$\Rightarrow a(4x-1)+b(3y-1)=0$

This passes through the intersection of the lines $4x-1=0$ and $3y-1=0$

i.e, $x=\frac{1}{4}, y=\frac{1}{3}$ i.e., $\left(\frac{1}{4},\frac{1}{3}\right)$.

43. Given lines are $3x+4y=5$, $5x+4y=4$, $\lambda x+4y=6$. These lines meet at a point if the point of intersection of first two lines lies on the third line.

From $3x+4y=5$ and $5x+4y=4$

We get $x=\frac{-1}{2}$, $y=\frac{13}{8}$

This lies on $\lambda x+4y=6$, if $\lambda\left(-\frac{1}{2}\right)+4\left(\frac{13}{8}\right)=6$

$\Rightarrow \lambda=1$

44. Equation of line through the point of intersection of lines $2x+3y+1=0$ and $3x-5y-5=0$ is given by

$(2+3k)x+(3-5k)y+(1-5k)=0$

Slope of line is given by

$\tan 45° = -\frac{(2+3k)}{3-5k}$

$\Rightarrow k=\frac{5}{2}$

$\therefore$ Equation of line is $19x-19y-23=0$

45. Equation of line passing through point of intersection of $u=0$ and $v=0$ is $u+kv=0$

$\therefore$ $(x+2y+5)+k(3x+4y+1)=0$

It is passing through (3, 2)

$\therefore$ $(3+2\times2+5)+k(3\times3+4\times2+1)=0$

$\therefore$ $k=-\frac{2}{3}$

$\therefore$ equation of line will be

$(x+2y+5)-\frac{2}{3}(3x+4y+1)=0$

$\Rightarrow 3x+2y-13=0$

46. Equation of line passing through point of intersection of $x+2y+3=0$ and $3x+4y+7=0$ is

$(x+2y+3)+k(3x+4y+7)=0$

$\Rightarrow (1+3k)x+(2+4k)y+3+7k=0$ .... (i)

Slope of equation (i) is $m_1=\frac{-(1+3k)}{2+4k}$

and slope of given line is $m_2=\frac{-1}{-1}=1$ ....(ii)

Since (i) and (ii) represent perpendicular lines.

$\therefore$ $m_1m_2=-1$

$\therefore$ $\frac{-(1+3k)}{(2+4k)}\times 1=-1$

$\therefore$ equation of required line is

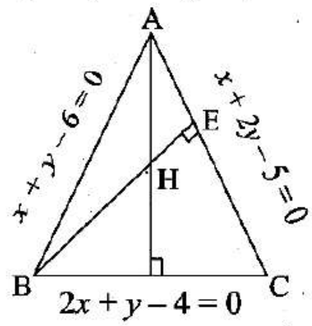
$(x+2y+3)-1(3x+4y+7)=0$

$\Rightarrow x+y+2=0$

47. Equation of AD is

$(x+y-6)+k(x+2y-5)=0$

$\Rightarrow (1+k)x+(1+2k)y-(6+5k)=0$ ....(i)



$\therefore$ Slope of AD $=m_1=\frac{-(1+k)}{(1+2k)}$

and Slope of BC $=m_2=-2$

$\therefore$ $=-1$ [$\because$ AD $\perp$ BC]

$\therefore$ $k=-\frac{3}{4}$

$\therefore$ From (i), equation of AD is

$x-2y=9$ .....(ii)

Similarly, equation of BE is

$2x-y=-12$ .....(iii)

By solving equation (ii) and (iii), we get

$x=-11, y=-10$

$\therefore$ $H\equiv(-11,-10)$

48. Any line through (1, –10) is given by
$y + 10 = m(x - 1)$
Since, it makes equal angle say 'α' with the given lines $7x - y + 3 = 0$ and $x + y - 3 = 0$

$\therefore \quad \tan\alpha = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)}$

$\Rightarrow m = \frac{1}{3}$ or -3

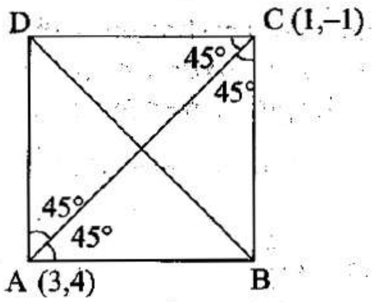
Hence, the two possible equations of third side are $3x + y + 7 = 0$, $x - 3y - 31 = 0$.

49. Putting k = 1, 2, we get
$3x + 2y = 12$ .....(i)
$4x + 3y = 19$ .....(ii)
The given lines are not parallel.
Hence on solving them, we get
$x = -2, y = 9$
Therefore, the lines pass through (–2, 9)

50. Since, the distance between the parallel lines $lx + my + n = 0$ and $lx + my + n' = 0$ is same as the distance between parallel lines $mx + y + n = 0$ and $mx + ly + n' = 0$. Therefore, the parallelogram is a rhombus. Since, the diagonals of a rhombus are at right angles, therefore the required angle is $\frac{\pi}{2}$.

51. Slope of AC = 5/2.
Let m be the slope of a line inclined at an angle of 45° to AC,

Then $\tan 45° = \pm\frac{m - \frac{5}{2}}{1 + m.\frac{5}{2}} \Rightarrow m = -\frac{7}{3}, \frac{3}{7}$



Thus, let the slope of AB or DC be $\frac{3}{7}$ and that of AD or BC be $-\frac{7}{3}$.
Then, equation of AB is $3x - 7y + 19 = 0$.
Also the equation of BC is $7x + 3y - 4 = 0$
On solving these equations, we get $B\left(-\frac{1}{2}, \frac{5}{2}\right)$

Now let the co-ordinates of the vertex D be (h, k). Since the middle points of AC and BD are same

$\therefore \quad \frac{1}{2}\left(h - \frac{1}{2}\right) = \frac{1}{2}(3+1) \Rightarrow h = \frac{9}{2}$

$\Rightarrow \frac{1}{2}\left(k + \frac{5}{2}\right) = \frac{1}{2}(4-1)$

$\Rightarrow k = \frac{1}{2}$

Hence, $D = \left(\frac{9}{2}, \frac{1}{2}\right)$

52. By the given condition of a + b + c = 0, the three lines reduce to

$x - y = \frac{p}{a}$ or $\frac{p}{b}$ or $\frac{p}{c} (p \neq 0)$.

All these lines are parallel. Hence, they do not intersect in finite plane.

53. Required line should be
$(3x - y + 2) + \lambda(5x - 2y + 7) = 0$ .....(i)
$\Rightarrow (3 + 5\lambda)x - (2\lambda + 1)y + (2 + 7\lambda) = 0$
$\Rightarrow \quad y = \frac{3+5\lambda}{2\lambda+1}x + \frac{2+7\lambda}{2\lambda+1}$ .....(ii)
As the equation (ii), has infinite slope,
$2\lambda + 1 = 0$
$\Rightarrow \lambda = -1/2$
Putting $\lambda = -1/2$ in equation (i) we have
$(3x - y + 2) + (-1/2)(5x - 2y + 7) = 0$
$\Rightarrow x = 3$

54. Here,

Slope of I[st] diagonal = $m_1 = \frac{2-0}{2-0} = 1$

$\Rightarrow \theta_1 = 45°$

Slope of II[nd] diagonal = $m_2 = \frac{2-0}{1-1} = \infty$

$\Rightarrow \theta_2 = 90°$

$\Rightarrow \theta_2 - \theta_1 = 45° = \frac{\pi}{4}$

55. Let the point be (h, k), then h + k = 4.....(i) and

$1 = \left|\frac{4h + 3k - 10}{\sqrt{4^2 + 3^2}}\right|$

$\Rightarrow 4h + 3k = 15$ .....(ii) and
$4h + 3k = 5$ .....(iii)
On solving (i) and (ii), and (i) and (iii), we get the required points (3, 1) and (–7, 11).

56. Here, $p = \left|\dfrac{-k}{\sqrt{\sec^2\alpha + \text{cosec}^2\alpha}}\right|$

and $p' = \left|\dfrac{-k\cos 2\alpha}{\sqrt{\cos^2\alpha + \sin^2\alpha}}\right|$

$\therefore\ 4p^2 + p'^2 = \dfrac{4k^2}{\sec^2\alpha + \text{cosec}^2\alpha} + \dfrac{k^2(\cos^2\alpha - \sin^2\alpha)^2}{1}$

$= 4k^2\sin^2\alpha\cos^2\alpha + k^2(\cos^4\alpha + \sin^4\alpha) - 2k^2\cos^2\alpha\sin^2\alpha$

$= k^2(\sin^2\alpha + \cos^2\alpha)^2$

$= k^2$

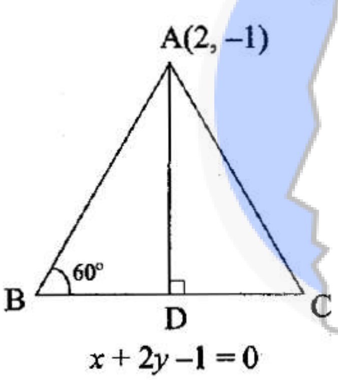
57. Let the distance of both lines be p_1 and p_2 from origin, then $p_1 = -\dfrac{8}{5}$ and $p_2 = -\dfrac{3}{5}$.

Hence, distance between both the lines

$= |p_1 - p_2| = \dfrac{5}{5} = 1$

58. $|AD| = \left|\dfrac{2-2-1}{\sqrt{1^2+2^2}}\right|$

$= \dfrac{1}{\sqrt{5}}$



$\tan 60° = \dfrac{AD}{BD}$

$\Rightarrow \sqrt{3} = \dfrac{1/\sqrt{5}}{BD}$

$\Rightarrow BD = \dfrac{1}{\sqrt{15}}$

$\therefore\ BC = 2BD = 2/\sqrt{15}$

59. $p_1.p_2 = \left(\dfrac{b\sqrt{a^2-b^2}\cos\theta + 0 - ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right) \times \left(\dfrac{-b\sqrt{a^2-b^2}\cos\theta - ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}\right)$

$= \dfrac{-[b^2(a^2-b^2)\cos^2\theta - a^2b^2]}{(b^2\cos^2\theta + a^2\sin^2\theta)}$

$= \dfrac{b^2[a^2 - a^2\cos^2\theta + b^2\cos^2\theta]}{b^2\cos^2\theta + a^2\sin^2\theta}$

$= \dfrac{b^2[a^2\sin^2\theta + b^2\cos^2\theta]}{b^2\cos^2\theta + a^2\sin^2\theta}$

$= b^2$

60. Lines $3x + 4y + 2 = 0$ and $3x + 4y + 5 = 0$ are on the same side of the origin. The distance between these lines is $d_1 = \left|\dfrac{2-5}{\sqrt{3^2+4^2}}\right| = \dfrac{3}{5}$.

Lines $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ are on the opposite sides of the origin. The distance between these lines is $d_2 = \left|\dfrac{2+5}{\sqrt{3^2+4^2}}\right| = \dfrac{7}{5}$.

Thus, $3x + 4y + 2 = 0$ divides the distance between $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ in the ratio $d_1 : d_2$ i.e., 3 : 7.

61. $2p = \left|\dfrac{0+0-1}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}}}\right|$

$\Rightarrow \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{4p^2} \Rightarrow \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{2}{8p^2}$

$\Rightarrow a^2, 8p^2, b^2$ are in H.P.

62. Lengths of perpendicular from (0,0) on the given lines are each equal to 2.

63. $L_{(-1,-1)} = 3(-1) - 8(-1) - 7 < 0$

$L_{(3,7)} = 3 \times 3 - 8 \times 7 - 7 < 0$

Hence, (–1, –1) and (3, 7) lie on the same side of line.

64. Let $L_1 \equiv 2x + 3y - 7 = 0$ and

$L_2 \equiv 2x + 3y - 5 = 0$

Here, slope of L_1 = slope of $L_2 = -\dfrac{2}{3}$

Hence, the lines are parallel.

65. Equation of any line through (0, a) is

$y - a = m(x - 0)$ or $mx - y + a = 0$ .....(i)

If the length of perpendicular from (2a, 2a) to the line (i) is 'a', then $a = \pm\dfrac{m(2a) - 2a + a}{\sqrt{m^2+1}}$

$\Rightarrow m = 0, \dfrac{4}{3}$

Hence, the required equations of lines are

$y - a = 0,\ 4x - 3y + 3a = 0$

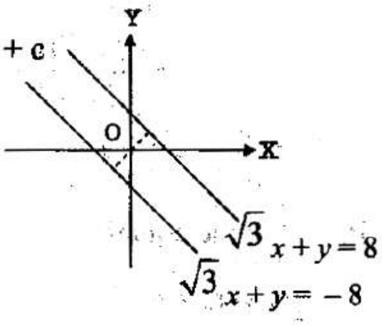
66. If the given lines represent the same line, then the length of the perpendiculars from the origin to the lines are equal, so that

$\dfrac{c}{\sqrt{1+m^2}} = \dfrac{p}{\sqrt{\cos^2\alpha + \sin^2\alpha}}$

$\Rightarrow c = p\sqrt{1+m^2}$

67. Point of intersection is (2, 3).
Therefore, the equation of line passing through (2, 3) is $y - 3 = m(x - 2)$
or $mx - y - (2m - 3) = 0$
According to the condition,
$$\left|\frac{3m - 2 - (2m - 3)}{\sqrt{1+m^2}}\right| = \frac{7}{5} \Rightarrow m = \frac{3}{4}, \frac{4}{3}$$
Hence, the equations are $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$.

68. Slope $= -\sqrt{3}$
∴ Line is $y = -\sqrt{3}\,x + c$
$\Rightarrow \sqrt{3}\,x + y = c$
Now $\frac{c}{2} = |4|$
$\Rightarrow c = \pm 8$
$\Rightarrow x\sqrt{3} + y = \pm 8$



69. Since, m (gradient) and x_1 are fixed and y_1 is variate, then they, form a set of parallel lines because gradient of every line remains 'm'.

70. The equation of lines passing through (1, 0) is given by $y = m(x - 1)$.
Its distance from origin is $\frac{\sqrt{3}}{2}$
$$\Rightarrow \left|\frac{-m}{\sqrt{1+m^2}}\right| = \frac{\sqrt{3}}{2} \Rightarrow m = \pm\sqrt{3}$$
Hence, the lines are $\sqrt{3}x + y - \sqrt{3} = 0$ and $\sqrt{3}x - y - \sqrt{3} = 0$

71. As $(\sin\theta, \cos\theta)$ and (3, 2) lie on the same side of $x + y - 1 = 0$, they should be of same sign.
∴ $\sin\theta + \cos\theta - 1 > 0$ as $3 + 2 - 1 > 0$
$$\Rightarrow \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) > 1$$
$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}} \Rightarrow 0 < \theta < \frac{\pi}{4}$$

72. Given form is $3x + 3y + 7 = 0$
$$\Rightarrow \frac{3}{\sqrt{3^2+3^2}}x + \frac{3}{\sqrt{3^2+3^2}}y + \frac{7}{\sqrt{3^2+3^2}} = 0$$
$$\Rightarrow \frac{3}{3\sqrt{2}}x + \frac{3}{3\sqrt{2}}y = \frac{-7}{3\sqrt{2}}$$
∴ $p = \left|\frac{-7}{3\sqrt{2}}\right| = \frac{7}{3\sqrt{2}}$

Competitive Thinking

1. The points are (1, 3) and (3, 15).
Hence, gradient $= \frac{15-3}{3-1} = \frac{12}{2} = 6$

2. $m_1 = \frac{6+4}{-2-3} = \frac{10}{-5} = -2$ and $m_2 = \frac{-18-6}{9-(-3)} = -2$
Hence, the lines are parallel.

3. Since, $m_1 m_2 = (2)\left(-\frac{1}{2}\right) = -1$
∴ the lines are perpendicular.

4. $m_1 m_2 = -1$
$$\Rightarrow \left(\frac{k-3}{2-4}\right)(2) = -1 \Rightarrow 2k - 6 = 2 \Rightarrow k = 4$$

5. The equation of a line perpendicular to $x - y = 0$ is $-x - y + c = 0$ ....(i)
Since, the line passes through (3, 2).
∴ $-3 - 2 + c = 0$
∴ $c = 5$
Putting $c = 5$ in (i), we get
$x + y = 5$

6. The given line is $bx - ay = ab$ ....(i)
It cuts X-axis at (a, 0).
The equation of a line perpendicular to (i) is $ax + by = k$.
Since, the line passes through (a, 0) $\Rightarrow k = a^2$
Hence, required equation of line is $ax + by = a^2$
i.e., $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$

7. The equation of a line perpendicular to $x + y + 1 = 0$ is $x - y + \lambda = 0$.
Since, the line passes through the point (1, 2).
∴ $1 - 2 + \lambda = 0$
$\Rightarrow \lambda = 1$
Hence, required equation of line is
$y - x - 1 = 0$

8. Let the equation be $\frac{x}{a} + \frac{y}{-a} = 1$.
$\Rightarrow x - y = a$ ....(i)
But, it passes through (–3, 2)
∴ $a = -3 - 2 = -5$
Putting the value of a in (i), we get
$x - y + 5 = 0$

9. The required equation passing through (–1, 1) and having gradient $\frac{3}{2}$ is

$y-1=\frac{3}{2}(x+1) \Rightarrow 2(y-1)=3(x+1)$

10. Midpoint ≡ (3, 2).

∴ the required equation is $y-2=2(x-3)$

$\Rightarrow 2x-y-4=0$

11. Any line through the middle point M(1, 5) of the intercept AB may be taken as

$\frac{x-1}{\cos\theta}=\frac{y-5}{\sin\theta}=r$ .....(i)

Since, the points A and B are equidistant from M and on the opposite sides of it.

Therefore, if the co-ordinates of A are obtained by putting r = d in (i), then the co-ordinates of B are given by putting r = –d.

Now, the point A(1 + d cosθ, 5 + d sinθ) lies on the line $5x-y-4=0$ and point B(1 – d cosθ, 5 – d sinθ) lies on the line $3x+4y-4=0$.

∴ $5(1+d\cos\theta)-(5+d\sin\theta)-4=0$

and $3(1-d\cos\theta)+4(5-d\sin\theta)-4=0$

Eliminating 'd', we get $\frac{\cos\theta}{35}=\frac{\sin\theta}{83}$

Hence, the required line is $\frac{x-1}{35}=\frac{y-5}{83}$ or

$83x-35y+92=0$.

12. Midpoint ≡ (2, 7)

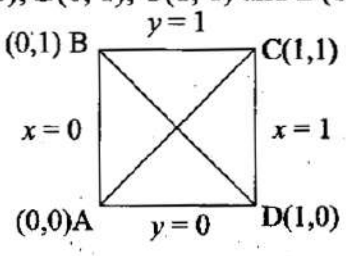
Slope of perpendicular = – 6

∴ the required equation is $y-7=-6(x-2)$

$\Rightarrow 6x+y-19=0$

13. The required diagonal passes through the midpoint of AB and is perpendicular to AB. So, its equation is $y-2=-3(x-2)$ or $y+3x-8=0$.

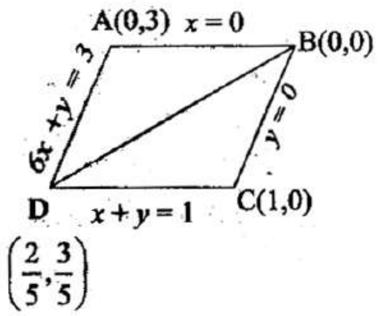
14. Co-ordinates of the vertices of the square are A(0, 0), B(0, 1), C(1, 1) and D(1, 0).



Now, the equation of AC is $y=x$ and of BD is

$y-1=-\frac{1}{1}(x-0) \Rightarrow x+y=1$

15.



From figure, diagonal BD is passing through origin, therefore its equation is given by

$\left(y-\frac{3}{5}\right)=\frac{-(3/5)}{-(2/5)}\left(x-\frac{2}{5}\right)$

$\Rightarrow 3x-2y=0$

16. Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$.

Given, a = b

So, equation of line is $x+y=a$

Since, this line passes through (2, 4).

∴ $2+4=a$

$\Rightarrow a=6$

∴ the required equation of line is $x+y=6$ i.e., $x+y-6=0$

17. Here, $a+b=-1$

∴ required line is $\frac{x}{a}-\frac{y}{1+a}=1$ .....(i)

Since, line (i) passes through (4, 3).

∴ $\frac{4}{a}-\frac{3}{1+a}=1$

$\Rightarrow 4+4a-3a=a+a^2$

$\Rightarrow a^2=4$

$\Rightarrow a=\pm 2$

∴ the required lines are $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$.

18. Equation of the line has its intercepts on the X-axis and Y-axis in the ratio 2 : 1 i.e., 2a and a

∴ $\frac{x}{2a}+\frac{y}{a}=1 \Rightarrow x+2y=2a$ .....(i)

Line (i) also passes through midpoint of (3, – 4) and (5, 2) i.e., (4, –1)

∴ $4+2(-1)=2a \Rightarrow a=1$

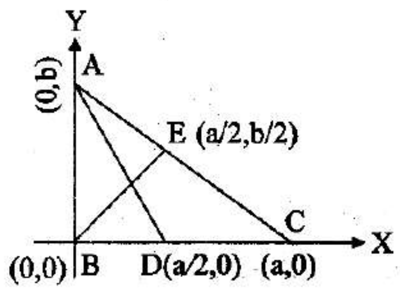
Hence, the equation of required line is

$x+2y=2$

19. $ax + by + c = 0$ always passes through $(1, -2)$.
$\therefore$ $a - 2b + c = 0 \Rightarrow 2b = a + c$
Therefore, a, b and c are in A.P.

20. Midpoint of the line joining the points $(4, -5)$ and $(-2, 9)$ is $\left(\frac{4-2}{2}, \frac{-5+9}{2}\right)$ i.e., $(1, 2)$
$\therefore$ Inclination of straight line passing through point $(-3, 6)$ and midpoint $(1, 2)$ is
$m = \frac{2-6}{1+3}$
$\Rightarrow \tan\theta = -1$
$\Rightarrow \theta = \frac{3\pi}{4}$

21. The equation of lines in intercept form are
$\frac{x}{-8/a} + \frac{y}{-8/b} = 1$
$\frac{x}{-3} + \frac{y}{2} = 1$
According to the given condition,
$-\frac{8}{a} = -(-3)$ and $-\frac{8}{b} = -2$
$\Rightarrow a = -\frac{8}{3}$ and $b = 4$

22.


From figure,
$\left(\frac{b/2}{a/2}\right)\left(\frac{b}{-a/2}\right) = -1$
$\Rightarrow a^2 = 2b^2 \Rightarrow a = \pm\sqrt{2}b$

23. Let the points of the required line on X-axis and Y-axis be A(a, 0) and B(0, b) respectively.
Since, $\left(\frac{3}{2}, \frac{5}{2}\right)$ is midpoint of AB.
$\therefore$ $\frac{a+0}{2} = \frac{3}{2}$ and $\frac{0+b}{2} = \frac{5}{2} \Rightarrow a = 3$ and $b = 5$
$\therefore$ the equation of line is $\frac{x}{3} + \frac{y}{5} = 1$
$\Rightarrow 5x + 3y - 15 = 0$

24. Since, the line makes an angle of measure 30° with Y-axis. Therefore, the line will make an angle of measure 60° or – 60° with X-axis.
$\therefore$ Slope of line = $\tan 60°$ or $\tan(-60°)$
$= \sqrt{3}$ or $-\sqrt{3} = \pm\sqrt{3}$

25. Since, l, m, n are in A.P.
$\therefore$ $2m = l + n$
Given equation of line is $lx + my = n = 0$
Consider, option (B),
If the point $(1, -2)$ satisfy the given equation.
$\therefore$ $l - 2m + n = 0 \Rightarrow 2m = l + n$
$\Rightarrow l$, m, n are A.P.

26. The required equation of line is
$\frac{x}{6} + \frac{y}{8} = 0 \Rightarrow 4x + 3y = 24$

27. Slope $= \frac{(2-1)}{1-\left(-\frac{1}{2}\right)} = \frac{1}{\left(\frac{3}{2}\right)} = \frac{2}{3}$
So, equation of the line is $y - 2 = \frac{2}{3}(x - 1)$
$\Rightarrow y = \frac{2}{3}x + \frac{4}{3}$
Putting $y = 0$, to find x-intercept, $\frac{2}{3}x + \frac{4}{3} = 0$
$\Rightarrow x = -2$
$\therefore$ x-intercept = – 2

28. Midpoint of given line segment = $(2, -1)$
Now, slope of the line segment $= \frac{-8}{8} = -1$
Slope of the required line segment is 1
$\therefore$ the required equation of line is $y + 1 = 1(x - 2)$
$\Rightarrow x - y = 3$

29. Here, the straight line is parallel to X-axis.
So, the slope of such a line = 0.

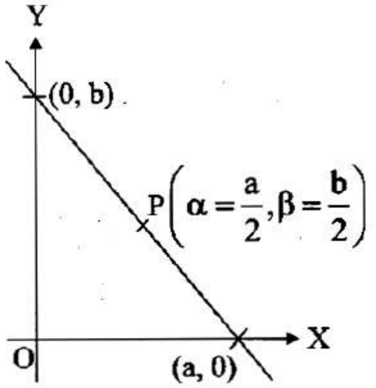
30. Since, the required line will be a line passing through A and B.
$\therefore$ $\frac{y-6}{6-(-4)} = \frac{x-1}{1-3}$
$\Rightarrow 10x - 10 = -2y + 12 \Rightarrow 5x + y - 11 = 0$

31. Since, $px - qy = r$ intersects at X-axis and Y-axis.
$\therefore$ $a = \frac{r}{p}$ and $b = -\frac{r}{q}$
$\therefore$ $a + b = \frac{r}{p} - \frac{r}{q} = r\left(\frac{q-p}{pq}\right)$

32. Let $P\left(\alpha=\frac{a}{2},\beta=\frac{b}{2}\right)$ be the midpoint of the line joining (a, 0) and (0, b).

$\therefore$ $\alpha=\frac{a}{2}\Rightarrow a=2\alpha$ ....(i)

and $\beta=\frac{b}{2}\Rightarrow b=2\beta$ ....(ii)



$\therefore$ Equation of a straight line cutting off intercepts a and b on X-axis and Y-axis respectively is

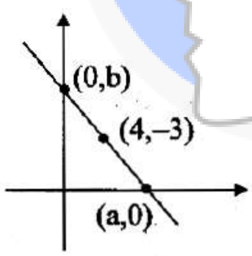
$\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{x}{2\alpha}+\frac{y}{2\beta}=1$ ....[From (i) and (ii)]

$\Rightarrow \frac{x}{\alpha}+\frac{y}{\beta}=2$

33. $\frac{a+0}{2}=4\Rightarrow a=8$

and $\frac{b+0}{2}=-3\Rightarrow b=-6$



$\therefore$ the required equation of the line is $\frac{x}{8}+\frac{y}{-6}=1$

$\Rightarrow \frac{3x-4y}{24}=1\Rightarrow 3x-4y=24$

34. Here, $m_1=-1$, $m_2=-\frac{1}{k}$.

For orthogonal lines,

$m_1m_2=-1\Rightarrow \frac{1}{k}=-1\Rightarrow k=-1$

35. Point of intersection of the lines is (3, –2)

Also, slope of perpendicular $=\frac{2}{7}$

Hence, the equation is $y+2=\frac{2}{7}(x-3)$

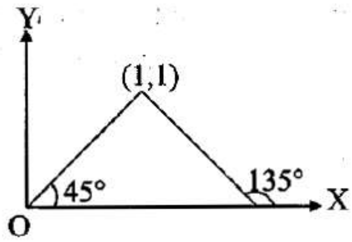
$\Rightarrow 2x-7y-20=0$

36. Point of intersection is $y=-\frac{21}{5}$ and $x=\frac{23}{5}$

$\therefore$ $3x+4y=\frac{3(23)+4(-21)}{5}=\frac{69-84}{5}=-3$

Hence, required line is $3x+4y+3=0$

37. Slopes of the lines are 1 and –1



Since, the point of intersection is (1, 1)

Hence, the required equations are

$y-1=\pm1(x-1)$

38. The lines are $bx+ay-ab=0$ and $bx-ay-ab=0$.

Hence, the required angle is

$\tan^{-1}\left|\frac{ab-(-ab)}{b^2+(-a^2)}\right|=\tan^{-1}\left|\frac{2ab}{b^2-a^2}\right|$

$=2\tan^{-1}\frac{b}{a}$ $\left[\because 2\tan^{-1}\frac{y}{x}=\tan^{-1}\left|\frac{2xy}{y^2-x^2}\right|\right]$

39. The given lines are perpendicular because $m_1m_2=(2)\left(\frac{-1}{2}\right)=-1$

Hence, the angle between the two lines is 90°.

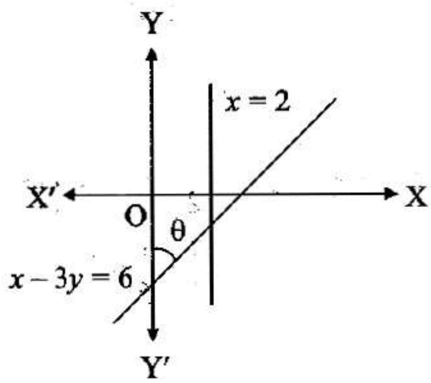
40. $a_1a_2+b_1b_2=\frac{1}{ab'}+\frac{1}{a'b}=0$

Therefore, the lines are perpendicular.

41. $\theta=90^\circ-\tan^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \tan\theta=\cot\left[\tan^{-1}\left(\frac{1}{3}\right)\right]=3$

$\Rightarrow \theta=\tan^{-1}(3)$



42. The slopes of the lines are $m_1 = \frac{-1}{2}$, $m_2 = 2$

$\therefore$ $m_1 m_2 = -1$

So, the lines are perpendicular i.e., $\theta = 90°$

43. Here, the given lines are

$ax + by + c = 0$

$bx + cy + a = 0$

$cx + ay + b = 0$

The lines will be concurrent, if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$

44. Here the lines are $x - 3 = 0$, $y - 4 = 0$ and $4x - 3y + a = 0$.

These will be concurrent, if

$\begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & -4 \\ 4 & -3 & a \end{vmatrix} = 0 \Rightarrow a = 0$

45. Given lines are concurrent, if $\begin{vmatrix} 2 & 1 & -1 \\ a & 3 & -3 \\ 3 & 2 & -2 \end{vmatrix} = 0$

$\Rightarrow - \begin{vmatrix} 2 & 1 & 1 \\ a & 3 & 3 \\ 3 & 2 & 2 \end{vmatrix} = 0$

This is true for all values of a because C_2 and C_3 are identical.

46. Lines are concurrent, if $\begin{vmatrix} 4 & 3 & -1 \\ 1 & -1 & 5 \\ b & 5 & -3 \end{vmatrix} = 0$

$\Rightarrow 4(3 - 25) - 3(-3 - 5b) - 1(5 + b) = 0$

$\Rightarrow -88 + 9 + 15b - 5 - b = 0$

$\Rightarrow -84 + 14b = 0$

$\Rightarrow b = 6$

47. $(a - 2b)x + (a + 3b)y + 3a + 4b = 0$

or $a(x + y + 3) + b(-2x + 3y + 4) = 0$, which represents a family of straight lines through point of intersection of $x + y + 3 = 0$ and $-2x + 3y + 4 = 0$ i.e, $(-1, -2)$.

48. a, b, c are in H. P., then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ .....(i)

Given, line is $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ .....(ii)

From (i) and (ii), we get

$\frac{1}{a}(x - 1) + \frac{1}{b}(y + 2) = 0$

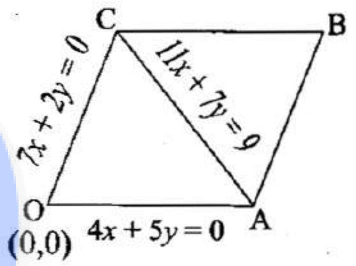
Since, $a \neq 0$, $b \neq 0$

So, $(x - 1) = 0$ and $(y + 2) = 0$

$\Rightarrow x = 1$ and $y = -2$

49. Two sides $x - 3y = 0$ and $3x + y = 0$ of the given triangle are perpendicular to each other. Therefore, its orthocentre is the point of intersection of $x - 3y = 0$ and $3x + y = 0$ i.e., $(0, 0)$.

50. Since, equation of diagonal $11x + 7y = 9$ does not pass through origin, so it cannot be the equation of the diagonal OB. Thus, on solving the equation AC with the equations OA and OC, we get $A\left(\frac{5}{3}, -\frac{4}{3}\right)$ and $C\left(\frac{-2}{3}, \frac{7}{3}\right)$



Therefore, the midpoint of AC is $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Hence, the equation of OB is $y = x$ i.e., $x - y = 0$.

51. The vertices of triangle are the intersection points of these given lines. The vertices of Δ are A(0, 4), B(1, 1), C(4, 0)

Now,

$AB = \sqrt{(0-1)^2 + (4-1)^2} = \sqrt{10}$

$BC = \sqrt{(1-4)^2 + (1-0)^2} = \sqrt{10}$

$AC = \sqrt{(0-4)^2 + (4-0)^2} = 4\sqrt{2}$

$\because$ AB = BC

$\therefore$ Δ is isosceles.

52. Dividing both sides of relation $3a + 2b + 4c = 0$ by 4, we get $\frac{3}{4}a + \frac{1}{2}b + c = 0$, which shows that for all values of a, b and c each member of the set of lines $ax + by + c = 0$ passes through the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

53. Given lines are $ax + by + c = 0$
and $x = \alpha t + \beta$, $y = \gamma t + \delta$
After eliminating t, we get
$\gamma x - \alpha y + \alpha\delta - \gamma\beta = 0$
For parallelism condition,
$\frac{a}{\gamma} = \frac{b}{-\alpha} \Rightarrow a\alpha + b\gamma = 0$

54. The equation of a straight line passing through (3, –2) is $y + 2 = m(x - 3)$ .....(i)
The slope of the line $\sqrt{3}x + y = 1$ is $-\sqrt{3}$
So, $\tan 60° = \pm \frac{m - (-\sqrt{3})}{1 + m(-\sqrt{3})}$
On solving, we get
$m = 0$ or $\sqrt{3}$
Putting the values of m in (i), the required equation of lines are $y + 2 = 0$ and
$\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

55. Since, the point (–4, 5) does not lie on the diagonal $7x - y + 8 = 0$, so point will lie on the other diagonal.
Also, diagonals are perpendicular.
∴ Slope of other diagonal = $\frac{-1}{7}$
∴ equation of the other diagonal is
$y - 5 = -\frac{1}{7}(x + 4) \Rightarrow 7y + x = 31$

56. Required equation of line which is parallel to $x + 2y = 5$ is $x + 2y + k = 0$ ....(i)
Given equation of lines are
$x + y = 2$ ....(ii)
$x - y = 0$ ....(iii)
Adding (ii) and (iii), we get $2x = 2 \Rightarrow x = 1$
From (iii), we get $y = 1$
∴ Point of intersection is (1, 1).
Putting $x = 1, y = 1$ in (i), we get $k = -3$
∴ the required equation of line is $x + 2y = 3$.

57. The lines passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ is
$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$
$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$ ...(i)
Line (i) is parallel to X-axis,
$a + b\lambda = 0 \Rightarrow \lambda = \frac{-a}{b}$
Putting the value of λ in (i), we get
$ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$
$\Rightarrow y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$
$\Rightarrow y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$
$\Rightarrow y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$
So, it is 3/2 unit below X-axis.

58. Here, $m_1 = -\cot\alpha$, $m_2 = \tan\beta$
∴ $\tan\theta = \left|\frac{-\cot\alpha - \tan\beta}{1 - \cot\alpha\tan\beta}\right|$
∴ $\tan\theta = -\cot(\alpha - \beta)$
∴ $\theta = \frac{\pi}{2} - \beta + \alpha$

59. The point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ are $\left(\frac{-4}{5}, \frac{7}{5}\right)$.
The equation of line which makes equal intercepts with the axes is $x + y = a$.
∴ $-\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$
∴ the required equation of the line is
$x + y - \frac{3}{5} = 0$ i.e., $5x + 5y - 3 = 0$

60. $x - 3y = 1$ ....(i)
and $x^2 - 4y^2 = 1$ ....(ii)
On solving (i) and (ii), we get
A(1,0) and B$\left(-\frac{13}{5}, -\frac{6}{5}\right)$
These are the points of intersection of the straight line and hyperbola.
∴ Length of straight line intercepted by the hyperbola
$= \sqrt{\left(-\frac{13}{5} - 1\right)^2 + \left(-\frac{6}{5}\right)^2}$
$= \sqrt{\left(-\frac{18}{5}\right)^2 + \left(-\frac{6}{5}\right)^2} = \sqrt{\frac{324 + 36}{25}}$
$= \sqrt{\frac{360}{25}} = \frac{6}{5}\sqrt{10}$ units

61. The point of intersection of the given lines are (–1, 1), (1, –1) and (2/3, 2/3) which is the vertices of an isosceles triangle.

62. Let the point be (h, 0), then $a = \left|\dfrac{bh + 0 - ab}{\sqrt{a^2+b^2}}\right|$

$\Rightarrow bh = \pm a\sqrt{a^2+b^2} + ab$

$\Rightarrow h = \dfrac{a}{b}(b \pm \sqrt{a^2+b^2})$

Hence, the points are $\left\{\dfrac{a}{b}(b \pm \sqrt{a^2+b^2}), 0\right\}$.

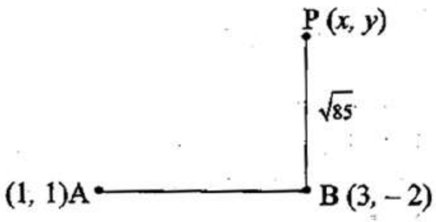
63. Here, the lines are $3x + 4y - 9 = 0$ and $6x + 8y - 15 = 0$ or $3x + 4y - \dfrac{15}{2} = 0$.

$\therefore$ Required distance $= \left|\dfrac{-9-\left(\dfrac{-15}{2}\right)}{\sqrt{3^2+4^2}}\right| = \left|\dfrac{-3}{10}\right| = \dfrac{3}{10}$

64. The line is $4x - 3y - 12 = 0$.

$\therefore$ Required length $= \left|\dfrac{-12}{\sqrt{4^2+(-3)^2}}\right| = \dfrac{12}{5} = 2\dfrac{2}{5}$

65. From option (C),



$BP = \sqrt{(5-3)^2 + (7+2)^2}$

$= \sqrt{4+81} = \sqrt{85}$

Hence, option (C) is correct.

66. Let p be the length of the perpendicular from the vertex (2, –1) to the base $x + y = 2$.

Then, $p = \left|\dfrac{2-1-2}{\sqrt{1^2+1^2}}\right| = \dfrac{1}{\sqrt{2}}$

If 'a' is the length of the side of triangle, then $p = a\sin 60^\circ$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{a\sqrt{3}}{2}$

$\Rightarrow a = \sqrt{\dfrac{2}{3}}$

67. Distance between lines $-x + y = 2$ and $x - y = 2$ is $\alpha = \left|\dfrac{2+2}{\sqrt{2}}\right| = 2\sqrt{2}$ ....(i)

Distance between lines $4x - 3y = 5$ and $6y - 8x = 1$ is

$\beta = \left|\dfrac{5-\left(\dfrac{-1}{2}\right)}{5}\right| = \dfrac{11}{10}$ ....(ii)

From (i) and (ii), we get

$\dfrac{\alpha}{\beta} = \dfrac{2\sqrt{2}}{11/10}$

$\Rightarrow 20\sqrt{2}\beta = 11\alpha$

68. $L = 2x + 3y - 4 = 0$,

$L_{(-6,2)} = -12 + 6 - 4 < 0$

$L' = 6x + 9y + 8 = 0$,

$L'_{(-6,2)} = -36 + 18 + 8 < 0$

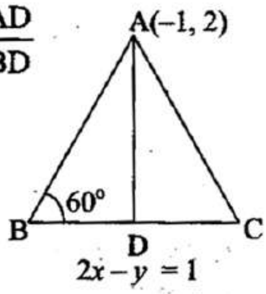
Hence, the point is below both the lines.

69. $AD = \left|\dfrac{-2-2-1}{\sqrt{(2)^2+(-1)^2}}\right| = \left|\dfrac{-5}{\sqrt{5}}\right| = \sqrt{5}$

Since, $\tan 60^\circ = \dfrac{AD}{BD}$

$\Rightarrow \sqrt{3} = \dfrac{\sqrt{5}}{BD}$

$\Rightarrow BD = \sqrt{\dfrac{5}{3}}$



$\therefore$ $BC = 2BD$

$= 2\sqrt{\dfrac{5}{3}} = \sqrt{\dfrac{20}{3}}$

70. $L_{12} \equiv x - 3y + 1 = 0$

$L_{23} \equiv 2x + y - 12 = 0$

$L_{13} \equiv 3x - 2y - 4 = 0$

Therefore, the required distances are

$D_3 = \left|\dfrac{4 - 3\times4 + 1}{\sqrt{10}}\right| = \dfrac{7}{\sqrt{10}}$

$D_1 = \left|\dfrac{4+1-12}{\sqrt{5}}\right| = \dfrac{7}{\sqrt{5}}$

$D_2 = \left|\dfrac{3\times5 - 2\times2 - 4}{\sqrt{9+4}}\right|$

$= \dfrac{7}{\sqrt{13}}$

71. Gradient of BC = –1 and its equation is $x + y + 4 = 0$. Therefore, the equation of line parallel to BC is $x + y + \lambda = 0$.

Also, it is $\frac{1}{2}$ unit distant from origin.

Thus, $\frac{\lambda}{\sqrt{2}} = \frac{1}{2} \Rightarrow \lambda = \frac{\sqrt{2}}{2}$

Hence, the required equation of line is $2x + 2y + \sqrt{2} = 0$

72. Line L passes through (13, 32).

∴ $\frac{13}{5} + \frac{32}{b} = 1$

$\Rightarrow b = -20$

So, equation of L is $\frac{x}{5} - \frac{y}{20} = 1 \Rightarrow 4x - y = 20$

Slope of L is $m_1 = 4$.

Slope of $\frac{x}{c} + \frac{y}{3} = 1$ is $m_2 = -\frac{3}{c}$

$\Rightarrow -\frac{3}{c} = 4$

$\Rightarrow c = -\frac{3}{4}$

Equation of line K is $-\frac{4x}{3} + \frac{y}{3} = 1$

$\Rightarrow 4x - y = -3$

Distance between L and K is $\left|\frac{20+3}{\sqrt{16+1}}\right| = \frac{23}{\sqrt{17}}$

73. Equation of straight line parallel to $4x - 3y = 5$ is $4x - 3y = \lambda$

According to the given condition,

$\frac{4(-1)-3(-4)-\lambda}{\sqrt{16+9}} = \pm 1$

$\Rightarrow 8 - \lambda = \pm 5$

$\Rightarrow \lambda = 3, 13$

∴ the equation of one of the lines is $4x - 3y - 3 = 0$

74. Given, equation of line is

$\frac{x\sin\alpha}{b} - \frac{y\cos\alpha}{a} - 1 = 0$

∴ perpendicular distance from origin

$= \left|\frac{0.\frac{\sin\alpha}{b} - \frac{0.\cos\alpha}{a} - 1}{\sqrt{\frac{\sin^2\alpha}{b^2} + \frac{\cos^2\alpha}{a^2}}}\right| = \frac{|ab|}{\sqrt{a^2\sin^2\alpha + b^2\cos^2\alpha}}$

75. Since $x + y = 4$ and $2x + 2y = 5$ are parallel.

Take (4, 0) on the line $x + y = 4$.

Distance of (4, 0) from the line $2x + 2y - 5 = 0$

$\frac{|2.4+2.0-5|}{\sqrt{2^2+2^2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4} > 1$

∴ Both lines are parallel and at a distance greater than unity.

∴ There is no point on the line $x + y = 4$.

76. Given equation of parallel lines are

$x - y + a = 0, x - y + b = 0$

∴ required distance $= \left|\frac{a-b}{\sqrt{(1)^2+(-1)^2}}\right| = \frac{|a-b|}{\sqrt{2}}$

77. Slope of given line $ax + by + c = 0$ is $-\frac{a}{b}$.

∴ $-\frac{a}{b} = \pm 1 \Rightarrow a = \pm b$ ....(i)

Distance of line $ax + by + c = 0$ from (1, –2)

$= \frac{|a-2b+c|}{\sqrt{a^2+b^2}}$

Distance of line $ax + by + c = 0$ from (3, 4)

$= \frac{|3a+4b+c|}{\sqrt{a^2+b^2}}$

According to the given condition,

$\frac{|a-2b+c|}{\sqrt{a^2+b^2}} = \frac{|3a+4b+c|}{\sqrt{a^2+b^2}}$

$\Rightarrow 3a + 4b + c = \pm(a - 2b + c)$

$\Rightarrow a + 3b = 0$ (taking +ve) ....(ii)

$\Rightarrow 2a + b + c = 0$ (taking –ve) ....(iii)

From, (i) and (ii), we get a = b = 0 which is not possible so taking (i) and (iii), (taking a = – b) we get

$a + c = 0 \Rightarrow c = -a$

$a : b : c = a : -a : -a = 1 : -1 : -1$

or $a = 1, b = -1, c = -1$

From (i) and (iii) (taking a = b), we get

$3a + c = 0 \Rightarrow c = -3a$

$a : b : c = a : a : -3a = 1 : 1 : -3$

∴ option (B) is the correct answer.

78. Equation of the line is

$y - 0 = \left(\frac{3-0}{-5}\right)(x-5)$

$\Rightarrow 3x + 5y - 15 = 0$

∴ $d = \left|\frac{3(4)+5(4)-15}{\sqrt{3^2+5^2}}\right| = \frac{17}{\sqrt{34}} = \sqrt{\frac{17}{2}}$