12 Sets, Relations and Functions

Formulae

Sets :

1. A set is well defined class or collection of objects.
   A set is described in the following two ways:
   i. Roster or Tabulation or Enumeration method
   ii. Set Builder or Rule or Property method.

2. **Types of sets:**

   i. **Null set or Empty set:** It is denoted by ϕ or{ }
      a. ϕ is unique
      b. ϕ is subset of every set
      c. ϕ is never written within brackets
         i.e., $\{\phi\}$ is not the null set.

   ii. **Singleton set :** The set $\{\phi\}$ is a singleton set.

   iii. **Finite set:** A = (a, e, i, o, u} is a finite set.
      a. Cardinal number of a finite set:
         It is denoted by n(A) or o(A)

   iv. **Infinite set:**
      A = (1, 2, 3, 4.....} is an infinite set.

   v. **Equivalent set:**
      Two finite sets A and B are equivalent if fn(A) = n(B)

   vi. **Equal sets:**
      Two sets A and B are equal iff A = B

   vii. **Universal set:** Superset of all the sets. It is usually denoted by Q or S or U or X.

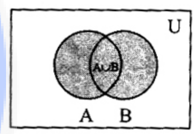
   viii. **Power set:** The family of all the subsets of set S is called the power set of S.
      It is denoted by P(S) i.e.,
      $P(S) = \{T : T \subseteq S\}$

   ix. **Subsets:** If A is subset of B, then
      $A \subseteq B \Rightarrow a \in A \Rightarrow a \in B$
      a. Every set is a subset of itself i.e.,
         $A \subseteq A$
      b. ϕ is a subset of every set.
      c. If $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$
      d. $A = B$ iff $a \subseteq B$ and $B \subseteq A$

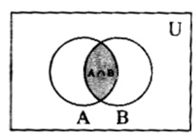
   Proper Subsets:
   If A is a proper subset of B, then $A \subset B$.
      a. If $A \subseteq B$, we may have $B \subseteq A$
      b. But if $A \subset B$, we cannot have $B \subset A$.

3. **Operations on Sets:**

   i. **Union of sets:** Let **A** and **B** be two sets.
      Then, $A \cup B$
      $=\{x : x \in A \text{ or } x \in B\}$
      $A \subseteq A \cup B$, $B \cup A$
      and $A \cup B = B \cup A$

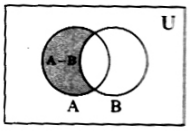


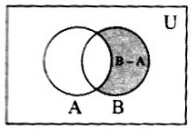
   ii. **Intersection of sets:** Intersection of two sets A and B is denoted by $A \cap B$
      $= \{x : x \in A \text{ and } x \in B)$
      $\therefore$ $A \cap B \subseteq A$, $A \cap B \subseteq B$ and
      $A \cap B = B \cap A$



   iii. **Disjoint sets:** Two sets A and B are s.t.b disjoint if $A \cap B = \phi$

   iv. **Difference of sets:** Let A and B be two sets, then $A - B = \{x : x \in A \text{ and } x \in B\}$ and
      $B - A = \{x : x \in B \text{ and } x \in A)$





a. $A - B \neq B - A$

b. The sets $A - B$, $B - A$ and $A \cap B$ are disjoint sets.

c. $A - B \subseteq A$ and $B - A \subseteq B$

d. $A - \phi = A$ and $A - A = \phi$

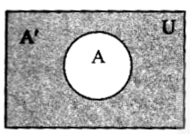
It is denoted by $A - B$ or $A \sim B$ or $A \backslash B$ or $C_A B$ (complement of B in A).

v. **Symmetric difference of two sets:** is denoted by $A \Delta B = (A - B) \cup (B - A)$
$= (A \cup B) - (A \cap B)$

vi. **Complement of a set:**

The complement of A with respect to U is denoted by A' or A^c or C(A) or U – A

i.e., $A' = \{x \in U : x \notin A\}$



4. If X is the universal set and A, B $\subseteq$ X, then

i. $(A')' = A$

ii. $X' = \phi$

iii. $\phi' = X$

iv. $A \cap A' = \phi$

v. $A \cup A' = X$

vi. If $A \subseteq B$, then $B' \subseteq A'$

5. If A, B, C are subsets of universal set X, then for

	Union of sets	**Intersection of sets**
i.	$A \cup \phi = A$	$A \cap \phi = \phi$
ii.	$A \cup X = X$	$A \cap X = A$
iii.	$A \cup B = B \cup A$	$A \cap B = B \cap A$
iv.	$(A \cup B) \cup C$ $= A \cup (B \cup C)$	$(A \cap B) \cap C$ $= A \cap (B \cap C)$
v.	$A \cup A = A$	$A \cap A = A$
vi.	$A \subseteq B$ $\Rightarrow A \cup B = B$	$A \subseteq B$ $\Rightarrow A \cap B = A$

6. Distributive Properties of union and intersection :

i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

7. De-Morgan's laws:

i. $(A \cup B)' = A' \cap B'$

ii. $(A \cap B)' = A' \cup B'$

iii. $A - (B \cap C) = (A - B) \cup (A - C)$

iv. $A - (B \cup C) = (A - B) \cap (A - C)$

8. If A, B and C are any three sets, then

i. $A \cap (B - C) = (A \cap B) - (A \cap C)$

ii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

iii. $P(A) \cap P(B) = P(A \cap B)$

iv. $P(A) \cup P(B) = P(A \cup B)$

v. If $P(A) = P(B) \Rightarrow A = B$

where, P(A) is the power set of A.

9. More Results on operations on sets:

For any sets A and B, we have

i. $A \subseteq A \cup B, B \subseteq A \cup B, A \cup B \subseteq A,$
$A \cap B \subseteq B$

ii. $A - B = A \cap B'$, $B - A = B \cap A'$

iii. $(A - B) \cap B = \phi$

iv. $(A - B) \cup B = A \cup B$

v. $A \subseteq B \Leftrightarrow B' \subseteq A'$

vi. $A - B = B' - A'$

vii. $(A \cup B) \cap (A \cup B') = A$

viii. $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

ix. $A - (A - B) = A \cap B$

x. $A - B = B - A \Leftrightarrow A = B$ and
$A \cup B = A \cap B \Rightarrow A = B$

10. Results on cardinal number of some sets:

If A, B and C are finite sets and U be the universal set, then

i. $n(A \cup B) = n(A) + n(B)$ if A and B are disjoint sets.

ii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

iii. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

iv. $n(A) = n(A-B) + n(A \cap B)$

$n(B) = n(B-A) + n(A \cap B)$

Here $n(A-B) = n(A) - n(A \cap B)$

and $n(A-B) = n(A \cup B) - n(B)$

v. $n(A') = n(U) - n(A)$

vi. $n(A' \cap B') = n(A \cap B)'$

vii. $n(A' \cap B') = n(A \cap B)' = n(U) - n(A \cap B)$

viii. $n(A \cap B') = n(A) - n(A \cap B)$

ix. $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$

x. $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

xi. If A_1, A_2, A_3 ..... A_n are disjoint sets, then $n(A_1) \cup A_2 \cup A_3$ ..... $\cup A_n)$ $= n(A_1) + n(A_2) + n(A_3)$ ......+ $n(A_n)$

xii. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

xiii. $n(A \cap B' \cap C')$

$= n(A) - n(A \cap B) - n(A \cap C)$

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$n(B \cap A' \cap C')$

$= n(B) - n(B \cap C) - n(B \cap A) + n(A \cap B \cap C)$

$n(C \cap A' \cap B')$

$= n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C)$

Relations :

1. Cartesian Product of sets :

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$. If A, B and C are three sets then,

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

iii. $A \times (B - C) = (A \times B) - (A \times C)$

iv. If $A \subseteq B$, then $(A \times C) \subseteq (B \times C)$

v. If $A \subseteq B$, then $(A \times B) \cap (B \times A) = A^2$

vi. If $A \subseteq B$ and $C \subseteq D$ then $A \times C \subseteq B \times D$

vii. $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$, where S and T are two sets.

2. Relation:

If R is a relation from A to B then $R \subseteq A \times B$ i.e.,$R \subseteq \{(a,b):a \in A, b \in B\}$

3. Number of possible relations from A to B $= 2^{mn}$ [if o(A) = m and o(B) = n]

4. Domain and Range of a Relation:

i. Domain of R = (a : (a, b) $\in$ R}

ii. Range of R = {b : (a, b) $\in$ R}

If R is a relation from A to B, then Dom (R) $\subseteq$ A and Range (R) $\subseteq$ B (i.e., Co-domain)

5. Inverse Relation:

The inverse of R, denoted by R^{-1} is a relation from B to A and is defined by

$R^{-1} = \{(b, a) : (a, b) \in R\}$

Thus,

i. $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1} \ \forall\ a \in A, b \in B$

ii. Dom (R^{-1}) = Range (R)

iii. Range (R^{-1}) = Dom (R)

iv. $(R^{-1})^{-1} = R$

6. Equivalence Relation:

A relation R on a set 'A' is said to be equivalence relation on a iff

i. It is reflexive i.e.,$(a, a) \in R \ \forall\ a \in A$

ii. It is symmetric i.e., $(a, b) \in R$ $\Rightarrow (b, a) \in R \ \forall\ a, b \in A$

iii. It is transitive i.e., $(a, b) \in R$ and $(b, c) \in R$ $\Rightarrow$ (a, c) e R $\forall$ a, b, c, $\in$ A

iv. It is antisymmetric i.e., $(a, b) \in R$ and $(b, a) \in R$, then a = b.

7. Composition of two Relations: If A, B and C are three sets such that $R \subseteq A \times B$ and $S \subseteq B \times C$, then $SoR \subseteq A \times C$ and $(SoR)^{-1} = R^{-1}oS^{-1}$. It is clear that aRb, bSc $\Rightarrow$ a(SoR)c

Functions:

1. $f : X \to Y : \Rightarrow$ f is a function from set X to set Y, if to each element $x \in X$, $\exists$ a unique element $y \in Y$.

2. Domain and Range:

i. Domain : All possible values of x for which f(x) exists.

ii. Range : All possible values of f(x), for all values of x.
i.e., $R_f = \{y \in Y : y = f(x)\}$

iii. $R_f \subseteq$ Co-domain

3. One-One function or Injection:

A function $f : X \to Y$ is one-one iff

i. $x \neq y$

$\Rightarrow f(x) \neq f(y)$

ii. $f(x) = f(y)$

$\Rightarrow x = y$

4. Onto function or Surjection:

A function $f : X \to Y$ is onto iff Range of f = Co-domain of f.

5. Into Function:

A function $f: X \to Y$ is an into function, if there exists an element in Y having no pre-image in X.

6. Many-One function:

$f : X \to Y$ is a many-one function, if x,

$\exists$ x, y $\in$ X such that $x \neq y$, but $f(x) = f(y)$

7. Bijective function:

A function both injective and surjective is called bijective function.

8. Inverse of a function:

If $f : X \to Y$ be a one-one, onto function, then the mapping $f^{-1} : Y \to X$ such that $f^{-1}(y) = x$ is called inverse of the function $f: X \to Y$.

9. Modulus or Absolute value or Numerical function:

$|x| = x$ if $x > 0$
$= -x$ if $x < 0$
$= 0$ if $x = 0$

for $f : R \to R$, $D_f = R$, $R_F = R^+$

10. Signum function:

$\text{sign}(x) = \frac{|x|}{x}$ if $x \neq 0$
$= 0$ if $x = 0$

Or, sign (x) = 1 if $x > 0$
$= -1$ if $x < 0$
$= 0$ if $x = 0$

For $f : R \to R$, $D_f = R$
$R_f = \{-1, 0, 1\}$

11. The Greatest Integer function or step function or floor function:

$\forall x \in R$ Let [x] denotes the greatest Integer in x.

i. $[x] = x$ when $x \in I$

ii. $[x] = 0$ when $0 \leq x < 1$

iii. $[x] < x$ when $x \notin I$

iv. $[x] = k$ when $k \leq x < k + 1$, if $x \in I$

v. $[x] \leq x < [x] + 1$

12. Even and Odd Function:

i. Even function:

If $f(-x) = f(x)$ $\forall x \in$ domain

ii. Odd function:

If $f(-x) = -f(x)$ $\forall x \in$ domain

13. Periodic function:

A function f(x) is s.t.b periodic function if $f(x + T) = f(x)$ $\forall x \in$ domain. Here the least + ve value of T is called the period of the function.

14. Operations on functions:

i. $(f + g)(x) = f(x) + g(x)$

ii. $(f - g)(x) = f(x) - g(x)$

iii. $(f.g)(x) = f(x).g(x)$

iv. $(f / g)(x) = \frac{f(x)}{g(x)}$; $g(x) \neq 0$

v. $(kf)(x) = kf(x)$

15. Some special functions:

i. if $f(x + y) = f(x) + f(y)$, then $f(x) = kx$

ii. if $f(xy) = f(x) + f(y)$, then $f(x) = \log x$

iii. if $f(x + y) = f(x).f(y)$, then $f(x) = e^x$

iv. if $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$,

then $f(x) = x^n \pm 1$

Shortcuts

1. If A has n elements, then P(A) has 2^n elements.
2. The total number of subsets of a finite set containing n elements is 2^n.
3. Number of proper subsets of A, containing n elements is $2^n - 1$.
4. Number of non-empty subsets of A, containing n elements is $2^n - 1$
5. Number of elements in exactly two of the sets A, B and C

$= n(A \cap B) + n(B \cap C) + n(C \cap A)$

$-3n(A \cap B \cap C)$

6. Number of elements in exactly one of the sets A, B and C.

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C)$

$-2n(A \cap C) + 3n(A \cap B \cap C)$

7. Number of elements which belong to exactly one of A or B

i.e., $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

8. The idnetiry relation on a set A is an anti-symmetric relation.

9. The relation is 'congruent to' on the set T of all triangles in a plane is a transitive relation.

10. If R and S are two equivalence relations on a set A, then R $\cap$ S is also an equivalence relation on A.

11. The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.

12. The inverse of an equivalence relation is an equivalence relation.

13. The number of functions from a finite set A into a finite set B = $[n(B)]^{n(A)}$

14. The number of one-one functions that can be defined from a set A into a finite set B is

$^{n(B)}P_{n(A)}$; if $n(B) \geq n(A)$

0 ; otherwise

15. The number of onto functions, that can be defined from a finite set A, containing n elements onto a finite set B, containing 2 elements = $2^n - 2$

16. The number of onto functions from A to B where, o(A) = m, o(B) = n and m $\geq$ n is

$$\sum_{r=1}^{n} (-1)^{n-r}\, {}^nC_r\, r^m$$

17. The number of bijections from a finite set A onto a finite set B is

n(A)!; if n(A) = n(B)

0 ; otherwise

18. If o(A $\cap$ B) = n then

$o[(A \times B) \cap (B \times A)] = n^2$

19. If any line parallel to X-axis, cuts the graph of the function almost one point, then function is one-one.

20. If there is even a single line parallel to X-axis, cuts the graph of the function atleast two points, then function is many-one.

21. For, Domain and Range:

If function is in the form:

i. $\sqrt{f(x)}$, take $f(x) \geq 0$

ii. $\frac{1}{\sqrt{f(x)}}$, take $f(x) > 0$

iii. $\frac{1}{f(x)}$, take $f(x) \neq 0$

22. **Periodic functions**

Functions	Period
$\sin^n x, \cos^n(x)$; if $(n = \text{even})$ $\sec^n x, \operatorname{cosec}^n x$; (if n is odd and fraction)	π 2π
$\lvert\sin x\rvert, \lvert\cos x\rvert, \lvert\tan x\rvert, \lvert\cot x\rvert, \lvert\operatorname{cosec} x\rvert, \lvert\sec x\rvert$	π
$x - \lvert x\rvert, \sin(x-[x]), \sin(x-[-x]), x-[-x]$	1
$\sin^{-1}(\sin x), \cos^{-1}(\cos x)$	2π
$\left(\frac{1}{2}\right)^{\sin x}, \left(\frac{1}{2}\right)^{\cos x}, \left(\frac{1}{2}\right)^{\sin x} + \left(\frac{1}{2}\right)^{\cos x}$	2π
$\sqrt{\cos x}, \sqrt{\frac{1+\cos x}{2}}$	2π
$(\lvert\sin x) + \lvert\cos x\rvert), \sin^4 x + \cos^4 x$	$\frac{\pi}{4}$
$\cos x + \cos\frac{\pi}{2} + \cos\left(\frac{x}{2^2}\right) + \cos\left(\frac{x}{2^3}\right)$...... $\cos\left(\frac{x}{2^{n-1}}\right) + \cos\left(\frac{x}{2^n}\right)$	$2^n\pi$
$\cos(\cos x) + \cos(\sin x)$	$\frac{\pi}{2}$
$\sin(\sin x) + \sin(\cos x)$	2π
$\frac{\lvert\sin x + \cos x\rvert}{\lvert\sin x\rvert + \lvert\cos x\rvert} = \frac{\left\lvert\sqrt{2}\sin\left(x + \frac{n}{4}\right)\right\rvert}{\lvert\sin x\rvert + \lvert\cos x\rvert}$	π
$2^{\sin x} + 2^{\cos x}$	2π

MULTIPLE CHOICE QUESTIONS

Classical Thinking

12.1 Sets

1. If B is the set whose elements are obtained by adding 1 to each of the even numbers, then the set builder notation of B is
a) B = {x : x is even}
b) B= {x : x is odd and x > 1}
c) B = {x : x is odd and x ∈ Z}
d) B = {x : x is an integer}

2. $A \cup B = A$ if
a) $A \subset B$ b) $B \subset A$
c) $A = B$ d) $A \cap B = \phi$

3. If A and B are finite sets (non-empty), then number of elements in A x B is
a) $n(A \cup B)$ b) $n(A \cap B)$
c) $n(A) \times n(B)$ d) none of these

4. If A= {1,2,3}, B = {3, 4, 5}, C = {4, 5, 6}, then $A \cup B \cup C$ =
a) {1, 2, 3, 4, 5, 6} b) {3}
c) {1, 2, 3, 4, 5} d) {1, 3, 5}

5. $A \subseteq B$ and $B \subseteq C$, then
a) $B \subseteq A$ b) $C \subseteq A$
c) $C \subseteq B$ d) $A \subseteq C$

6. Which of the following is a true statement ?
a) $O \in \{\}$ b) $0 \in \{\{0\}\}$
c) $O \in \{0\}$ d) $O \in \{0\}$

7. $A - (B \cup C)$ is equal to
a) $(A - B) \cup (A - C)$
b) $(A \cup B) - (A \cup C)$
c) (A – B) – (A – C)
d) $(A - B) \cap (A - C)$

8. Let X = {x/x ∈ N, I ≤ x ≤ 8}, A = {1,2,3},
B = {2, 4, 6}, C = {1, 3, 5, 7}, then A' =
a) {1, 3, 5, 7, 8}
b) {4, 5, 6, 7, 8}
c) {2, 4, 6, 8}
d) {4, 5,7,8}

9. If A, B, C are any three sets, then $A \cup (B \cap C)$ is equal to
a) $(A \cup B) \cap (A \cup C)$
b) $(A \cup B) \cup (A \cup C)$
c) $(A \cap B) \cap (A \cap C)$
d) none of these

10. If A ≡ {2x/x ∈ N} and B = {4x/x ∈ N}, then $A \cup B$ =
a) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20....}
b) {4, 8, 12, 16, 20....}
c) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
d) {4, 8, 12, 16, 20}

11. $(A \cup B)'$ is equal to
a) $A' \cup B'$ b) $A' \cap B'$
c) $A \cap B$ d) $A \cup B$

12. Let X = {a, b, c, p, q, r, x,y, z},
A = {b, q, y}, B = {a, p, r, x, y] then (A ∩ B)'
a) {a, b, c, p, q, r,x, z}
b) {a, c, p, r, x, z}
c) {b, c, q, z}
d) {a, b, p, q, x, z}

13. If Q is the set of rational numbers and P is the set of irrational numbers, then
a) $P \cap Q = \phi$ b) $P \subset Q$
c) $Q \subset P$ d) $P - Q = \phi$

14. The set of all prime numbers is
a) a finite set b) a singleton set
c) an infinite set d) a null set

15. Two sets A and B are disjoint iff
a) $A \cup B = \phi$ b) $A \cap B = \phi$
c) $A - B = \phi$ d) $B - A = \phi$

16. If A, B, C are any three sets, then A × (B ∪ C) is equal to
a) (A × B) ∪ (A × C)
b) (A × B) ∩ (A × C)
c) A × B – A × C
d) A × (B – C)

17. Let X= {1,2, 3,4, 5, 6, 7, 8, 9}
A = {2, 4, 5, 7, 8}, B = {1, 3, 5, 7},
C = {4, 6, 8, 9}, then $A \cap (B \cup C)$
a) $A \cap (B \cap C)$
b) $A \cup (B \cap C)$
c) $(A \cap B) \cup (A \cap C)$
d) $(A \cup B) \cup (A \cup C)$

18. In a class of 100 students, 60 play cricket, 50 play volleyball and 28 play both. Find the number of students who play atleast one of the two games.
a) 18 b) 32
c) 110 d) 82

19. If A={x x is a natural number},
B = {x | x is an even number},
$A \cap B =$
a) {2, 4, 6, 8} b) {1, 3, 5, 7}
c) {2, 4, 6, 8....} d) {1, 3, 5, 7,...}

20. $A - B = \phi$ iff
a) $A \subset B$ b) $B \subset A$
c) $A = B$ d) $A \cap B = \phi$

21. $A \cap B =$ iff
a) $A \subset B$ b) $B \subset A$
c) $A = B$ d) $A \cap B = \phi$

22. If B = {x | x is an even number},
C = {x | x is an odd number}, then $B \cap C =$
a) 4 b) {2,4,6,8....}
c) {1,3,5,7,...} d) {0}

23. Sets A and B are such that A has 25 members, B has 20 members and $A \cup B$ has 35 members. The number of members in the set $A \cap B$ is
a) 10 b) 5
c) 15 d) 20

24. If A and B are disjoint, then $n(A \cup B)$ is equal to
a) n(A) b) n(B)
c) n(A) + n(B) d) n(A). n(B)

25. Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. How many like only tea and not coffee?
a) 9 b) 5
c) 11 d) 6

26. If A = {x | x is a multiple of 2, x ∈ N},
B = {x | x is a multiple of 5, x ∈ N},
C = {x | x is a multiple of 10, x ∈ N}, then
$(A \cap B) \cap C =$
a) {5, 15, 25,...} b) {10,15,20,...}
c) {5,10,15,20,...} d) {10, 20,30,...}

27. If n(A) = 10, n(B) = 6 and n(C) = 5 for three disjoint sets A, B, C, then $n(A \cup B \cup C)$ equals
a) 11 b) 21
c) 1 d) 9

28. Let X = {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {2, 4, 5, 7, 8}, B = {1, 3, 5, 7},
C = {4, 6, 8, 9}, then $A \cup (B \cap C) =$
a) $(A \cup B)$ n $(A \cup C)$
b) $A \cap (B \cup C)$
c) $(A \cap B) \cup (A \cap C)$
d) $(A \cup B) \cup (A \cup C)$

29. A – B is equal to
a) $(A \cup B) - (A \cap B)$
b) $A \cap B'$
c) $A \cap B$
d) B – A

12.2 Relations

30. If A = {x : $x^2 - 5x + 6 = 0$}, B = {2, 4},
C = {4, 5}, then $A \times (B \cap C)$ is
a) {(2, 4), (3, 4)}
b) {(4, 2), (4, 3)}
c) {(2, 4), (3, 4), (4, 4)}
d) {(2, 2), (3, 3), (4, 4), (5, 5)}

31. Y = {1, 2, 3, 4, 5}, A = {1, 2}, B = {3, 4, 5} and ϕ denote the null set. If $A \times B$ denotes the cartesian product of sets A and B, then $(Y \times A) \cap (Y \times B)$ is
a) Y b) A
c) B d) ϕ

32. If A= {0, 1} and B = {1, 0}, then $A \times B =$
a) {(0,1), (1,0)} b) {(0,0), (1,1)}
c) $A \times A$ d) {(0, 1), (0, 0) (1, 1)}

33. If A= {1,2} and B= {0, 1}, then $A \times B =$
a) {(1,0), (1,1), (2,0), (2,1)}
b) {(1,0), (2,1)}
c) {(1,1), (1,2), (0,1) (0,2)}
d) {(1,0), (2,0) (0,0)}

34. If n(A) = 3, n(B) = 4, then n(A × A × B) is

a) 12 b) 9

c) 16 d) 36

35. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then (A – B) × (B – C) =

a) {(1,2), (1,5), (2, 5)}

b) {(1,4)}

c) (1,4)

d) {(1,2), (1,4)}

36. If n(A) = 5 and n(B) = 2. If a, b, c, d, e are distinct and (a, 2), (b, 3), (c, 2), (d, 3), (e, 2) are in A × B, find A and B.

a) A={a,b,c,d,e},B={2, 3}

b) A={a,b,d,c,e},B={3, 1}

c) A={a,b,c,d,e},B={2, 2}

d) A={a,b,c,e,d},B={3, 3}

37. If A = {a, b} and B = {1, 2, 3} then

(A × B) ∩ (B × A) =

a) {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}

b) {(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)}

c) {(a, a), (b, b), (a, b), (b, a)}

d) ϕ

38. If A × B = {(a, 1), (a, 5), (a, 2), (b, 2), (b, 5), (b, l)},then B × A =

a) {(a, 1), (a, 5), (a, 2), (b, 2), (b, 5), (b, 1)}

b) {(1, a), (5, a), (2, a), (2, b), (5, b), (1, b)}

c) {(1, a), (a, 5), (2, a), (2, b), (5, b), (1, b)}

d) does not exist

39. If A, B and C are any three sets, then A × (B – C) is equal to

a) (A × B) ∪ (A × C)

b) (A × B) ∩ (A × C)

c) (A × B) – (A × C)

d) none of these

40. The domain of the relation

R= {(1,3), (3, 5), (2, 6)} is

a) 1,3 and 2 b) {1,3,2}

c) {3,5,6} d) 3,5 and 6

41. If A ={1,2, 3}, B ={3,4, 5}, then (A ∩ B) × A is

a) {(1,3), (2, 3) (3, 3)}

b) {(3, 1), (3, 2), (3, 3)}

c) {(1,3), (3,1), (3, 2)}

d) {(1,3), (2, 4), (3, 5)}

42. Let R = {(a, a)} be a relation on a set A. Then R is

a) Symmetric

b) Antisymmetric

c) Symmetric and antisymmetric

d) Neither symmetric nor anti-symmetric

43. In the set A = {1, 2, 3, 4, 5}, a relation R is defined by R = {(x, y)| x, y ∈ A and x < y}. Then R is

a) Reflexive

b) Symmetric

c) Transitive

d) An equivalence relation

44. If R ⊂ A × B and S ⊂ B × C be two relations, then $(SoR)^{-1}$ =

a) S^{-1} oR^{-1} b) R^{-1} oS^{-1}

c) SoR d) RoS

12.3 Functions

45. If a function f(x) is given as f(x) = x – 3x + 2 for all x ∈ R, then f(– l) =

a) 6 b) 0

c) 2 d) 8

46. If a function f(x) is given as f(x) = $x^2 - 3x + 2$ for all x ∈ R, then f(a + h) =

a) $a^2 + (2a + 3)h - 3a + 2 + h^2$

b) $a^2 + (2a - 3)h + 3a + 2 + h^2$

c) $a^2 + (2a - 3)h - 3a + 2 + h^2$

d) $a^2 + (2a + 3)h + 3a + 2 + h^2$

47. If f(x) = ax + 6 and f(l) = 11, then a =

a) 6 b) 17

c) 11 d) 5

48. If f(x) = $x^2 - 6x + 5$, $0 \le x \le 4$ then f(8) =

a) 5 b) 21

c) 11 d) does not exist

49. If f(x) = 3x – 1, g(x) = $x^2 + 1$ then f [g(x)] =

a) $3x^2 + 2$ b) $9x^2$

c) $3x^2 - 2$ d) $9x^2$

50. A function f is said to be even, if

a) f(x) = – f(x)

b) f(– c) f(– x) = – f(x)

c) f(– x) = – f(x)

d) none of these

51. Which of the following is a polynomial function?

a) $\frac{x^2-1}{x}, x \neq 0$

b) $x^3+3x^2-4x+\sqrt{2}\,x^{-2}$

c) $\frac{3x^2+7x-1}{3}$

d) $2x^2+\sqrt{x}+1$

52. If f (x) = 4x – x^2, then f(a + 1) – f(a – 1) =

a) 4(2 – a) b) 2(4 – a)

c) 4(2 + a) d) 2(4 + a)

53. The function

$f: R \to R: f(x) = 1$; if $x > 0$

$= 0$; if $x = 0$

$= -1$; if $x < 0$ is a

a) rational function

b) modulus function

c) signum function

d) sine function

54. If $f(x)=\frac{x-1}{x+1}$, then $f\left(\frac{1}{f(x)}\right)$ equals

a) 0 b) 1

c) x d) $\frac{1}{x}$

55. If f(x) = x^2, g(x) = 5x – 6, then g[f(x)] =

a) $25x^2 - 60x + 36$ b) $5x^2 + 6$

c) $25x^2 + 60x - 36$ d) $5x^2 - 6$

56. If $f(x)=x^2+\frac{1}{x}, x \neq 0$ then $\left(\frac{1}{x}\right)=$

a) $\frac{1}{x^2}+x$ b) $\frac{1}{x}+x^2$

c) $\frac{1}{x^2}-x$ d) $\frac{1}{x}-x^2$

57. Let f : R $\to$ R and g : R $\to$ R be given by f(.x) = x^2 and g(x) = x^3 + 1, then (fog) (x)

a) $x^6 + 1$ b) $x^6 - 1$

c) $(x^3 - 1)^2$ d) $(x^3 + 1)^2$

58. If f(x) = x^2 – 6x + 9, 0 $\leq$ x $\leq$ 4, then f(3) =

a) 4 b) 1

c) 0 d) does not exist

59. If $f(x)=\frac{1}{\sqrt{5x-7}}$, then dom(f) =

a) $R-\left\{\frac{7}{5}\right\}$ b) $\left[\frac{7}{5}, \infty\right)$

c) $\left[\frac{5}{7}, \infty\right)$ d) $\left(\frac{7}{5}, \infty\right)$

60. Find [2.75], if [x] denotes greatest integer not greater than x ?

a) 2 b) 3

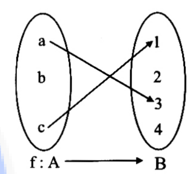
c) 0.75 d) 1.75

61. If f(x) = x^2 + 1, then the value of (fof) (x) is equal to

a) $x^4 + 1$ b) $x^4 + 2x^2 + 2$

c) $x^4 + x^2 + 1$ d) none of these

62. The diagram given below shows that



a) f is a function from A to B

b) f is a one-one function from A to B

c) f is a bijection from A to B

d) f is not a function.

63. If $f(x)=1-\frac{1}{x}$, then $f\left(f\left(\frac{1}{x}\right)\right)$ is

a) $\frac{1}{x}$ b) $\frac{1}{1+x}$

c) $\frac{x}{x-1}$ d) $\frac{1}{x-1}$

64. If for two functions g and f, gof is both injective and surjective, then which of the following is true?

a) g and f should be injective and surjective

b) g should be injective and surjective

c) f should be injective and surjective

d) None of them may be surjective and injective

65. Domain of function f(x) = $\sin^{-1} 5x$ is

a) $\left(-\frac{1}{5}, \frac{1}{5}\right)$ b) $\left[-\frac{1}{5}, \frac{1}{5}\right]$

c) R d) $\left(0, \frac{1}{5}\right)$

66. Domain of the function log $|x^2 - 9|$ is
a) R b) R– [– 3, 3]
c) R– {– 3, 3} d) {– 3, 3}

67. Domain of the function $\sqrt{\log\{(5x - x^2)/6\}}$ is
a) (2, 3) b) [2, 3]
c) [1, 2] d) [1, 3]

68. Inverse of the function y = 2x – 3 is x + 3 is
a) $\frac{x+3}{2}$ b) $\frac{x-3}{2}$
c) $\frac{1}{2x-3}$ d) $\frac{1}{x+3}$

69. Domain of the function $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$ is
a) $\{x : x \in R, x \neq 3\}$
b) $\{x : x \in R, x \neq 2\}$
c) $\{x : x \in R\}$
d) $\{x : x \in R, x \neq 2, x \neq -3\}$

70. If $f(x) = \frac{3x+4}{5x-7}, g(x) = \frac{7x+4}{5x-3}$ then $f[g(x)] =$
a) – 41 b) x
c) – x d) 41

Critical Thinking

12.1 Sets

1. A– B = B– A if
a) $A \subset B$ b) $B \subset A$
c) $A \cap B = \phi$ d) A = B

2. Which of the following is the empty set ?
a) {x : x is a real number and $x^2 - 1 = 0$}
b) {x : x is a real number and $x^2 + 1 = 0$}
c) {x : x is a real number and $x^2 - 9 = 0$}
d) {x : x is a real number and $x^2 = x + 2$}

3. Which of the following is not true ?
a) $(A \cap B) \subset A$ b) $A \subset A \cup B$
c) $(A - B) \subset A$ d) $A \subset (A - B)$

4. If A $\equiv$ {x/$6x^2 + x - 15 = 0$},
B = {x/$2x^2 - 5x + 3 = 0$} and
C = {x/$2x^2 - x - 3 = 0$}, then $A \cap B \cap C =$
a) $\left\{-\frac{5}{3}, \frac{3}{2}\right\}$ b) $\left\{1, \frac{3}{2}\right\}$
c) $\left\{-1, \frac{3}{2}\right\}$ d) $\left\{\frac{3}{2}\right\}$

5. Let X = {x/x $\in$ N, 1 $\leq$ x $\leq$ 8}, A= {1,2,3}, B = {2, 4, 6}, C = {1, 3, 5, 7}, then $(A \cup B)' =$
a) {5,7,8} b) {1,3,5,6,7,8}
c) {2,4,6,8} d) {1,3,5,7,8}

6. Which of the following is an empty set ?
a) The set of prime numbers which are even.
b) The solution set of the equation
$$\frac{2(2x+3)}{x+1} - \frac{2}{x+1} + 3 = 0, x \in R$$
c) (A × B) $\cap$ (B × A) where A and B are disjoint.
d) The set of reals which satisfy
$x^2 + k + i - 1 = 0$

7. A – B is equal to
a) B – A b) $A \cup B$
c) $A \cap B$ d) $A - (A \cap B)$

8. In a group of 20 adults, there are 8 males and 9 vegetarians. Find the number of female non-vegetarians, if the group contains 5 male vegetarians?
a) 4 b) 8
c) 12 d) 10

9. If A = {x | x is a multiple of 2, $x \in N$},
B = {x | x is a multiple of 5, $x \in N$},
C = {x | x is a multiple of 10, $x \in N$}, then
$A \cap (B \cup C) =$
a) {10,20,30,...}
b) {5,10,20,...}
c) {4,8,10,12,...}
d) {2,4,5,15,...}

10. If X is the universal set and A, B are subsets of X such that n(X) = 99, n(A') = 80, n(B') = 85 and n(A $\cap$ B)' = 94, then n(A $\cup$ B) =
a) 33 b) 14
c) 28 d) 29

11. If A= {(a, b): 2a + b = 5, a, b e W} then A =
a) {(0,5), (1,3), (2, 2)}
b) {(0,5), (1,4), (2,1)}
c) {(0, 5), (1, 3), (2, 1)}
d) {(1,1), (2, 2), (3, 3)}

12. A= $\{x/x^2 - 7x + 12 = 0\}$,
B = $\{x/x^2 - x - 12 = 0\}$, then A $\cap$ B =
a) {3} b) {4}
c) {–3,3,4} d) {3,4,5}

13. If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is
a) 120 b) 30
c) 31 d) 32

14. Which of the following is not true?
a) $0 \in \{0, \{0\}\}$
b) $\{0\} \in \{0, \{0\}\}$
c) $\{0\} \subset \{0, \{0\}\}$
d) $O \subset \{0, \{0\}\}$

15. In a class of 120 students, 46 play chess, 30 play table tennis and 40 play carrom, 14 play chess and table tennis, 10 play table tennis and carrom, 8 play chess and carrom, and 30 students do not play any of these games. How many play chess, table tennis and carrom?
a) 8 b) 6
c) 10 d) 4

16. Which of the following set is not a null set?
a) P = {x/x $\in$ N, 2x + 1 is even}
b) Q = {x/x $\in$ I, x^2 is not positive}
c) R = {x/x $\in$ N, x is odd and x^2 is even}
d) S = {x/x $\in$ R, $x^2 + 1 = 0$}

17. If A and B are any two sets, then
$(A \cup B) - (A \cap B) =$
a) A – B b) B – A
c) (A– B) $\cup$ (B – A) d) none of these

18. B = $\{x/x^2 - x - 12 = 0\}$
C = $\{x/x^2 - 8x + 15 = 0\}$, then B $\cup$ C =
a) {3,4,5} b) {3,4}
c) {–3,3,4,5} d) {–3,4,5}

19. If A is any set, then
a) $A \cup A' = \phi$ b) $A \cap A' = X$
c) $A \cap A' = \phi$ d) none of these

20. In a consumer - preference survey of an item, fifteen were found to use Brand A, twenty were found to use Brand B, five were found to be in the habit of using both brands A and B. Find the number of consumers using at least one of the two brands of the item.
a) 30 b) 20
c) 15 d) 35

21. A = $\{x \mid x^2 - 9x + 20 = 0\}$,
B = $\{x \mid x^2 + 13x + 42 = 0\}$
C = $\{x \mid x^2 - 3x - 70 = 0\}$ and the universal set X = {–7, – 6, 4, 5, 10, 12}, then $A \cap (B \cap C) =$
a) {– 7, – 6, 4, 5, 10}
b) {4,5,10}
c) {–7,4,5,10}
d) ϕ

22. In a group of 100 children, 62 like pizza, 47 like burger and 36 like both. Find the number of students who like pizza but not burger.
a) 26 b) 15
c) 36 d) 30

23. If A $\equiv$ {a, e, i, o, u},C $\equiv$ {p, q, r, ..., z} and X $\equiv$ {a, b, c, ..., z} is the universal set, then (A $\cup$ C)' =
a) $A \cap C$ b) $A' \cap C'$
c) $A' \cup C'$ d) $(A \cap C)'$

24. In a group of 50 persons, everyone takes either tea or coffee. If 35 take tea and 25 take coffee, then the number of persons who take tea only (and not coffee) is
a) 10 b) 25
c) 35 d) 30

25. Let $A = \{(x, y) : y = e^x,\ x \in R\}$
$B = \{x, y) : y = e^{-x},\ x \in R\}$ Then
a) $A \cap B = \phi$ b) $A \cap B \neq \phi$
c) $A \cup B = R^2$ d) None of these

26. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to
a) A b) B
c) ϕ d) $A \cap B$

27. If A and B are two sets then
$(A - B) \cup (B - A) \cup (A \cap B)$ is equal to
a) $A \cup B$ b) $A \cap B$
c) A d) B'

28. Let U be the universal set and $A \cup B \cup C = U$, then $\{(A - B) \cup (B - C) \cup (C - A)\}'$ is equal to
a) $A \cup B \cup C$ b) $A \cup (B \cap C)$
c) $A \cap B \cap C$ d) $A \cap (B \cup C)$

29. In a battle 70 % of the combatants lost one eye, 80 % an ear, 75 % an arm, 85 % a leg, x % lost all the four limbs. The minimum value of x is
a) 10 b) 12
c) 15 d) 9

30. A survey shows that 63 % of the Americans like cheese whereas 76 % like apples. If x % of the Americans like both cheese and apples, then
a) $x = 39$ b) $x = 63$
c) $39 \leq x \leq 63$ d) $39 < x < 63$

31. If (1, 3), (2, 5) and (3, 3) are three elements of $A \times B$ and the total number of elements in $A \times B$ is 6, then the remaining elements of $A \times B$ are
a) (1,5); (2, 3); (3, 5) b) (5, 1); (3, 2); (5, 3)
c) (1,5); (2, 3); (5, 3) d) (1,3); (2, 5); (3, 3)

32. $A - B = A$ iff
a) $A \subset B$ b) $B \subset A$
c) $A = B$ d) $A \cap B = \phi$

33. If $C = \{x/ x$ is an odd number$\}$,
$D = \{x/ x$ is a prime number$\}$, then $C \cap D =$
a) $\{2,4,6,8....\}$ b) ϕ
c) $\{1,2,3,4,5,...\}$ d) $\{3,5,7,11,....\}$

34. If $X = \{8^n - 7n - 1 : n \in N\}$ and
$Y = \{49(n - 1) : n \in N\}$ then
a) $X \subseteq Y$ b) $Y \subseteq X$
c) $X = Y$ d) None of these

12.2 Relations

35. If R is a set of all real numbers then what does cartesian product $R \times R \times R$ represent ?
a) set of all points in space
b) set of all points in XY plane
c) set of points, only in 1st Quadrant of XY plane
d) R^2

36. If A = {a, b, c, d} and B = {1, 2, 3}, then which of the following is a relation from A to B ?
a) R_1 = {(a, l),(2,b),(c,3)}
b) R_2={(a, 1), (d, 3), (b, 2), (b, 3)}
c) R_3={(l,a),(2,b),(3,c)}
d) R_4={(a, 1), (b, 2), (c, 3), (3, d)}

37. Let A and B be two sets such that $A \times B$ = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}, Then,
a) A ={1,2, 3}and B = {a,b}
b) A= {a,b} and B = {1,2, 3}
c) A = {1,2, 3} and $B \subset$ {a, b}
d) A c {a, b} and $B \subset$ {1,2, 3}

38. The cartesian product $A \times A$ has 9 elements among which are found (– 1,0) and (0, 1). Then the set A is
a) {– 1,0,1} b) {– 1,0,2}
c) {– 1,11,10} d) {– 2,0,2}

39. If A = {a, b}, B = {c, d}, C = {d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} =
a) $A \cap (B \cup C)$ b) $A \cup (B \cap C)$
c) $A \times (B \cup C)$ d) $A \times (B \cap C)$

40. Let A and B be two sets such that $A \times B$ has 6 elements. If three elements of $A \times B$ are {(1,4), (2, 6), (3, 6)}, then
a) A = {1, 2} and B = {3,4, 6}
b) A ={4, 6} and B = {1,2, 3}
c) A={l,2,3}and B ={4,6}
d) A= {1,2,4} and B = {3, 6}

41. Let A = {1, 2, 3}. The total number of distinct relations, that can be defined over A is
a) 2^9 b) 6
c) 8 d) 9

42. Let $P = \{(x, y) \mid x^2 + y^2 = 1,\ x, y \in R\}$. Then P is
a) Reflexive b) Symmetric
c) Transitive d) Anti-symmetric

43. Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is

a) Less than n

b) Greater than or equal to n

c) Less than or equal to n

d) not equal to n

44. If R be a relation < from A = {1,2, 3, 4} to B = {1, 3, 5} i.e., $(a, b) \in R \Leftrightarrow a < b$, then Ro R^{-1} is

a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}

b) {(3,1) (5,1), (3, 2), (5, 2), (5, 3), (5, 4)}

c) {(3, 3), (3, 5), (5, 3), (5, 5)}

d) {(3, 3) (3, 4), (4, 5)}

45. Let n(A) = n, then the number of all relations on A is

a) 2^n b) $2^{(n)!}$

c) 2^n d) n^2

46. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is

a) 2^{mn} b) $2^{mn}-1$

c) 2 mn d) m^n

47. If $R = \{(x, y)| x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in Z, then domain of R is

a) {0,1,2} b) {0,–1,– 2}

c) {– 2, – 1,0, 1,2} d) {– 2, – 1,0,1}

48. The relation R defined in N as $aRb \Leftrightarrow b$ is divisible by a is

a) Reflexive but not symmetric

b) Symmetric but not transitive

c) Symmetric and transitive

d) Symmetric

49. The relation "is subset of " on the power set P(A) of a set A is

a) Symmetric

b) Anti-symmetric

c) Equivalency relation

d) None of these

50. R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x – 3. Then R^{-1} is

a) {(8, 11), (10, 13)} b) {(11, 18), (13, 10)}

c) {8,11} d) {10,13}

51. Let R be a relation on a set A such that $R = R^{-1}$,then R is

a) Reflexive b) Symmetric

c) Transitive d) Not symmetric

52. Which one of the following relations on R is an equivalence relation

a) aR, b $\Leftrightarrow$ |a| = |b|

b) $aR_2b \Leftrightarrow a \geq b$

c) $aR_3 b \Leftrightarrow$ a divides b

d) $aR_4 \Leftrightarrow a < b$

53. The relation "congruence modulo m" is

a) Reflexive only

b) Transitive only

c) Symmetric only

d) An equivalence relation

12.3 Functions

54. Let A ={1,2, 3} and

B = {2, 3, 4}, then which of the following is a function from A to B ?

a) {(1,2), (1,3), (2, 3), (3, 3)}

b) {(0, 3), (2, 4)}

c) {(1,3), (2, 3), (3, 3)}

d) {(1,2), (2, 3), (3, 4), (3, 2)}

55. If $f(x) = ax^2 + bx + 2$ and f(l) = 3, f(4) = 42, then a and b respectively are

a) – 3, 2 b) 3, 2

c) – 2, 3 d) 3, – 2

56. If f ={(1,4), (2, 5), (3, 6)} and g ={(4, 8), (5, 7), (6, 9)}, then gof is

a) {}

b) {(1,8), (2, 7), (3, 9)}

c) {(1,7), (2, 8), (3, 9)}

d) {(1,8), (2, 5), (3, 9)}

57. If for non-zero x, $a.f(x) + b.f\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then f (2) =

a) $\frac{3(2b+3a)}{2(a^2-b^2)}$ b) $\frac{3(2b-3a)}{2(a^2-b^2)}$

c) $\frac{3(3a-2b)}{2(a^2-b^2)}$ d) $\frac{6}{a+b}$

58. The range of the function $f(x)=\dfrac{x}{1+x^2}$ is

a) $\left[0,\dfrac{1}{2}\right]$ b) $\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$

c) $\left[-\dfrac{1}{2},0\right]$ d) $\left[-\dfrac{1}{2},0\right)\cup\left(0,\dfrac{1}{2}\right]$

59. If $f: R \to R$ is defined as $f(x) = x^2 - 3x + 4$ for all $x \in R$, then $f^{-1}(2)$ is equal to

a) {1,2} b) (1,2)

c) [1,2] d) none of these

60. The range of the function, $f(x)=\dfrac{1+x^2}{x^2}$ is

a) (0,1) b) [0,1]

c) $(1, \infty)$ d) $[1, \infty)$

61. If $f(x)=\dfrac{x-1}{x+1}$, then $f(\alpha x)=$

a) $\dfrac{f(x)+\alpha}{1+\alpha f(x)}$

b) $\dfrac{(\alpha-1)f(x)+\alpha+1}{(\alpha+1)f(x)+(\alpha-1)}$

c) $\dfrac{(\alpha+1)f(x)+\alpha-1}{(\alpha-1)f(x)+(\alpha+1)}$

d) $\dfrac{f(\alpha x)-1}{f(\alpha x)+1}$

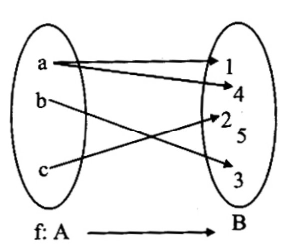
62. If $f(x)=x+\dfrac{1}{x}$, such that

$[f(x)]^3 = f(x^3) + \lambda f\left(\dfrac{1}{x}\right)$, then $\lambda =$

a) 1 b) 3

c) – 3 d) – 1

63. The diagram given below shows that



a) f is a function from A to B

b) f is a one-one function from A to B

c) f is an onto function from A to B

d) f is not a function.

64. Range of the function $f(x)=\dfrac{1}{3x+2}$ is

a) R b R– {0}

c) $(0, \infty)$ d) $R-\left\{-\dfrac{2}{3}\right\}$

65. The range of the function $f(x)=\dfrac{x^2-3x+2}{x^2+x-6}$ is

a) $R-\left[\dfrac{1}{5},1\right]$ b) R

c) R – {1} d) R – {– 3, 2}

66. The domain of the function

$f(x)=\sqrt{x^2-5x+6}+\sqrt{2x+8-x^2}$, is

a) [2,3] b) [– 2,4]

c) $[-2, 2] \cup [3, 4]$ d) $[-2, 1] \cup [2, 4]$

67. Let f be a real valued function, satisfying $f(x + y) = f(x) f(y)$ for all $x,y \in R$ Such that, $f(1) = a$. Then , $f(x) =$

a) a^x b) ax

c) x^a d) log x

68. Which of the following functions is an odd function ?

a) $f(x)=\sqrt{1+x+x^2}-\sqrt{1-x+x^2}$

b) $f(x)=x\left(\dfrac{a^x+1}{a^x-1}\right)$

c) $f(x)=\log_{10}\left(\dfrac{1-x^2}{1+x^2}\right)$

d) f(x) = k (constant)

69. If $f(x)=\dfrac{x+3}{4x-5}$ and $t=\dfrac{3+5x}{4x-1}$, then $f(t)$ is

a) – x b) 17x

c) x d) – 17x

70. If $f(x) = x^2 - 2x + 3$, then the value of x for which $f(x) = f(x + 1)$ is

a) 1/2 b) 1/3

c) 1 d) 3

71. Domain of the function $f(x)=\sqrt{\frac{x}{1+x}}$ is

a) $(-\infty,-1)\cup[0,\infty)$ b) R

c) $[0,\infty)$ d) $(-\infty,-1)$

72. If $f: R-\{3\}\to R-\{1\}$ be defined by

$f(x)=\frac{x-2}{x-3}$, then f is

a) one-one into b) one-one onto

c) many-one into d) many-one onto

73. Let $f(x)=x, g(x)=\frac{1}{x}$ and

$h(x) = f(x)\,g(x)$, then $h(x) = 1$ iff

a) x is a real number

b) x is a rational number

c) x is an irrational number

d) x is a real number $\neq 0$

74. Range of the function $f(x) = \sqrt{x^2+x+1}$ equal to

a) $[0,\infty]$ b) $\left[\frac{\sqrt{3}}{2},\infty\right)$

c) $\left(\frac{\sqrt{3}}{2},\frac{-\sqrt{3}}{2}\right)$ d) $(0, 0)$

75. Which of the following is an even function ?

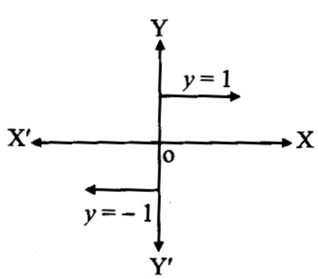
a) $\sqrt{x}$ b) $x^2+\sin^2 x$

c) $\sin^3 x$ d) all of above

76. If $f(x)=\frac{1}{1-x}$, then $f[f\{f(x)\}]$ is equal to

a) $\frac{x-1}{x}$ b) $f(x)$

c) x d) $-x$

77. 

is the graph of

a) Modulus function b) Step function

c) Signum function d) Rational function

78. Mapping $f: R\to R$ which is defined as $f(x) = \cos x$, $x\in R$ will be

a) Neither one-one nor onto

b) One-one

c) Onto

d) One-one onto

79. If $f: R\to R$, then $f(x) = |x|$ is

a) One-one but not onto

b) Onto but not one-one

c) One-one and onto

d) Many-one

80. $f:[0,\infty]\to[0,\infty]$ and $f(x)=\frac{x}{1+x}$, then f is

a) One-one and onto

b) One-one but not onto

c) Onto but not one-one

d) Neither one-one nor onto

81. Domain and range of $f(x)=\frac{|x-3|}{x-3}$ are respectively

a) $R,[-1,1]$ b) $R-\{3\},\{1,-1\}$

c) R^+, R d) None of these

82. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$

a) ≥ 0 only when $\theta\geq 0$

b) ≤ 0 for all real θ

c) ≥ 0 for all real θ

d) ≤ 0 only when $\theta\leq 0$

83. If f be the greatest integer function and g be the modulus function, then

$(gof)\left(-\frac{5}{3}\right)-(fog)\left(-\frac{5}{3}\right)=$

a) 1 b) – 1

c) 2 d) 4

84. If R denotes the set of all real numbers, then the function $f: R\to R$ defined by $f(x) = [x]$ is

a) One-one only

b) Onto only

c) Both one-one and onto

d) Neither one-one nor onto

85. If in greatest integer function, the domain is a set of real numbers, then range will be set of

a) Real numbers
b) Rational numbers
c) Imaginary numbers
d) Integers

86. If the domain of function $f(x) = x^2 - 6x + 1$ is $(-\infty, \infty)$, then the range of function is

a) $(-\infty, \infty)$ b) $[-2, \infty)$
c) $(-2, 3)$ d) $(-\infty, -2)$

87. Domain of the function $\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$ is

a) $(-1, 1)$ b) $(-1, 1) - \{0\}$
c) $[-1, 1]$ d) $[-1, 1] - \{0\}$

88. The interval for which

$\sin^{-1} x\sqrt{x} + \cos^{-1}\sqrt{x} = \dfrac{\pi}{2}$ holds

a) $[0, \infty)$ b) $[0, 3]$
c) $[0, 1]$ d) $[0, 2]$

89. The domain of $\sin^{-1}\left[\log\left(\dfrac{x}{3}\right)\right]$ is

a) $[1, 9]$ b) $(-1, 9)$
c) $[-9, 1]$ d) $[-9, -1]$

90. Given the function $f(x) = \dfrac{a^x + a^{-x}}{2}\ (a > 2)$.

Then, $f(x + y) + f(x - y) =$

a) $2f(x).f(y)$ b) $f(x).f(y)$
c) $\dfrac{f(x)}{f(y)}$ d) $f(x) + f(y)$

Competitive Thinking

12.1 Sets

1. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are

a) 7, 6 b) 6, 3
c) 5, 1 d) 8, 7

2. Suppose $A_1, A_2, A_3, \ldots\ldots\ldots A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots\ldots\ldots B_n$ are n sets each with 3 elements.

Let $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_j = S$ and each elements of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's. Then n is equal to

a) 15 b) 3
c) 45 d) 35

3. The set $A = \{x : x \in R,\ x^2 = 16 \text{ and } 2x = 6\}$ equals

a) ϕ b) {14,3,4}
c) {3} d) {4}

4. If the sets A and B are defined as

$A = \{(x,y) : y = e^x, x \in R\}$;
$B = \{(x, y) : y = x, x \in R\}$, then

a) $B \subseteq A$
b) $A \subseteq B$
c) $A \cap B = \phi$
d) $A \cup B = A$

5. If $X = \{4^n - 3n - 1 : n \in N$ and $Y = \{9(n-1) : n \in N\}$, then $X \cup Y$ is equal to

a) X
b) Y
c) N
d) None of these

6. In a town of 10,000 families it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is

a) 3100 b) 3300
c) 2900 d) 1400

7. If A, B and C are non-empty sets, then $(A - B) \cup (B - A)$ equals

a) $(A \cup B) - B$

b) $A - (A \cap B)$

c) $(A \cup B) - (A \cap B)$

d) $(A \cap B) \cup (A \cup B)$

8. If P, Q and R are subsets of a set A, then $R \times (P^c \cup Q^c)^c =$

a) $(R \times P) \cap (R \times Q)$

b) $(R \times Q) - (R \times P)$

c) $(R \times P) \cup (R \times Q)$

d) (a) and (b)

9. In rule method the null set is represented by

a) $\{\}$ b) ϕ

c) $\{x : x = x\}$ d) $\{x : x \neq x\}$

10. The number of non-empty subsets of the set $\{1, 2, 3, 4\}$ is

a) 15 b) 14

c) 16 d) 17

11. If A, B, C be three sets such that

$A \cup B = A \cup C$ and $A \cap B = A \cap C$, then

a) A = B b) B = C

c) A = C d) A = B = C

12. If $N_a = \{an : n \in N\}$, then $N_5 \cap N_7 =$

a) N_7 b) N

c) N_{35} d) N_5

13. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

a) 128 b) 216

c) 240 d) 160

14. In a class of 30 pupils, 12 take needle work, 16 take physics and 18 take history. If all the 30 students take at least one subject and no one takes all three then the number of pupils taking 2 subjects is

a) 16 b) 6

c) 8 d) 20

15. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$, where Z is the set of all integers, is

a) 2 b) 4

c) 8 d) 12

16. A set contains 2n + 1 elements. The number of subsets of this set containing more than n elements is equal to

a) 2^{n-1} b) 2^n

c 2^{n+1} d) 2^{2n}

17. Which of the following is a true statement ?

a) $\{a\} \in \{a, b, c\}$ b) $\{a\} \subseteq \{a, b, c\}$

c) $\phi \in \{a, b, c\}$ d) None of these

18. If A = {x : x is a multiple of 4} and B = {x : x is a multiple of 6} then $A \subset B$ consists of all multiples of

a) 16 b) 12

c) 8 d) 4

19. A class has 175 students. The following data shows the number of students obtaining one or more subjects. Mathematics 100, Physics 70, Chemistry 40; Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone

a) 35 b) 48

c) 60 d) 22

20. Consider the following relations :

1. $A - B = A - (A \cap B)$

2. $A = (A \cap B) \cup (A - B)$

3. $A - (B \cup C) = (A - B) \cup (A - C)$

which of these is/are correct

a) 1 and 3 b) 2 only

c) 2 and 3 d) 1 and 2

21. If two sets A and B are having 99 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are

a) 2^{99} b) 99^2

c) 100 d) 18

22. Given n(U) = 20, n(A) = 12, n(B) = 9, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n((A \cup B)^c) =$

a) 17 b) 9

c) 14 d) 3

23. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$

a) 3 b) 6

c) 9 d) 18

24. If $A = [(x, y) : x^2 + y^2 = 25]$ and $B = [(x, y) : x^2 + 9y^2 = 144$, then $A \cap B$ contains

a) One point b) Three points

c) Two points d) Four points

25. In a college of 300 students, every student reads 5 newspaper and every newspaper is read by 60 students. The no. of newspaper is

a) At least 30 b) At most 20

c) Exactly 25 d) Exactly 30

26. If $n(A) = 8$ and $n(A \cap B) = 2$, then $n[(A \cap B)' \cap A]$ is equal to

a) 2 b) 4

c) 6 d) 8

27. For any two sets A and B, $A - (A - B)$ equals

a) B b) $A - B$

c) $A \cap B$ d) $A^c \cap B^c$

28. Let X and Y be the sets of all positive divisors of 400 and 1000 respectively (including 1 and the number). Then, $n(X \cap Y) =$

a) 4 b) 6

c) 8 d) 12

29. Three sets A, B, C are such that $A = B \cap C$ and $B = C \cap A$, then

a) $A \subset B$ b) $A \supset B$

c) $A \equiv B$ d) $A \subset B'$

30. For any two sets A and B if

$A \cap X = B \cap X \neq \phi$ and $A u X = B \cup X$ for some set X, then

a) $A - B = A \cap B$ b) $A = B$

c) $B - A = A \cap B$ d) None of these

31. If the set A contains 5 elements, then the number of elements in the power set P(A) is equal to

a) 32 b) 25

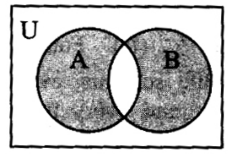
c) 16 d) 8

32. 25 people for programme A, 50 people for progamme B, 10 people for both. So, the number of employee employed only for programme A are

a) 15 b) 20

c) 35 d) 40

33. The shaded region in the figure represents



a) $A \cap B$

b) $A \cup B$

c) $B - A$

d) $(A - B) \cup (B - A)$

34. If $A = \{x \mid x \text{ is a root of } x^2 - 1 = 0\}$, $B = \{x \mid x \text{ is a root of } x^2 - 2x + 1 = 0\}$, then

a) $A \cap B = A$ b) $A \cup B = \phi$

c) $A \cup B = A$ d) $A \cap B = \phi$

35. The value of

$(A \cup B \cup C) n (A \cap B^c \cap C^c)^c nC^c$, is

a) $B \cap C^c$

b) $B^c \cap C^c$

c) $B \cap C$

d) $A \cap B \cap C$

36. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games, then the number of students who play neither is

a) 45 b) 0

c) 25 d) 35

37. The set $A = \{x : |2x + 3| < 7\}$ is equal to the set

a) $D = \{x : 0 < x + 5 < 7\}$

b) $B = \{x : -3 < x < 7\}$

c) $E = \{x : -7 < x < 7\}$

d) $C = \{x : -13 < 2x < 4\}$

38. There is a group of 265 persons who like either singing or dancing or painting. In this group 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of persons who like only dancing and painting is

a) 10 b) 20

c) 30 d) 40

12.2 Relations

39. Let A = {2, 4, 6, 8}. A relation R on A is defined by R = {(2, 4), (4, 2), (4, 6), (6, 4)}. Then R is

a) Anti-symmetric b) Reflexive

c) Symmetric d) Transitive

40. Let A = {a, b, c} and B = {1, 2}. Consider a relation R defined from set A to set B. Then R | is equal to set

a) A b) B

c) A × B d) B × A

41. Let R_1 be a relation defined by

$R_1 = \{(a, b)| a \geq b, a, b \in R\}$. Then R_1 is

a) An equivalence relation on R

b) Reflexive, transitive but not symmetric

c) Symmetric, transitive but not reflexive

d) Neither transitive nor reflexive but symmetric

42. In order that a relation R defined on a not empty set A is an equivalence relation, it sufficient, if R

a) Is reflexive

b) Is symmetric

c) Is transitive

d) Possesses all the above three properties

43. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12) (3, 9), (3, 12), (3, 6)} be a relation on the A = {3, 6, 9, 12}. Then relation is

a) An equivalence relation

b) Reflexive and symmetric only

c) Reflexive and transitive only

d) Reflexive only

44. Let R = {(1,3), (4,2), (2,4), (2,3), (3,1)} be a relation on the set A = {1, 2, 3, 4}. Then relation R is

a) Reflexive b) Transitive

c) Not symmetric d) A function

45. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

a) 2^9 b 9^2

c) 9 d) 2^{9-1}

46. Let R be the relation on the set R of all real numbers defined by a R b iff $|a - b| \leq 1$. Then R is

a) Reflexive and Symmetric

b) Symmetric only

c) Transitive only

d) Anti-symmetric only

47. The number of reflexive relations of a set with four elements is equal to

a) 2^{16} b) 2^{12}

c) 2^8 d) 2^4

48. If A = (a,b, c},B = {b, c, d} and

C = {a, d, c}, then $(A - B) \times (B \cap C) =$

a) {(a, c), (a, d)}

b) {(a,b),(c,d)}

c) {(c, a), (a, d)}

d) {(a, c), (a, d), (b, d)}

49. The relation R defined in N as aRb $\Leftrightarrow$ b is divisible by a is

a) Reflexive but not symmetric

b) Symmetric but not transitive

c) Symmetric and transitive

d) None of these

50. Let r be a relation from R (set of real numbers) to R defined by r = {(a, b) a, b $\in$ R and

$a - b + \sqrt{3}$ is an irrational number}. The relation r is

a) an equivalence relation

b) reflexive only

c) symmetric only

d) transitive only

51. Let a relation R in the set N of natural numbers be defined as

$(x, y) \Leftrightarrow x^2 - 4xy + 3y^2 = 0 \ \forall x, y \in N$. The relation R is

a) reflexive

b) symmetric

c) transitive

d) an equivalence relation

52. Let A = {x, y, z} and B = {a, b, c, d}. Which one of the following is not a relation from A to B?

a) {(x, a), (x, c)}

b) {(y, c), (y, d)}

c) {(z, a), (z, d)}

d) {(z, b), (y, b), (a, d)}

53. $R \subseteq A \times A$ (where $A \neq 0$) is an equivalence relation if R is

a) Reflexive, symmetric but not transitive

b) Reflexive, niether symmetric nor transitive

c) Reflexive, symmetric and transitive

d) None of these

54. On the set N of all natural numbers define the relation R by aRb iff the G. C. D. of a and b is 2. Then R is

a) reflexive but not symmetric
b) symmetric only
c) reflexive and transitive
d) reflexive, symmetric and transitive

55. Let W denote the words in English dictionary. Define the relation R by R = {(x, y) $\in$ W $\times$ W: the words x and y have at least one letter in common), then R is

a) reflexive, not symmetric and transitive
b) not reflexive, symmetric and transitive
c) reflexive, symmetric and not transitive
d) reflexive, symmetric and transitive

56. For any two real numbers θ and ϕ, we define $\theta R \phi$ if and only if $\sec^2\theta - \tan^2\phi = 1$. The relation R is

a) Reflexive but not transitive
b) Symmetric but not reflexive
c) Both reflexive and symmetric but not transitive
d) An equivalence relation

12.3 Functions

57. If f(x) = cos(log x), then

$$f(x)f(y) - \frac{1}{2}[f(x/y) + f(xy)] =$$

a) -1 b) $\frac{1}{2}$
c) -2 d) 0

58. If $f(x) = \frac{1-x}{1+x}$, then f [f(cos 2θ)] =

a) tan 2θ b) sec 2θ
c) cos 2θ d) cot 2θ

59. The value of b and c for which the identity f(x + 1) – f(x) = 8x + 3 is satisfied, where $f(.x) = bx^2 + cx + d$, are

a) b = 2, c = 1 b) b = 4, c = –1
c) b = –1, c = 4 d) b = –1, c = 1

60. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, then

a) $f\left(\frac{\pi}{4}\right) = 2$ b) $f(-\pi) = 2$
c) $f(\pi) = 1$ d) $f\left(\frac{\pi}{2}\right) = -1$

61. The graph of the function y = f(x) is symmetrical about the line x = 2, then

a) f(x) = – f(– x) b) f(2 + x) = f(2 – x)
c) f(x) = f(– x) d) f(x + 2) = f(x – 2)

62. If $f(x) = \frac{x^2-1}{x^2+1}$, for every real numbers, then the minimum value of f

a) Does not exist because f is bounded
b) Is not attained even through f is bounded
c) Is equal to +1
d) Is equal to – 1

63. The function f : R → R defined by f(x) = (x – 1) (x – 2) (x – 3) is

a) One-one but not onto
b) Onto but not one-one
c) Both one-one and onto
d) Neither one-one nor onto

64. Set A has 3 elements and set B has 4 elements. The number of injection that can be defined from A to B is

a) 144 b) 12
c) 24 d) 64

65. The function f: R → R defined by $f(x) = e^x$ is

a) Onto b) Many-one
c) One-one and into d) Many one and onto

66. Let the function f : R → R be defined by f(x) = 2x + sin x, x $\in$ R. Then f is

a) One-to-one and onto
b) One-to-one but not onto
c) Onto but not one-to-one
d) Neither one-to-one nor onto

67. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, \text{ when n is odd} \\ -\frac{n}{2}, \text{ when n is even} \end{cases}, \text{ is}$$

a) One-one but not onto
b) Onto but not one-one
c) One-one and onto both
d) Neither one-one nor onto

68. The period of f(x) = x – [x], if it is periodic, is

a) f(x) is not periodic b) $\frac{1}{2}$
c) 1 d) 2

69. The domain of $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}$ is

a) R– {– 2} b) (– 2,+ ∞)
c) R– {– 1,– 2,– 3} d) (– 3, ∞) – {– 1,– 2}

70. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

a) $(-3, -1) \cup (1, \infty)$
b) $[-3, -1] \cup [1, \infty)$
c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$
d) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

71. Domain of the function $f(x)=\sqrt{2-2x-x^2}$ is

a) $-\sqrt{3} \le x \le \sqrt{3}$
b) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$
c) $-2 \le x \le 2$
d) $-2+\sqrt{3} \le x \le -2-\sqrt{3}$

72. The domain of the function

$\sqrt{\log(x^2-6x+6)}$

a) $(-\infty, \infty)$
b) $(-\infty, 3-\sqrt{3}) \cup (3+\sqrt{3}, \infty)$
c) $(-\infty, 1) \cup [5, \infty)$
d) $[0, \infty]$

73. Domain of the function
$f(x) = \sin^{-1}(1 + 3x + 2x^2)$ is

a) $(-\infty, \infty)$ b) $(-1, 1)$
c) $\left[-\frac{3}{2}, 0\right]$ d) $\left(-\infty, \frac{-1}{2}\right) \cup (2, \infty)$

74. The domain of the function $f(x)=\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

a) [1, 2) b) [2, 3)
c) [1, 2] d) [2, 3]

75. The range of the function $f(x)=\frac{x+2}{|x+2|}$ is

a) {0,1} b) {– 1,1}
c) R d) R– {– 2}

76. The range of f(x) = cos x – sin x is

a) (– 1, 1) b) [– 1, 1)
c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ d) $[-\sqrt{2}, \sqrt{2}]$

77. Range of $f(x)=\frac{x^2+34x-7i}{x^2+2x-7}$ is

a) [5, 9]
b) $(-\infty, 5] \cup [9, \infty)$
c) (5, 9)
d) $(-\infty, 5) \cup (9, \infty)$

78. If $f(x)=\log\frac{1+x}{1-x}$, then f(x) is

a) Even function
b) $f(x_1) f(x_2) = f(x_1 + x_2)$
c) $\frac{f(x_1)}{f(x_2)}=f(x_1-x_2)$
d) Odd function

79. If $y=f(x)=\frac{x+2}{x-1}$, then x =

a) f(y) b) 2f(y)
c) $\frac{1}{f(y)}$ d) – f(y)

80. If the function $f:[1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ is

a) $\left(\frac{1}{2}\right)^{x(x-1)}$
b) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$
c) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$
d) Not defined

81. If f(x) = 3x – 5,, then $f^{-1}(x)$ is

a) $\dfrac{1}{3x-5}$

b) Is given by $\dfrac{x+5}{3}$

c) Does not exist because f is not one-one

d) Does not exist because f is not onto

82. If $f(x)=\dfrac{\alpha x}{x+1}$, $x\neq -1$, for what value of α is f(f(x)) = x

a) $\sqrt{2}$ b) $-\sqrt{2}$

c) 1 d) – 1

83. If $f(x)=\log\left[\dfrac{1+x}{1-x}\right]$, then $f\left[\dfrac{2x}{1+x^2}\right]$ is equal to

a) $[f(x)]^2$ b) $[f(x)]^3$

c) 2f(x) d) 3f(x)

84. Let f : N → N defined by f(x) = $x^2 + x + 1$, x ∈ N,then f is

a) One-one onto

b) Many one onto

c) One-one but not onto

d) None of these

85. Which one of the following is a bijective function on the set of real numbers?

a) 2x – 5 b) |x|

c) x^2 d) $x^2 + 1$

86. If f : R → S defined by

f(x) = sin x – $\sqrt{3}$ cosx + 1 is onto, then the interval of S is

a) [– 1, 3] b) [1, 1]

c) [0, 1] d) [0, –1]

87. If f(x) is periodic function with period T, then the function f(ax + b) where a > 0, is periodic with period

a) T/b b) aT

c) bT d) T/a

88. Domain of f(x) = log|log x| is

a) (0, ∞) b) (1, ∞)

c) (0, 1) ∪ (1, ∞) d) (–∞, 1)

89. The domain of the function

$f(x)=\sqrt{\log\dfrac{1}{|\sin x|}}$ is

a) R– {2nπ, n ∈ I} b) R – {nπ, n ∈ I)

c) R– (– π, π) d) (–∞, ∞)

90. The function $f(x)=\sin(\log(x+\sqrt{x^2+1}))$ is

a) Even function

b) Odd function

c) Neither even nor odd

d) Periodic function

91. $f(x)=\dfrac{2x-1}{x+5}$ $(x\neq 5)$, then $f^{-1}(x)$ is equal to

a) $\dfrac{x+5}{2x-1}, x\neq\dfrac{1}{2}$ b) $\dfrac{5x+1}{2-x}, x\neq 2$

c) $\dfrac{5x-1}{2-x}, x\neq 2$ d) $\dfrac{x-5}{2x+1}, x\neq\dfrac{1}{2}$

92. The inverse of the function $f(x)=\dfrac{e^x+e^{-x}}{e^x+e^{-x}}+2$ is

a) $\log_e\left(\dfrac{x-2}{x-1}\right)^{\frac{1}{2}}$ b) $\log_e\left(\dfrac{x-1}{3-x}\right)^{\frac{1}{2}}$

c) $\log_e\left(\dfrac{x}{3-x}\right)^{\frac{1}{2}}$ d) $\log_e\left(\dfrac{x-1}{x+1}\right)^{-2}$

93. If $e^{f(x)}=\dfrac{10+x}{10-x}$, $x\in(-10,10)$ and

$f(x)=kf\left(\dfrac{200x}{100+x^2}\right)$, then k =

a) 0.5 b) 0.6

c) 0.7 d) 0.8

94. If the real valued function $f(x)=\dfrac{a^x-1}{x^n(a^x+1)}$ is even, then n equals

a) 2 b) $\dfrac{-2}{3}$

c) $\dfrac{1}{4}$ d) $-\dfrac{1}{3}$

95. If [x] denotes the greatest integer $\le$ x then

$$\left[\frac{2}{3}\right]+\left[\frac{2}{3}+\frac{1}{99}\right]+\left[\frac{2}{3}+\frac{2}{99}\right]+....+\left[\frac{2}{3}+\frac{98}{99}\right]=$$

a) 99 b) 98
c) 66 d) 65

96. If the function

$$f(x)=\cos^2 x+\cos^2\left(\frac{\pi}{3}+x\right)-\cos x\cos\left(\frac{\pi}{3}+\pi\right)$$

is constant (independent of x), then the value of this constant is

a) 0 b) $\frac{3}{4}$
c) 1 d) $\frac{4}{3}$

97. The domain of the function

$$y=f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2} \text{ is}$$

a) [– 2, 1), excluding 0
b) [– 3, – 2], excluding – 2.5
c) [0, 1], excluding 0
d) none of these

98. The domain of the function

$$f(x)=\exp\,(\sqrt{5x-3-2x^2}) \text{ is}$$

a) $\left[1, \frac{-3}{2}\right]$ b) $\left[\frac{3}{2}, \infty\right]$
c) $(-\infty, 1]$ d) $\left[1, \frac{3}{2}\right]$

99. The composite map fog of the functions $f : R \to R$, $f(x) = \sin x$ and $g : R \to R$, $g(x) = x^2$ is

a) $(\sin x)^2$ b) $\sin x^2$
c) x^2 d) $x^2(\sin x)$

100. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$, then

$$f\left(f\left(\frac{1}{2}\right)\right)=$$

a) 2^{-4} b) 2^{-3}
c) 2^{-2} d) 2^{-1}

101. Two functions $f : R \to R$ and $g : R \to R$ are defined as follows:

$$f(x)=\begin{cases}0; (x \text{ rational})\\ 1; (x \text{ irratonal})\end{cases}$$

$$g(x)=\begin{cases}-1; (x \text{ rational})\\ 0; (x \text{ irratonal})\end{cases}$$

then (gof)(e) + (fog)(π) =

a) – 1 b) 0
c) 1 d) 2

102. If the functions f, g, h are defined from the sets of real numbers R to R such that

$$f(x)=x^2-1,\ g(x)=\sqrt{x^2+1},\ h(x)=\begin{cases}0, \text{ if } x\le 0\\ x, \text{ if } x>0\end{cases}$$

then the composite function (hofog)(x) =

a) $\begin{cases}0, x=0\\ x^2, x>0\\ -x^2, x<0\end{cases}$ b) $\begin{cases}0, x=0\\ x^2, x\ne 0\end{cases}$

c) $\begin{cases}0, x\le 0\\ x^2, x>0\end{cases}$ d) none of these

103. If $f : R \to R$ and $g : R \to R$ are given by $f(x) = |x|$ and $g(x) = [x]$ for each $x \in R$, then $\{x \in R : g(f(x)) \le f(g(x)\} =$

a) $Z \cup (-\infty, 0)$ b) $(-\infty, 0)$
c) Z d) R

104. If $f(x) = ax + b$ and $g(x) = cx + d$, then $f(g(x)) = g(f(x))$ is equivalent to

a) f(c) = g(a) b) f(d) = g(b)
c) f(a) = g(c) d) f(b) = g(b)

105. If

$$f(x) = \sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}\right)\cos x$$

and $g\left(\frac{5}{4}\right)=1$, then gof(x) is

a) a polynomial of first degree in sin x and cosx
b) a constant function
c) a polynomial of second degree in sin x and cos x
d) none of these

106. If $f(x) = x^3 + 5x + 1$ for real x, then

a) f is one-one and onto in R

b) f is one-one but not onto in R

c) f is onto in R but not one-one

d) f is neither one-one nor onto in R

107. Let $g(x) = 1 + x - [x]$ and

$f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$, then for all x, f(g(x)) is equal to

a) x　　b 1

c) f(x)　　d) g(x)

108. If $f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 0, & \text{if } x \text{ is irrational}\end{cases}$

and $g(x)=\begin{cases}0, & \text{if } x \text{ is rational}\\ x, & \text{if } x \text{ is irrational}\end{cases}$

then f – g is

a) one-one and onto

b) one-one and into

c) many one and onto

d) neither one-one nor onto

109. If $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$ for all real x,

then $(fog)^{-1}\left(\frac{1}{x}\right)$ is equal to

a) x　　b) $\frac{1}{x}$

c) – x　　d) $-\frac{1}{x}$

110. If $x \neq 1$ and $f(x)=\frac{x+1}{x-1}$ is a real function, then f(f(f(2))) is

a) 1　　b) 2

c) 3　　d) 4

111. The function $f : X \to Y$ defined by $f(x) = \sin x$ is one-one but not onto if X and Y are respectively equal to

a) R and R

b) $[0,\pi]$ and $[-1,1]$

c) $\left[0, \frac{\pi}{2}\right]$ and $[-1, 1]$

d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$

112. If the function $f : N \to N$ is defined by

$f(x)=\sqrt{x}$, then $\frac{f(25)}{f(16)+f(1)}$ is equal to

a) $\frac{5}{6}$　　b) $\frac{5}{7}$

c) $\frac{5}{3}$　　d) 1

113. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying (fogogof) (x) = (gogof) (x), where (fog) (x) = f(g(x)) is

a) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, ...\}$

b) $\pm\sqrt{n\pi}, n \in \{1, 2, ...\}$

c) $\frac{\pi}{2}+2n\pi,\ n \in \{....., -2, -1, 0, 1, 2, ...\}$

d) $2n\pi,\ n \in \{....., -2, -1, 0, 1, 2, ...\}$

114. The domain of the function $\cos^{-1}(\log_2(x^2 + 5x + 8))$ is

a) [2,3]　　b) [– 2,2]

c) [3, 1]　　d) [– 3,– 2]

115. If $f : R \to R$ is defined by $f(x) = |x|$ then

a) $f^{-1}(x) = -x$

b) $f^{-1}(x)=\frac{1}{|x|}$

c) The function $f^{-1}(x)$ does not exist

d) $f^{-1}(x)=\frac{1}{x}$

116. The range of function $f(x)=\log_e\sqrt{4-x^2}$ is given by

a) $(0, \infty)$　　b) $(-\infty, \infty)$

c) $(-\infty, \log_e 2)$　　d) $(\log_e 2, \infty)$

117. The range of the function $f(x)=\tan\sqrt{\frac{\pi^2}{9}-x^2}$ is

a) $[0, 3]$ b) $[0, \sqrt{3}]$

d) $[3, \sqrt{3}]$ d) $[\sqrt{3}, 3]$

118. Equation cos 2x + 1 = a(2 – sin x) can have a real solution for

a) all values of a b) $a \in [2, 6]$

c) $a \in (-\infty, 2)$ d) $a \in (0, \infty)$

119. The range of the function $f(x) = \log_e(3x^2 + 4)$ is equal to

a) [logo 2, ∞] b) $[\log_e 3, \infty)$

c) $[2\log_e 3, \infty)$ d) $[2\log_e 2, \infty)$

120. If $f(x) = \sin x + \cos x, x \in (-\infty, \infty)$ and $g(x) = x^2, x \in (-\infty, \infty)$, then (fog) (x) is equal to

a) 1 b) 0

c) $\sin^2(x) + \cos(x^2)$ d) $\sin(x^2) + \cos(x^2)$

121. Number of bijective function from a set of 10 elements to itself is

a) 5! b) 10!

c) 15! d) 8!

122. If g(y) is inverse of function $f : R \to R$ given by f(x) = x + 3, then g(y) =

a) y + 3 b) y – 3

c) $\frac{y}{3}$ d) 3y

123. Let R be the set of real numbers and the mapping $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 5 - x^2$ and g(x) = 3x – 4, then the value of (fog) (–1) is

a – 44 b) – 54

c) – 32 d) – 64

124. A = {1, 2, 3, 4}, B = (1, 2, 3, 4, 5, 6} are two sets and function $f : A \to B$ is- defined by $f(x) = x + 2;\ \forall x \in A$, then the function f is

a) Bijective b) Onto

c) One-one d) Many-one

125. The function $f(x)=\sec\left[\log(x+\sqrt{1+x^2})\right]$ is

a) Odd

b) Even

c) Neither odd nor even

d) Constant

126. The domain of the function

$f(x)\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$ is

a) $(-3,3)$ b) $[-3, 3]$

c) $(-\infty,-3)\cup(3,\infty)$ d) $(-\infty,-3)\cup[3,\infty)$

127. Let f = {(1, 1), (2, 4), (0, – 2), (– 1, – 5)} be a linear function from Z into Z. Then, f(x) is

a) f(x) = 3x – 2 b) f(x) = 6x – 8

c) f(x) = 5x – 2 d) f(x) = 7x + 2

128. Let $f : R - \left\{\frac{5}{4}\right\} \to R$ be a function defined as $f(x)=\frac{5x}{4x+5}$. The inverse of f is the map $g : \text{Range } f \to R - \left\{\frac{5}{4}\right\}$ given by

a) $g(y)=\frac{y}{5-4y}$ b) $g(y)=\frac{5y}{5+4y}$

c) $g(y)=\frac{5y}{5-4y}$ d) None of these

129. Let A = {– 1, 0, 1, 2}, B = {– 4, – 2, 0, 2} and $f, g : A \to B$ be function defined by

$f(x) = x^2 - x$ and $g(x) = 2\left|x-\frac{1}{2}\right|-1$. Then

a) f = g b) f = 2g

c) g = 2f d) None of these

130. If $f(x)=\frac{x-1}{x+1}$, then f(2x) is

a) $\frac{f(x)+1}{f(x)+3}$ b) $\frac{3f(x)+1}{f(x)+3}$

c) $\frac{f(x)+3}{f(x)+1}$ d) $\frac{f(x)+3}{3f(x)+1}$

131. Let $f : N \to N$ defined by

$$f(n)=\begin{cases}\frac{n+1}{2} & \text{if } n \text{ is odd}\\ \frac{n}{2} & \text{if } n \text{ is even}\end{cases}$$ then f is

a) onto but not one-one

b) one-one and onto

c) neither one-one nor onto

d) one-one but not onto

Evaluation Test

1. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}$ $(gof)(x) = 2x^2 - 5x + 2$, then f(x) is equal to

a) $2x + 3$ b) $2x - 3$
c) $2x^2 + 3x + 1$ d) $2x^2 - 3$

2. If f(x) and g(x) are two functions with

$g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then $f'(x) =$

a) $3x^2 + 3$ b) $x^2 - \frac{1}{x^2}$
c) $1 + \frac{1}{x^2}$ d) $3x^2 + \frac{3}{x^4}$

3. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$ for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

a) $\frac{7n(n+1)}{2}$ b) $\frac{7n}{2}$
c) $\frac{7(n+1)}{2}$ d) $7n(n + 1)$

4. If $f :[- 6, 6] \to R$ is defined by $f(x) = x^2 - 3$ for $x \in R$, then $(fofof)(-1) + (fofof)(0) + (fofof)(1)$ is equal to

a) $f(4\sqrt{2})$ b) $f(3\sqrt{2})$
c) $f(2\sqrt{2})$ d) $r(\sqrt{2})$

5. Let [x] denote the greatest integer less than or equal to x. If $x = (\sqrt{3}+1)^5$, then [x] is equal to

a) 75 b) 50
c) 76 d) 152

6. If f is a real valued function such that $f(x + y) = f(x) + f(y)$ and $f(1) = 5$, then the value of f(100) is

a) 200 b) 300
c) 350 d) 500

7. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$,
$T = \{(x,y) : x - y \text{ is an integer}\}$
Which one of the following is true?

a) S is an equivalence relation on R but T is not
b) T is an equivalence relation on R but S is not
c) Neither S nor T is an equivalence relation on R
d) Both S and T are equivalence relations on R

8. Consider the following relations:
$R = \{(x, y) : x, y$ are real numbers and $x = \omega y$ for some rational number $\omega\}$

$S\left\{\left(\frac{m}{n}, \frac{p}{q}\right)\right.$: m, n, p and q are integers such that n, $q \neq 0$ and $qm = pn\}$ Then,

a) S is an equivalence relation but R is not an equivalence
b) R and S both are equivalence relations
c) R is an equivalence relation but S is not an equivalence relation
d) neither R nor S is an equivalence relation

9. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then,

a) $P \subset Q$ and $Q - P = \phi$
b $Q \not\subset P$
c) $P \not\subset Q$
d) $P = Q$

10. The domain of the function

$f(x) = \sin^{-1}\left(\frac{8,3^{x-2}}{1-3^{2(x-1)}}\right)$ is

a) $(-\infty, 0]$ b) $[2, -\infty)$
c) $(-\infty, 0) \cup [2, \infty)$ d) $(-\infty, -1) \cup (1, \infty)$

11. A real valued function f(x) satisfies the functional equation
$f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to

a) $f(-x)$ b) $f(a) + f(a - x)$
c) $f(x)$ d) $f(x)$

12. If n(A) denotes the number of elements in set A and if $n(A) = 4$, $n(B) = 5$ and $n(A \cap B) = 3$, then $n[(A \times B) \cap (B \times A)] =$

a) 8 b) 9
c) 10 d) 11

13. If A = {θ : 2 cos² θ + sin θ ≤ 2} and

$B=\left\{\theta:\frac{\pi}{2}\le\theta\le\frac{3\pi}{2}\right\}$, then A ∩ B is equal to

a) $\left\{\theta:\pi\le\theta\le\theta\le\frac{3\pi}{2}\right\}$

b) $\left\{\theta:\frac{\pi}{2}\le\theta\le\frac{5\pi}{6}\right\}$

c) $\left\{\theta:\frac{\pi}{2}\le\theta\le\frac{5\pi}{6}\right\}\cup\left\{\theta:\pi\le\theta\le\frac{3\pi}{2}\right\}$

d) none of these

14. The domain of the function

$f(x)=\sqrt{\log_{10}\left\{\frac{\log_{10}x}{2(3-\log_{10}x)}\right\}}$ is

a) $(10, 10^3)$

b) $(10^2, 10^3)$

c) $[10^2, 10^3)$

d) $[10^2, 10^3]$

Classical Thinking

1. (C)	2. (B)	3. (C)	4. (A)	5. (D)	6. (C)	7. (D)	8. (B)	9. (A)	10. (A)
11. (B)	12. (A)	13. (A)	14. (C)	15. (B)	16. (A)	17. (C)	18. (D)	19. (C)	20. (A)
21. (A)	22. (A)	23. (A)	24. (C)	25. (D)	26. (D)	27. (B)	28. (A)	29. (B)	30. (A)
31. (D)	32. (C)	33. (A)	34. (D)	35. (B)	36. (A)	37. (D)	38. (B)	39. (C)	40. (B)
41. (B)	42. (C)	43. (C)	44. (B)	45. (A)	46. (C)	47. (D)	48. (D)	49. (A)	50 (B)
51. (C)	52. (A)	53. (B)	54. (D)	55. (D)	56. (A)	57. (D)	58. (C)	59. (D)	60. (A)
61. (B)	62. (D)	63. (C)	64. (A)	65. (B)	66. (C)	67. (B)	68. (A)	69. (D)	70. (B)

Critical Thinking

1. (D)	2. (B)	3. (D)	4. (D)	5. (A)	6. (C)	7. (D)	8. (B)	9. (A)	10. (C)
11. (C)	12. (B)	13. (C)	14. (D)	15. (B)	16. (B)	17. (C)	18. (C)	19. (C)	20. (A)
21. (D)	22. (A)	23. (B)	24. (B)	25. (B)	26. (C)	27. (A)	28. (C)	29. (A)	30. (C)
31. (A)	32. (D)	33. (D)	34. (A)	35. (A)	36. (B)	37. (B)	38. (A)	39. (C)	40. (C)
41. (A)	42. (B)	43. (B)	44. (C)	45. (C)	46. (A)	47. (C)	48. (A)	49. (B)	50. (A)
51. (B)	52. (A)	53. (D)	54. (C)	55. (D)	56. (B)	57. (B)	58. (B)	59. (A)	60. (C)
61. (C)	62. (B)	63. (D)	64. (B)	65. (C)	66. (C)	67. (A)	68. (A)	69. (C)	70. (A)
71. (A)	72. (B)	73. (D)	74. (B)	75. (B)	76. (C)	77. (C)	78. (A)	79. (D)	80. (B)
81. (B)	82. (C)	83. (A)	84. (D)	85. (D)	86. (B)	87. (D)	88. (C)	89. (A)	90. (A)

Competitive Thinking

1. (B)	2. (C)	3. (A)	4. (C)	5. (B)	6. (B)	7. (C)	8. (A)	9. (D)	10. (A)
11. (B)	12. (C)	13. (D)	14. (A)	15. (C)	16. (D)	17. (A)	18. (B)	19. (C)	20. (D)
21. (B)	22. (D)	23. (B)	24. (D)	25. (C)	26. (C)	27. (C)	28. (D)	29. (C)	30. (B)
31. (A)	32. (A)	33. (D)	34. (C)	35. (A)	36. (C)	37. (A)	38. (A)	39. (C)	40. (C)
41. (B)	42. (D)	43. (C)	44. (C)	45. (A)	46. (A)	47. (D)	48. (A)	49. (A)	50. (B)
51. (A)	52. (D)	53. (C)	54. (B)	55. (C)	56. (D)	57. (D)	58. (C)	59. (B)	60. (D)
61. (B)	62. (D)	63. (B)	64. (C)	65. (C)	66. (A)	67. (C)	68 (C)	69. (D)	70. (C)
71. (B)	72. (C)	73. (C)	74. (B)	75. (B)	76. (D)	77. (B)	78. (D)	79. (A)	80. (B)
81. (B)	82. (D)	83. (C)	84. (A)	85. (A)	86. (A)	87. (D)	88. (C)	89. (B)	90. (B)
91. (B)	92. (B)	93. (A)	94. (D)	95. (C)	96. (B)	97. (A)	98. (D)	99. (B)	100. (D)
101. (A)	102. (B)	103. (D)	104. (B)	105. (B)	106. (A)	107. (B)	108. (A)	109. (B)	110. (C)
111. (C)	112. (D)	113. (A)	114. (D)	115. (C)	116. (C)	117. (B)	118. (B)	119. (D)	120. (D)
121. (B)	122. (B)	123. (A)	124. (C)	125. (B)	126. (B)	127. (A)	128. (C)	129. (A)	130. (B)
131. (A)									

Answers to Evaluation Test

1. (B)	2. (A)	3. (A)	4.(A)	5. (D)	6. (D)	7. (B)	8.(A)	9. (D)	10.(C)
11.(D)	12.(B)	13.(C)	14.(C)						

Answer Key

Classical Thinking

1. (C)	2. (B)	3. (C)	4. (A)	5. (D)	6. (C)	7. (D)	8. (B)	9. (A)	10. (A)
11. (B)	12. (A)	13. (A)	14. (C)	15. (B)	16. (A)	17. (C)	18. (D)	19. (C)	20. (A)
21. (A)	22. (A)	23. (A)	24. (C)	25. (D)	26. (D)	27. (B)	28. (A)	29. (B)	30. (A)
31. (D)	32. (C)	33. (A)	34. (D)	35. (B)	36. (A)	37. (D)	38. (B)	39. (C)	40. (B)
41. (B)	42. (C)	43. (C)	44. (B)	45. (A)	46. (C)	47. (D)	48. (D)	49. (A)	50 (B)
51. (C)	52. (A)	53. (B)	54. (D)	55. (D)	56. (A)	57. (D)	58. (C)	59. (D)	60. (A)
61. (B)	62. (D)	63. (C)	64. (A)	65. (B)	66. (C)	67. (B)	68. (A)	69. (D)	70. (B)

Critical Thinking

1. (D)	2. (B)	3. (D)	4. (D)	5. (A)	6. (C)	7. (D)	8. (B)	9. (A)	10. (C)
11. (C)	12. (B)	13. (C)	14. (D)	15. (B)	16. (B)	17. (C)	18. (C)	19. (C)	20. (A)
21. (D)	22. (A)	23. (B)	24. (B)	25. (B)	26. (C)	27. (A)	28. (C)	29. (A)	30. (C)
31. (A)	32. (D)	33. (D)	34. (A)	35. (A)	36. (B)	37. (B)	38. (A)	39. (C)	40. (C)
41. (A)	42. (B)	43. (B)	44. (C)	45. (C)	46. (A)	47. (C)	48. (A)	49. (B)	50. (A)
51. (B)	52. (A)	53. (D)	54. (C)	55. (D)	56. (B)	57. (B)	58. (B)	59. (A)	60. (C)
61. (C)	62. (B)	63. (D)	64. (B)	65. (C)	66. (C)	67. (A)	68. (A)	69. (C)	70. (A)
71. (A)	72. (B)	73. (D)	74. (B)	75. (B)	76. (C)	77. (C)	78. (A)	79. (D)	80. (B)
81. (B)	82. (C)	83. (A)	84. (D)	85. (D)	86. (B)	87. (D)	88. (C)	89. (A)	90. (A)

Competitive Thinking

1. (B)	2. (C)	3. (A)	4. (C)	5. (B)	6. (B)	7. (C)	8. (A)	9. (D)	10. (A)
11. (B)	12. (C)	13. (D)	14. (A)	15. (C)	16. (D)	17. (A)	18. (B)	19. (C)	20. (D)
21. (B)	22. (D)	23. (B)	24. (D)	25. (C)	26. (C)	27. (C)	28. (D)	29. (C)	30. (B)
31. (A)	32. (A)	33. (D)	34. (C)	35. (A)	36. (C)	37. (A)	38. (A)	39. (C)	40. (C)
41. (B)	42. (D)	43. (C)	44. (C)	45. (A)	46. (A)	47. (D)	48. (A)	49. (A)	50. (B)
51. (A)	52. (D)	53. (C)	54. (B)	55. (C)	56. (D)	57. (D)	58. (C)	59. (B)	60. (D)
61. (B)	62. (D)	63. (B)	64. (C)	65. (C)	66. (A)	67. (C)	68 (C)	69. (D)	70. (C)
71. (B)	72. (C)	73. (C)	74. (B)	75. (B)	76. (D)	77. (B)	78. (D)	79. (A)	80. (B)
81. (B)	82. (D)	83. (C)	84. (A)	85. (A)	86. (A)	87. (D)	88. (C)	89. (B)	90. (B)
91. (B)	92. (B)	93. (A)	94. (D)	95. (C)	96. (B)	97. (A)	98. (D)	99. (B)	100. (D)
101. (A)	102. (B)	103. (D)	104. (B)	105. (B)	106. (A)	107. (B)	108. (A)	109. (B)	110. (C)
111. (C)	112. (D)	113. (A)	114. (D)	115. (C)	116. (C)	117. (B)	118. (B)	119. (D)	120. (D)
121. (B)	122. (B)	123. (A)	124. (C)	125. (B)	126. (B)	127. (A)	128. (C)	129. (A)	130. (B)
131. (A)									

Hints

Classical Thinking

1. Adding 1 to even integers give odd integers.

2. $A \cup B = A$ if every element of B is contained in A i.e $B \subset A$

7. $A - (B \cup C) = (A - B) \cap (A - C)$

10. $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, \ldots\}$

    $B = \{4, 8, 12, 16, 20, \ldots\}$

    $A \cup B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20 \ldots\}$

12. $(A \cap B)' = A' \cup B'$

13. There is no real number which is both rational as well as irrational.

17. $B \cup C = \{1, 3, 4, 5, 6, 7, 8, 9\}$
$A \cap B = \{5, 7\}$, $A \cap C = \{4, 8\}$
$A \cap (B \cup C) = \{4, 5, 7, 8\}$
$(A \cap B) \cup (A \cap C) = \{4, 5, 7, 8\}$

18. $n(U) = 100$
A = Students who play cricket, $n(A) = 60$
B = Students who play volleyball, $n(B) = 50$
$A \cap B$ = Students who play both the games,
$n(A \cap B) = 28$
$\therefore$ Number of students who play atleast one game
$= n(A \cup B) = n(A) + n(B) - n(A \cap B) = 82$

19. $A = \{1, 2, 3, 4, 5, \ldots\}$, $B = \{2, 4, 6, 8, \ldots\}$
$\therefore$ $A \cap B = \{2, 4, 6, 8 \ldots\}$

22. $B = \{2, 4, 6, 8, \ldots\}$, $C = \{1, 3, 5, 7, \ldots\}$
$B \cap C = \phi$

23. $n(A) = 25$, $n(B) = 20$ and $n(A \cup B) = 35$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$35 = 25 + 20 - n(A \cap B)$
$\Rightarrow n(A \cap B) = 10$

24. Since A and B are disjoint,
$\therefore$ $A \cap B = \phi$
$\therefore$ $n(A \cap B) = 0$
Now $n(A \cup B) = n(A) + n(B) - n(A \cup B)$
$= n(A) + n(B) - 0$
$= n(A) + n(B)$.

25. T = Set of members who like tea, $n(T) = 11$
C = Set of members who like coffee, $n(C) = 14$
$\therefore$ $n(T \cup C) = 20$
$T \cap C'$ = Set of members who like only tea and not coffee.
$\therefore$ $n(T \cup C') = n(T) - n(T \cap C)$
$T \cap C$ = Set of members who like both tea and coffee
$\therefore$ $n(T \cap C) = n(T) + n(C) - n(T \cup C) = 5$
$\therefore$ $n(T \cap C) = 5$
$\therefore$ $n(T \cup C') = n(T) - n(T \cap C) = 11 - 5 = 6$

26. $A = \{2, 4, 6, 8, 10, \ldots\}$, $B = \{5, 10, 15, 20, \ldots\}$
$C = \{10, 20, 30, 40, \ldots\}$
and $(A \cap B) = \{10, 20, 30, \ldots\}$
$\therefore$ $(A \cap B) \cap C = \{10, 20, 30, \ldots\}$

27. Since A, B, C are disjoint sets.
$\therefore$ $n(A \cup B \cup C) = n(A) + n(B) + n(C) = 21$

28. $B \cap C = \{\ \}$
$A \cup (B \cap C) = \{2, 4, 5, 7, 8\}$
$(A \cup B) \cap (A \cup C) = \{2, 4, 5, 7, 8\}$

29. $A - B = \{x: x \in A \text{ and } x \notin B\} = A \cap B'$

30. $A = \{2, 3\}$, $B = \{2, 4\}$, $C = \{4, 5\}$
$\therefore$ $(B \cap C) = \{4\}$
$\therefore$ $A \times (B \cap C) = \{2, 3\} \times \{4\} = \{(2, 4), (3, 4)\}$

31. $(Y \times A) \cap (Y \times B) = Y \times (A \cap B) = Y \times \phi = \phi$

32. Here, $B = A$

33. $A \times B = \{(1, 0), (1, 1), (2, 0), (2, 1)\}$

34. $n(A \times A \times B) = n(A).\ n(A).\ n(B) = 3 \times 3 \times 4$
$= 36$

35. $A - B = \{1\}$, $B - C = \{4\}$
$\therefore$ $(A - B) \times (B - C) = \{(1, 4)\}$

36. Since (a, 2), (b, 3), (c, 2), (d, 3), (e, 2) are elements of $A \times B$
$\therefore$ a, b, c, d, e $\in$ A and 2, 3 $\in$ B

37. $A = \{a, b\}$, $B = \{1, 2, 3\}$
$\therefore$ $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$
$B \times A = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, c)\}$
$\therefore$ $(A \times B) \cap (B \times A) = \phi$

40. Dom (R) = {1, 2, 3}

41. $A \cap B = \{3\}$ and $A = \{1, 2, 3\}$

43. Since $x \not< x$ therefore R is not reflexive. Also $x < y$ does not imply that $y < x$, So R is not symmetric. Let x Ry and y Rz. Then, x, y and $y < z \Rightarrow x < z$ i.e., x Rz. Hence, R is transitive.

45. $f(x) = x^2 - 3x + 2 \Rightarrow f(-1) = (-1)^2 - 3(-1) + 2$

46. $f(x) = x^2 - 3x + 2$
$f(a + h) = (a + h)^2 - 3(a + h) + 2$
$= a^2 + (2a - 3)h - 3a + 2 + h^2$

47. $f(x) = ax + 6 \Rightarrow f(1) = a(1) + 6 = a + 6$
$f(1) = 11 \Rightarrow 11 = a + 6 \Rightarrow a = 5$

48. $f(x) = x^2 - 6x + 5,\ 0 \le x \le 4$
f(8) does not exist (since $x = 8$ does not belong to the domain of f).

49. Since $f(x) = 3x - 1$, $g(x) = x^2 + 1$
$\therefore$ $f[g(x)] = 3[g(x)] - 1 = 3[x^2 + 1] - 1 = 3x^2 + 2$

51. $\dfrac{3x^2 + 7x - 1}{3} = x^2 + \dfrac{7}{3}x - \dfrac{1}{3}$ is a polynomial function.

52. $f(a+1) - f(a-1)$
$= 4(a+1) - (a+1)^2 - [4(a-1) - (a-1)^2]$
$= 4(2-a)$

54. $f(x) = \dfrac{x-1}{x+1}$
$\Rightarrow f\left(\dfrac{1}{f(x)}\right) = f\left(\dfrac{x+1}{x-1}\right) = \dfrac{\dfrac{x+1}{x-1}-1}{\dfrac{x+1}{x-1}+1} = \dfrac{1}{x}$

55. $g[f(x)] = 5[f(x)] - 6 = 5x^2 - 6$

56. $f(x) = x^2 + \dfrac{1}{x}$
$f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^2 + \dfrac{1}{\left(\dfrac{1}{x}\right)} = \dfrac{1}{x^2} + x$

57. $(fog)(x) = f[g(x)] = f(x^3+1) = (x^3+1)^2$

58. $f(x) = x^2 - 6x + 9,\ 0 \le x \le 4$
$f(3) = (3)^2 - 6(3) + 9 = 0$

59. For Dom(f), $5x - 7 > 0 \Rightarrow x > \dfrac{7}{5}$
Hence, $D_f = \left(\dfrac{7}{5}, \infty\right)$

61. $f(f(x)) = f(x^2+1) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$

62. As f (b) is not defined, f is not a function.

63. $f\left(f\left(\dfrac{1}{x}\right)\right) = f\left(1 - \dfrac{1}{1/x}\right) = f(1-x) = \dfrac{x}{x-1}$

65. $-1 \le 5x \le \Rightarrow \dfrac{-1}{5} \le x \le \dfrac{1}{5}$
Hence, domain is $\left[\dfrac{-1}{5}, \dfrac{1}{5}\right]$.

66. For $x = -3, 3$, $|x^2 - 9| = 0$
Therefore, $\log|x^2 - 9|$ does not exist at $x = -3, 3$.
Hence, domain of function is $R - \{-3, 3\}$

67. $\log\left\{\dfrac{5x - x^2}{6}\right\} \ge 0 \Rightarrow \dfrac{5x - x^2}{6} \ge 1$
$\Rightarrow x^2 - 5x + 6 \le 0$ or $(x-2)(x-3) \le 0$.
Hence, $2 \le x \le 3$.

68. $y = 2x - 3 \Rightarrow x = \dfrac{y+3}{2}$
$\Rightarrow f^{-1}(y) = \dfrac{y+3}{2} \Rightarrow f^{-1}(x) = \dfrac{x+3}{2}$

69. $f(x) = \dfrac{(x-2)(x-1)}{(x-2)(x+3)}$
Hence, domain is $\{x : x \in R, x \ne 2, x \ne -3\}$.

70. $f[g(x)] = \dfrac{3[g(x)]+4}{5[g(x)]-7} = \dfrac{3\left[\dfrac{7x+4}{5x-3}\right]+4}{5\left[\dfrac{7x+4}{5x-3}\right]-7} = x$

Critical Thinking

1. A – B and B – A are always disjoint and hence A – B = B – A only if either of these is ϕ i.e., if $A \subset B$ and $B \subset A$ i.e., if A = B.

2. There is no real number x such that $x^2 + 1 = 0$.

4. $A = \{x / 6x^2 + x - 15 = 0\}$
$\therefore$ $6x^2 + x - 15 = 0$
$\Rightarrow (3x+5)(2x-3) = 0$
$\therefore$ $x = -\dfrac{5}{3}$ or $x = \dfrac{3}{2}$
$\Rightarrow A = \left\{-\dfrac{5}{3}, \dfrac{3}{2}\right\}$
Similarly, $B = \left\{1, \dfrac{3}{2}\right\}$ and $C = \left\{-1, \dfrac{3}{2}\right\}$
$A \cap B \cap C = \left\{\dfrac{3}{2}\right\}$

5. $A \cup B = \{1, 2, 3, 4, 6\}$
$\Rightarrow (A \cup B)' = \{5, 7, 8\}$

6. For any $(a, b) \in A \times B$, $a \in A$ and $b \in B$.
Now (a, b) will belong to B × A only if $a \in B$ and $b \in A$ and that can happen only if $A \cap B \ne \phi$. But, in this case $A \cap B = \phi$.
$\therefore$ $(A \times B) \cap (B \times A) = \phi$

7. A – B is the set of those elements of A which are not common with B.

8. U = Universal set of all adults
M = Set of all males, F = Set of all females
V = Set of all vegetarians
Total number of adults = 20
Total number of males = 8
$\therefore$ Total number of females = 20 – 8 = 12
Total number of vegetarian = 9
Total number of male vegetarian = 5
$\therefore$ Total number of female vegetarian = 9 – 5 = 4
$\therefore$ Total number of female non-vegetarian
= 12 – 4 = 8

9. $A = \{2, 4, 6, 8, 10, \ldots\}$, $B = \{5, 10, 15, 20, \ldots\}$, $C = \{10, 20, 30, 40, \ldots\}$
and $(B \cup C) = \{5, 10, 15, 20, \ldots\}$
$\therefore$ $A \cap (B \cup C) = \{10, 20, 30, \ldots\}$

10. $n(A) = n(X) - n(A') = 19$
$n(B) = n(X) - n(B') = 14$
$n(A \cap B) = n(X) - n(A \cap B)' = 5$
$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 28$

11. $W \rightarrow$ denotes whole numbers
$2a + b = 5$
$\therefore$ $a = 0, b = 5$ or $a = 1, b = 3$ or $a = 2, b = 1$
For $a \geq 3$, the values of b will not be whole numbers $\Rightarrow A = \{(0, 5), (1, 3), (2, 1)\}$

12. $A = \{3, 4\}$, $B = \{-3, 4\}$, $A \cap B = \{4\}$

13. Number of proper subsets of $A = 2^n - 1$
$= 2^5 - 1$ ....[$\because$ o(A) = 5]
$= 32 - 1 = 31$

14. $\subset$ is a relation between two sets and 0 is not a set.

15. C = Set of students who play chess
T = Set of students who play table tennis
M = Set of students who play carrom
$\therefore$ $n(X) = 120$, $n(C) = 46$, $n(T) = 30$, $n(M) = 40$
$n(C \cap T) = 14$, $n(T \cap M) = 10$, $n(C \cap M) = 8$,
$n(C \cup T \cup M)' = 30$
$\therefore$ $n(C \cup T \cup M) = n(X) - n(C \cup T \cup M)' = 90$
$(C \cap T \cap M)$ = Set of students who play chess, table tennis and carrom.
$\therefore$ $n(C \cup T \cup M)$
$= n(C) + n(T) + n(M) - n(C \cap T) - n(T \cap M) - n(C \cap M) + n(C \cap T \cap M)$
$\therefore$ $90 = 46 + 30 + 40 - 14 - 10 - 8 + n(C \cap T \cap M)$
$\therefore$ $n(C \cap T \cap M) = 6$

16. Q is not a null set because $Q = \{0\}$

17. $A - B$, $B - A$ and $A \cap B$ are pairwise disjoint and their union is $A \cup B$.

18. $B = \{-3, 4\}$, $C = \{3, 5\}$, $B \cup C = \{-3, 3, 4, 5\}$

20. A = Set representing no. of consumers using Brand A, $n(A) = 15$
B = Set representing no. of consumers using Brand B, $n(B) = 20$
$A \cap B$ = Set representing no. of consumers using both the brands, $n(A \cap B) = 5$
$A \cup B$ = Set representing no. of consumers using atleast one brand.
$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 30$

21. $A = \{4, 5\}$, $B = \{-6, -7\}$, $C = \{-7, 10\}$
$(B \cap C) = \{-7\} \Rightarrow A \cap (B \cap C) = \phi$

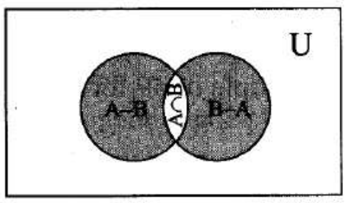
22. P = Set of children who like pizza
B = Set of children who like burger
$n(P) = 62$, $n(B) = 47$, $n(P \cap B) = 36$
$(P \cap B')$ = Set of children who like pizza but not burger
$\therefore$ $n(P \cap B') = n(P) - n(P \cap B) = 62 - 36 = 26$.

24. Let $A \equiv$ set of persons who take tea and
$B \equiv$ set of persons who take coffee
$n(A \cup B) = 50$, $n(A) = 35$, $n(B) = 25$
$\therefore$ $n(A \cap B) = 10$
Hence, $n(A - B) = n(A) - n(A \cap B)$
$= 35 - 10 = 25$

25. Since, $y = e^x$, $y = e^{-x}$ will meet, when $e^x = e^{-x}$
$\Rightarrow e^{2x} = 1$,
$\therefore$ $x = 0, y = 1$
$\therefore$ A and B meet on (0, 1)
$\therefore$ $A \cap B \neq \phi$

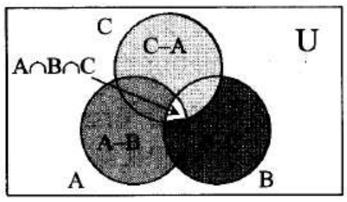
26. $A \cap (A \cup B)' = A \cap (A' \cap B')$, ...[$\because (A \cup B)' = A' \cap B'$]
$= (A \cap A') \cap B'$ ....[by associative law]
$= \phi \cap B'$, ...[$\because A \cap A' = \phi$]
$= \phi$.

27. From Venn-Euler's diagram,



$\therefore$ $(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$

28. From Venn-Euler's Diagram,



Clearly, $\{(A - B) \cup (B - C) \cup (C - A)\}' = A \cap B \cap C$.

29. Minimum value of $x = 100 - (30+20+25+15)$
$= 100 - 90 = 10$.

30. Let A denote the set of Americans, who like cheese and let B denote the set of Americans, who like apples.
Let Population of Americans be 100.
Then $n(A) = 63$, $n(B) = 76$
Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 63 + 76 - n(A \cap B)$
$\therefore$ $n(A \cup B) + n(A \cap B) = 139$
$\Rightarrow n(A \cap B) = 139 - n(A \cup B)$
But, $n(A \cup B) \leq 100$
$\therefore$ $-n(A \cup B) \geq -100$
$\therefore$ $139 - n(A \cup B) \geq 139 - 100 = 39$
$\therefore$ $n(A \cap B) \geq 39$ i.e., $39 \leq n(A \cap B)$
Again, $A \cap B \subseteq A$, $A \cap B \subseteq B$
$\therefore$ $n(A \cap B) \leq n(A) = 63$ and
$n(A \cap B) \leq n(B) = 76$
$\therefore$ $n(A \cap B) \leq 63$
Then, $39 \leq n(A \cap B) \leq 63 \Rightarrow 39 \leq x \leq 63$

31. Here $1, 2, 3 \in A$ & $3, 5 \in B$
$\therefore$ $A \times B = \{1, 2, 3\} \times \{3, 5\}$
$\therefore$ The remaining elements are : (1, 5), (2, 3), (3, 5)

32. A – B = A iff A and B have no element in common.

33. $C = \{1, 3, 5, 7, \ldots\}$, $D = \{2, 3, 5, 7, 11, \ldots\}$
$C \cap D = \{3, 5, 7, 11, \ldots\}$

34. Since, $8^n - 7n - 1 = (7 + 1)^n - 7n - 1$
$= 7^n + {}^nC_1 7^{n-1} + {}^nC_2 7^{n-2} + \ldots$
$+ {}^nC_{n-1} 7 + {}^nC_n - 7n - 1$
$= {}^nC_2 7^2 + {}^nC_3 7^3 + \ldots + {}^nC_n 7^n$,
$({}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc.)
$= 49[{}^nC_2 + {}^nC_3(7) + \ldots + {}^nC_n 7^{n-2}]$
$\therefore$ $8^n - 7n - 1$ is a multiple of 49 for $n \geq 2$
For $n = 1$, $8^n - 7n - 1 = 8 - 7 - 1 = 0$
For $n = 2$, $8^n - 7n - 1 = 64 - 14 - 1 = 49$
$\therefore$ $8^n - 7n - 1$ is a multiple of 49 for $n \in N$.
$\therefore$ X contains elements which are multiples of 49 and clearly Y contains all multiples of 49.
$\therefore$ $X \subseteq Y$

36. $R_2 \subseteq A \times B$, so it is a relation from A to B.

37. Clearly, A is the set of all first elements in ordered pairs in A × B and B is the set of all second elements in A × B.

38. Since, $(-1, 0) \in A \times A$ and $(0, 1) \in A \times A$
$\therefore$ $(-1, 0) \in A \times A \Rightarrow -1, 0 \in A$
and $(0, 1) \in A \times A \Rightarrow 0, 1 \in A$
$\therefore$ $\{-1, 0, 1\} \in A$

39. The given set is a cartesian product containing 6 elements. Only $A \times (B \cup C)$ contains 6 elements.

40. $(1, 4), (2, 6), (3, 6) \in A \times B$
$\Rightarrow \{1, 2, 3\} \subset A$ and $\{4, 6\} \subset B$
$\therefore$ A has 3 elements and B has 2 elements.

41. $n(A \times A) = n(A).\, n(A) = 3^2 = 9$
So, the total number of subsets of A × A is 2^9 and a subset of A × A is a relation over the set A.

42. The given relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$.

43. Since R is an equivalence relation on set A, therefore $(a, a) \in R$ for all $a \in A$. Hence, R has at least n ordered pairs.

44. We have, R = {(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4, 5)}
R^{-1} = {(3, 1), (5, 1), (3, 2), (5, 2); (5, 3); (5, 4)}
Hence, RoR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}

45. Number of relations on the set A = Number of subsets of $(A \times A) = 2^{n^2}$, $[\because n(A \times A) = n^2]$.

46. Number of relations from A to B = $2^{o(A) \cdot o(B)}$

47. Since, $R = \{(x, y) | x, y \in Z, x^2 + y^2 \leq 4\}$
$\therefore$ R = {(–2, 0), (–1, 0), (–1, 1), (0 –1) (0, 1), (0, 2), (0, –2) (1, 0), (1, 1), (2, 0)}
Hence, Domain of R = {– 2, – 1, 0, 1, 2}.

48. For any $a \in N$, we find that a|a. therefore R is reflexive but R is not symmetric, because aRb does not imply that bRa.

49. The relation is not symmetric, because $A \subseteq B$ does not imply that $B \subset A$. But it is anti-symmetric because $A \subset B$ and $B \subset A \Rightarrow A = B$

50. R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3 \Rightarrow x - y = 3$
$\therefore$ R = {11, 8},{13, 10}.
Hence, R^{-1} = {(8, 11), (10, 13)}

51. Let $(a, b) \in R$
Then, $(a, b) \in R \Rightarrow (b, a) \in R^{-1}$
$\Rightarrow (b, a) \in R$ ....$[\because R = R^{-1}]$
So, R is symmetric.

55. $f(x) = ax^2 + bx + 2$

$\therefore$ $f(1) = a(1)^2 + b(1) + 2 = a + b + 2$

But $f(1) = 3 \Rightarrow 3 = a + b + 2 \Rightarrow a + b = 1$ ....(i)

and $f(4) = a(4)^2 + b(4) + 2 = 16a + 4b + 2$

But $f(4) = 42 \Rightarrow 42 = 16a + 4b + 2$

$\therefore$ $40 = 16a + 4b \Rightarrow 4a + b = 10$ ....(ii)

By solving, (i) & (ii) a = 3 and b = – 2

56. $(gof)(1) = g(f(1)) = g(4) = 8$,

$(gof)(2) = g(f(2)) = g(5) = 7$

and $(gof)(3) = g(f(3)) = g(6) = 9$

57. $a.f(x) + b.f\left(\frac{1}{x}\right) = \frac{1}{x} - 5$

On replacing x by $\frac{1}{x}$, $b.f(x) + a.f\left(\frac{1}{x}\right) = x - 5$

Solving two equations,

$$f(x) = \frac{1}{a^2 - b^2}\left(\frac{a}{x} - bx\right) - \frac{5}{a+b}$$

$\therefore$ $f(2) = \dfrac{3(2b - 3a)}{2(a^2 - b^2)}$

58. $f(x)$ is defined for all $x \in R$. So, dom(f) = R.

Let $y = f(x) \Rightarrow y = \dfrac{x}{1+x^2}$

$\therefore$ $x = \dfrac{1 \pm \sqrt{1 - 4y^2}}{2y}$

For x to be real, $1 - 4y^2 \geq 0$ and $y \neq 0$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$ and $y \neq 0$

59. $f^{-1}(y) = \{x \in R : y = f(x)\}$

$\Rightarrow f^{-1}(2) = \{x \in R : 2 = f(x)\}$

$= \{x \in R : x^2 - 3x + 4 = 2\}$

$= \{x \in R : x^2 - 3x + 2 = 0\} = \{1, 2\}$

60. $f(x)$ is defined for all $x \in R - \{0\}$.

So, dom(f) = $R - \{0\}$

Let $y = \dfrac{1+x^2}{x^2} \Rightarrow x = \pm\sqrt{\dfrac{1}{y-1}}$

For x to be real, $y - 1 > 0 \Rightarrow y \in (1, \infty)$

61. $f(x) = \dfrac{x-1}{x+1} \Rightarrow \dfrac{f(x)+1}{f(x)-1} = \dfrac{2x}{-2}$

$\Rightarrow x = \dfrac{1+f(x)}{1-f(x)}$

$\therefore$ $f(\alpha x) = \dfrac{\alpha x - 1}{\alpha x + 1} = \dfrac{(\alpha+1)f(x) + \alpha - 1}{(\alpha-1)f(x) + \alpha + 1}$

62. $f(x) = x + \dfrac{1}{x} \Rightarrow f(x^3) = x^3 + \dfrac{1}{x^3}$

$\therefore$ $[f(x)]^3 = \left(x + \frac{1}{x}\right)^3 = \left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$

$\therefore$ $[f(x)]^3 = f(x^3) + 3f(x)$

$\therefore$ $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right) \Rightarrow \lambda = 3$

63. As f (a) is not unique,

$\therefore$ f is not a function.

64. $\text{Dom}(f) = R - \left\{-\frac{2}{3}\right\}$

For Range(f), let $y = f(x) = \dfrac{1}{3x+2}$

$\therefore$ $3x + 2 = \dfrac{1}{y} \Rightarrow x = \dfrac{1}{3}\left(2 - \dfrac{1}{y}\right)$

x is real if $y \neq 0$. Hence, $R_f = R - \{0\}$

65. $f(x)$ is defined for $x^2 + x - 6 \neq 0$, i.e., $x \neq -3, 2$

$\therefore$ Dom(f) = $R - \{-3, 2\}$

Let $y = \dfrac{x^2 - 3x + 2}{x^2 + x - 6} = \dfrac{x-1}{x+3}$

$\Rightarrow x = \dfrac{3y+1}{y-1}$

x is real for $y - 1 \neq 0$, i.e., $y \neq 1$

Hence, range(f) = $R - \{1\}$

66. $f(x)$ is defined, if

$x^2 - 5x + 6 \geq 0$ and $2x + 8 - x^2 \geq 0$

$\Rightarrow (x-2)(x-3) \geq 0$ and $(x-4)(x+2) \leq 0$

$\therefore$ $x \in (-\infty, 2] \cup [3, \infty)$ and $x \in [-2, 4]$

$\therefore$ $x \in [-2, 2] \cup [3, 4]$

67. The general expression for the function satisfying $f(x + y) = f(x) f(y)$ for all $x, y \in R$ is $f(x) = [f(1)]^x = a^x$ for all $x, y \in R$. [$\because$ $f(1) = a$]

68. If $f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$, then

$f(-x) = \sqrt{1 - x + x^2} - \sqrt{1 + x + x^2}$

$\therefore$ $f(-x) = -f(x)$

So, $f(x)$ is an odd function.

69. $f(x) = \dfrac{x+3}{4x-5}$

$\therefore$ $f(t) = \dfrac{t+3}{4t-5} = \dfrac{\left(\frac{3+5x}{4x-1}\right)+3}{4\left(\frac{3+5x}{4x-1}\right)-5} = x$

70. $f(x) = f(x+1)$

$\therefore$ $x^2 - 2x + 3 = (x+1)^2 - 2(x+1) + 3$

$\therefore$ $x^2 - 2x = x^2 + 2x + 1 - 2x - 2 \Rightarrow x = 1/2$

71. For domain, take $\frac{x}{1+x} \geq 0$

$\therefore$ $D_f = (-\infty, -1) \cup [0, \infty)$

72. $f(x) = \frac{x-2}{x-3}, x \neq 3$

Let $y = f(x) \Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow y \neq 1 \Rightarrow$ Range of $f(x)$ is $R - \{1\}$

So, f is onto

For one-one, let $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1 = x_2$

Hence, f is one-one.

74. Here, $f(x) = \sqrt{x^2 + x + 1}$

$\Rightarrow y^2 = x^2 + x + 1$

$\Rightarrow x^2 + x + (1 - y^2) = 0$

$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(1-y^2)}}{2}$

$\Rightarrow x = \frac{-1 \pm \sqrt{4y^2 - 3}}{2}$

For x real, $4y^2 - 3 \geq 0$

$\therefore$ $y \geq \pm \frac{\sqrt{3}}{2}$

$\therefore$ $R_f = \left[\frac{\sqrt{3}}{2}, \infty\right)$

75. Let $f(x) = x^2 + \sin^2 x$

Here, $f(-x) = f(x)$

$\therefore$ $f(x)$ is an even function.

76. $f(f(x)) = \frac{1}{1 - f(x)} = \frac{1}{1 - \frac{1}{1-x}} = \frac{x-1}{x}$

$\therefore$ $f[f(f(x))] = f\left(\frac{x-1}{x}\right) = \frac{x}{x - x + 1} = x$

78. Let $x_1, x_2 \in R$, then $f(x_1) = \cos x_1$,

& $f(x_2) = \cos x_2$, Now $f(x_1) = f(x_2)$

$\Rightarrow \cos x_1 = \cos x_2 \Rightarrow x_1 = 2n\pi \pm x_2$

$\Rightarrow x_1 \neq x_2$,

$\therefore$ it is not one-one.

Again the value of f-image of x lies in between -1 to 1

$\Rightarrow f[R] = \{f(x) : -1 \leq f(x) \leq 1\}$

So other numbers of co-domain (besides -1 and 1) is not f-image. $f[R] \in R$, so it is also not onto. So this mapping is neither one-one nor onto.

79. $f(-1) = f(1) = 1$

$\therefore$ function is many-one function.

$\therefore$ f is neither one-one nor onto.

80. $f'(x) = \frac{1}{(1+x)^2} > 0 \ \forall x \in [0, \infty)$

and range $\in [0, 1)$

$\Rightarrow$ function is one-one but not onto.

81. Domain of $f(x) = R - \{3\}$,

and for Range : $x \neq 3 \Rightarrow x < 3$ or $x > 3$

Now, $x < 3 \Rightarrow x - 3 < 0 \Rightarrow |x - 3| = -(x - 3)$

$\Rightarrow f(x) = \frac{-(x-3)}{x-3} = -1$

Similarly, for $x > 3$, $f(x) = 1$

$\therefore$ Range (f) = $\{1, -1\}$.

82. Here, $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= \sin\theta(\sin\theta + 3\sin\theta - 4\sin^3\theta)$

$= 4\sin^2\theta(1 - \sin^2\theta)$

$= 4\sin^2\theta\cos^2\theta = (\sin 2\theta)^2$

$\therefore$ $f(\theta) \geq 0$ for all real θ.

83. Given, $(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right)$

$= g\left\{f\left(\frac{-5}{3}\right)\right\} - f\left\{g\left(\frac{-5}{3}\right)\right\}$

$= g(-2) - f\left(\frac{5}{3}\right) = 2 - 1 = 1$

84. Let $f(x_1) = f(x_2) \Rightarrow [x_1] = [x_2] \not\Rightarrow x_1 = x_2$

{For example, if $x_1 = 1.4$, $x_2 = 1.5$, then $[1.4] = [1.5] = 1$}

$\therefore$ f is not one-one.

Also, f is not onto as its range I (set of integers) is a proper subset of its co-domain R.

85. $[x] = I$ (Integers only).

86. $x^2 - 6x + 7 = (x-3)^2 - 2$

Here, minimum value is -2 and maximum ∞.

Hence, range of function is $[-2, \infty)$.

87. $1 + x \geq 0$

$\Rightarrow x \geq -1;\ 1 - x \geq 0$

$\Rightarrow x \leq 1, x \neq 0$

Hence, domain is $[-1, 1] - \{0\}$.

88. $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \dfrac{\pi}{2}$ holds,

for x lying in $[0, 1]$

89. $y = \sin^{-1}\left[\log_3\left(\dfrac{x}{3}\right)\right]$

$\therefore \quad -1 \leq \log_3\left(\dfrac{x}{3}\right) \leq 1$

$\therefore \quad \dfrac{1}{3} \leq \dfrac{x}{3} \leq 3$

$\therefore \quad 1 \leq x \leq 9$

$\therefore \quad x \in [1, 9]$

90. We have $f(x + y) + f(x - y)$

$= \dfrac{1}{2}\left[a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}\right]$

$= \dfrac{1}{2}\left[a^{x}(a^{y} + a^{-y}) + a^{-x}(a^{y} + a^{-y})\right]$

$= \dfrac{1}{2}(a^{x} + a^{-x})\,(a^{y} + a^{-y}) = 2f(x)f(y)$

Competitive Thinking

1. Since $2^m - 2^n = 56 = 8 \times 7 = 2^3 \times 7$

$\Rightarrow 2^n(2^{m-n} - 1) = 2^3 \times 7$

$\therefore \quad n = 3$ and $2^{m-n} = 8 = 2^3$

$\Rightarrow m - n = 3 \Rightarrow m - 3 = 3 \Rightarrow m = 6$

$\therefore \quad m = 6, n = 3$

2. $O(S) = O\left(\bigcup_{i=1}^{30} A_i\right) = \dfrac{1}{10}(5 \times 30) = 15$

Since, element in the union S belongs to 10 of A_i's

Also, $O(S) = O\left(\bigcup_{j=1}^{n} B_j\right) = \dfrac{3n}{9} = \dfrac{n}{3}$

$\therefore \quad \dfrac{n}{3} = 15 \Rightarrow n = 45$

3. $x^2 = 16 \Rightarrow x = \pm 4$

and $2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations. Thus, $A = \phi$.

4. Since, $y = e^x$ and $y = x$ do not meet for any $x \in R$

$\therefore \quad A \cap B = \phi$

5. Since, $4^n - 3n - 1 = (3 + 1)^n - 3n - 1$

$= 3^n + {}^nC_1 3^{n-1} + {}^nC_2 3^{n-2} + \ldots\ldots +$

${}^nC_{n-1}3 + {}^nC_n - 3n - 1$

$= {}^nC_2 3^2 + {}^nC_3 .3^3 + \ldots\ldots + {}^nC_n 3^n$

$\ldots.[{}^nC_0 = {}^nC_n,\ {}^nC_1 = {}^nC_{n-1}$ etc$]$

$= 9[{}^nC_2 + {}^nC_3(3) + \ldots\ldots + {}^nC_n 3^{n-1}]$

$\therefore \quad 4^n - 3n - 1$ is a multiple of 9 for $n \geq 2$.

For $n = 1$, $4^n - 3n - 1 = 4 - 3 - 1 = 0$,

For $n = 2$, $4^n - 3n - 1 = 16 - 6 - 1 = 9$

$\therefore \quad 4^n - 3n - 1$ is a multiple of 9 for all $n \in N$

$\therefore$ X contains elements, which are multiples of 9, and clearly Y contains all multiples of 9.

$\therefore \quad X \subseteq Y$ i.e., $X \cup Y = Y$.

6. $n(A) = 40\%$ of $10{,}000 = 4{,}000$

$n(B) = 20\%$ of $10{,}000 = 2{,}000$

$n(C) = 10\%$ of $10{,}000 = 1{,}000$

$n(A \cap B) = 5\%$ of $10{,}000 = 500$

$n(B \cap C) = 3\%$ of $10{,}000 = 300$

$n(C \cap A) = 4\%$ of $10{,}000 = 400$

$n(A \cap B \cap C) = 2\%$ of $10{,}000 = 200$

We want to find,

$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$

$= n(A) - n[A \cap (B \cup C)]$

$= n(A) - n[(A \cap B) \cup (A \cap C)]$

$= n(A) - [n(A \cap B) + n(A \cap C)$

$- n(A \cap B \cap C)]$

$= 4000 - [500 + 400 - 200]$

$= 4000 - 700 = 3300$.

7. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

8. $R \times (P^c \cup Q^c)^c = R \times [(P^c)^c \cap (Q^c)^c]$

$= R \times (P \cap Q) = (R \times P) \cap (R \times Q)$

10. The number of non- empty subsets $= 2^n - 1$

$= 2^4 - 1 \qquad \ldots.[\because\ n = 4]$

$= 15$.

12. $N_5 \cap N_7 = N_{35}$,

$[\because$ 5 and 7 are relatively prime numbers$]$.

13. $n(C) = 224$, $n(H) = 240$, $n(B) = 336$

$n(H \cap B) = 64$, $n(B \cap C) = 80$

$n(H \cap C) = 40$, $n(C \cap H \cap B) = 24$

$n(C^c \cap H^c \cap B^c) = n(C \cup H \cup B)^c$

$= n(U) - n(C \cup H \cup B)$

$= 800 - [n(C) + n(H) + n(B) - n(H \cap C)$

$- n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

$= 800 - [224 + 240 + 336 - 64 - 80 - 40 + 24]$

$= 800 - 640 = 160$

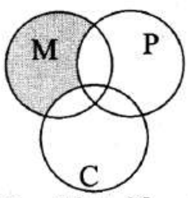
14. Given n(N) = 12, n(P) = 16, n(H) = 18,
$n(N \cup P \cup H) = 30$ and $n(N \cap P \cap H) = 0$
From, $n(N \cup P \cup H)$
$= n(N) + n(P) + n(H) - n(N \cap P) - n(P \cap H) - n(N \cap H) + n(N \cap P \cap H)$
$\therefore$ $n(N \cap P) + n(P \cap H) + n(N \cap H) = 16$
Now, number of pupils taking two subjects
$= n(N \cap P) + n(P \cap H) + n(N \cap H) - 3n(N \cap P \cap H)$
$= 16 - 0 = 16.$

15. Given set is $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$
We can see that, $2(\pm 2)^2 + 3(\pm 3)^2 = 35$
and $2(\pm 4)^2 + 3(\pm 1)^2 = 35$
$\therefore$ (2, 3), (2, –3), (–2, –3), (–2, 3), (4, 1), (4, –1), (–4, –1), (–4, 1) are 8 elements of the set.
$\therefore$ n = 8.

16. Let the original set contains (2n + 1) elements, then subsets of this set containing more than n elements, i.e., subsets containing (n + 1) elements, (n + 2) elements, ……. (2n + 1) elements.
$\therefore$ Required number of subsets
$= {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$
$= {}^{2n+1}C_{n} + {}^{2n+1}C_{n-1} + ... + {}^{2n+1}C_{1} + {}^{2n+1}C_{0}$
$= {}^{2n+1}C_{0} + {}^{2n+1}C_{1} + {}^{2n+1}C_{2} + ... + {}^{2n+1}C_{n-1} + {}^{2n+1}C_{n}$
$= \frac{1}{2}\left[(1+1)^{2n+1}\right] = \frac{1}{2}[2^{2n+1}] = 2^{2n}.$

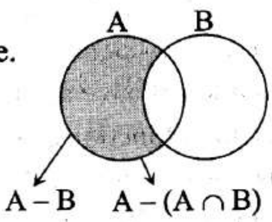
18. A = {4, 8, 12, 16, 20, 24, …..}
B = {6, 12, 18, 24, 30, …….}
$\therefore$ $A \subset B = \{12, 24, ...\} = \{x : x \text{ is a multiple of } 12\}.$

19. n(M alone)
$= n(M) - n(M \cap C) - n(M \cap P) + n(M \cap P \cap C)$



$= 100 - 28 - 30 + 18 = 60$

20. $A - B = A - (A \cap B)$ is correct.
$A = (A \cap B) \cup (A - B)$ is correct.
(3) is false.
$\therefore$ (1) and (2) are true.

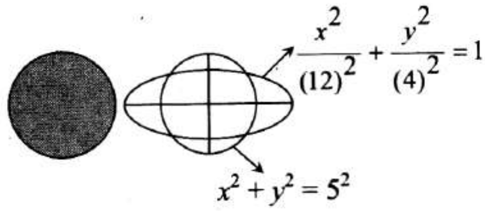


21. $n((A \times B) \cap (B \times A)) = n^2 = 99^2.$

22. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 12 + 9 - 4 = 17$
Now, $n((A \cup B)^c) = n(U) - n(A \cup B)$
$= 20 - 17 = 3$

23. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 3 + 6 - n(A \cap B)$
Since, maximum number of elements in $A \cap B = 3$
$\therefore$ Minimum number of elements in $A \cup B = 9 - 3 = 6$

24. A = Set of all values $(x, y) : x^2 + y^2 = 25 = 5^2$



$B = \frac{x^2}{144} + \frac{y^2}{16} = 1$ i.e., $\frac{x^2}{(12)^2} + \frac{y^2}{(4)^2} = 1$
Clearly, $A \cap B$ consists of four points.

25. Let number of newspapers be x. If every students reads one newspaper, the number of students would be $x(60) = 60x$
Since, every students reads 5 newspapers
$\therefore$ Numbers of students $= \frac{x \times 60}{5} = 300, x = 25$

26. $n[(A \cap B)' \cap A]$
$= n[(A' \cup B') \cap A]$ ....[By DeMorgan's law]
$= n(A' \cap A) \cup n(B' \cap A)$ ....[By distributive law]
$= n(A) - n(A \cap B) = 8 - 2 = 6$

27. $A - (A - B)$
$= A \cap (A \cap B^c)^c = A \cap (A^c \cup B)$
$= \phi \cup (A \cap B) = A \cap B$

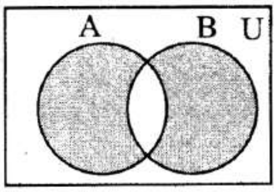
28. $n(X \cap Y) = 12$ and these are 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200

29. $A = B \cap C$, $B = C \cap A$
$\Rightarrow$ A, B are equivalent sets.
....[$\because$ A and B are interchangeable in both equations]

30. $A \cap X = B \cap X = \phi$
$\therefore$ A and X, B and X are disjoint sets
Also, $A \cup X = B \cup X \Rightarrow A = B$

31. Power set is the set of all subsets.
$n(A) = 5 \Rightarrow n(P(A)) = 2^5 = 32$

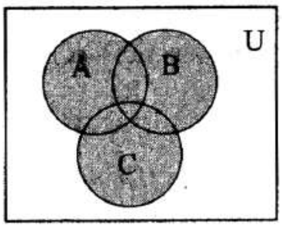
32. $n(A - B) = n(A) - n(A \cap B) = 25 - 10 = 15$

33. 

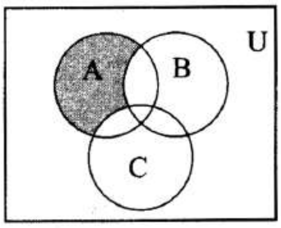
Since, $A - B = A - (A \cap B)$
and $B - A = B - (A \cap B)$
Option (D) is the correct answer.

34. $A = \{x \mid x \text{ is a root of } x^2 - 1 = 0\}$
$= \{x \mid x \text{ is a root of } (x - 1)(x + 1) = 0\}$
$\Rightarrow x = \pm 1$
$B = \{x \mid x \text{ is a root of } x^2 - 2x + 1 = 0\}$
$= \{x \mid x \text{ is a root of } (x - 1)^2 = 0\}$
$\Rightarrow x = 1$
$\Rightarrow A \cup B = A$

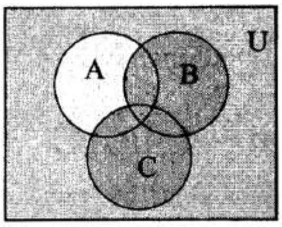
35. i. $A \cup B \cup C$



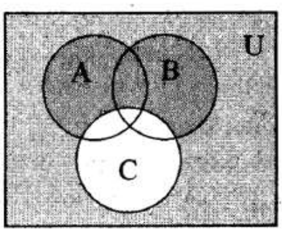
ii. $(A \cap B^c \cap C^c)$



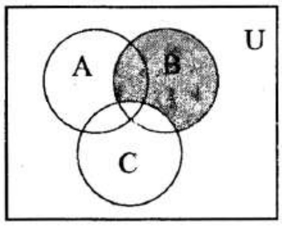
iii. $(A \cap B^c \cap C^c)^c$



iv. C^c



v. $(A \cup B \cup C) \cap (A \cap B^c \cap C^c)^c \cap C^c = B \cap C^c$



36. $n(X) = 60, n(C) = 25, n(T) = 20, n(C \cap T) = 10$
$\therefore$ $n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$= 25 + 20 - 10 = 35$
$\therefore$ $n(C \cup T)' = n(X) - n(C \cup T) = 60 - 35 = 25$

37. $|2x + 3| < 7 \Rightarrow -7 < 2x + 3 < 7$
$\Rightarrow -10 < 2x < 4 \Rightarrow -5 < x < 2 \Rightarrow 0 < x + 5 < 7$

38. $n(S \cup P \cup D) = 265, n(S) = 200, n(D) = 110,$
$n(P) = 55, n(S \cap D) = 60, n(S \cap P) = 30,$
$n(S \cap D \cap P) = 10,$
$n(S \cup P \cup D) = n(S) + n(D) + n(P) - n(S \cap D)$
$- n(D \cap P) - n(P \cap S) + n(S \cap D \cap P)$
$\therefore$ $265 = 200 + 110 + 55 - 60 - 30$
$- n(P \cap D) + 10$
$\therefore$ $n(P \cap D) = 285 - 265 = 20$
$\therefore$ $n(P \cap D) - n(P \cap D \cap S) = 20 - 10 = 10$

39. Given $A = \{2, 4, 6, 8\}$;
$R = \{(2, 4)(4, 2)(4, 6)(6, 4)\}$
$(a, b) \in R \Rightarrow (b, a) \in R$ and also $R^{-1} = R$.
Hence, R is symmetric.

40. $R = A \times B$.

41. For any $a \in R$, we have $a \geq a$. Therefore the relation R is reflexive, but it is not symmetric, as $(2, 1) \in R$ but $(1, 2) \notin R$. The relation R is transitive also, because $(a, b) \in R$, $(b, c) \in R$ imply that $a \geq b$ and $b \geq c$ which is turn imply that $a \geq c$.

43. Here, (3, 3), (6, 6), (9, 9), (12, 12), [Reflexive]; (3, 6), (6, 12), (3, 12), [Transitive].
Hence, reflexive and transitive only.

44. Given $A = \{1, 2, 3, 4\}$
$R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$
$(2, 3) \in R$ but $(3, 2) \notin R$.
Hence, R is not symmetric.
R is not reflexive as $(1, 1) \notin R$.
R is not a function as $(2, 4) \in R$ and $(2, 3) \in R$.
R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

45. $A = \{2, 4, 6\}$; $B = \{2, 3, 5\}$

$\therefore$ $A \times B$ contains $3 \times 3 = 9$ elements.

Hence, number of relations from A to B = 2^9.

46. $|a - a| = 0 < 1$

$\therefore$ $a\,R\,a \;\forall\, a \in R$

$\therefore$ R is reflexive.

Again, $a\,R\,b \Rightarrow |a-b| \le 1 \Rightarrow |b-a| \le 1 \Rightarrow bRa$

$\therefore$ R is symmetric,

Again, $1R\frac{1}{2}$ and $\frac{1}{2}R1$ but $\frac{1}{2} \ne 1$

$\therefore$ R is not anti-symmetric.

Further, 1 R 2 and 2 R 3 but $1 \not{R}\, 3$,

$[\because |1-3| = 2 > 1]$

$\therefore$ R is not transitive.

47. Total number of reflexive relations in a set with n elements = 2^n.

Therefore, total number of reflexive relation set with 4 elements = 2^4.

48. $A - B = \{a\}$, $B \cap C = \{c, d\}$

$\therefore$ $(A - B) \times (B \cap C) = \{a\} \times \{c, d\} = \{(a, c), (a, d)\}$

49. For any $a \in N$, we find $a|a$, therefore R is reflexive.

But, R is not symmetric, because aRb does not imply that bRa.

50. $r = \{(a, b) |\; a, b \in R$ and $a - b + \sqrt{3}$ is an irrational no.$\}$

Here, r is reflexive as $aRa = a - a + \sqrt{3} = \sqrt{3}$ which is an irrational no.

$\sqrt{3}\,r\,1 = \sqrt{3} - 1 + \sqrt{3} = 2\sqrt{3} - 1$, which is an irrational number.

But $1\,r\,\sqrt{3} = 1 - \sqrt{3} + \sqrt{3} = 1$ which is not an irrational number.

$\therefore$ $\sqrt{3}\,r\,1 \Rightarrow \frac{1}{\sqrt{3}}$

$\therefore$ r is not symmetric.

Also, r is not transitive.

Since, $\sqrt{3}\,r\,1$ and $1\,r\,2\sqrt{3} \not\Rightarrow \sqrt{3}\,r\,2\sqrt{3}$

$\therefore$ Option (B) is the correct answer.

51. $x^2 - 4x^2 + 3x^2 = 0$

$\therefore$ $xRx \Rightarrow$ Reflexive

52. In option (D), ordered pair $(a, d) \notin A \times B$. Thus it is not a relation from A to B.

53. A relation is equivalence if it is reflexive, symmetric and transitive.

54. Since, G. C. D. of a and a is 'a'

$\therefore$ if $a \ne 2$, then G. C. D. $\ne 2$

$\therefore$ R is not reflexive.

Let aRb

$\therefore$ G. C. D. of a, b = 2

i.e., (a, b) = 2

$\Rightarrow$ (b, a) = 2

$\Rightarrow$ G. C. D. of b, a = 2

$\therefore$ R is symmetric.

Again, let aRb and bRc

$\therefore$ $\left.\begin{array}{l}\text{G. C. D. of a, b} = 2\\ \text{and G. C. D. of b, c} = 2\end{array}\right\} \not\Rightarrow$ G. C. D of (a, c) = 2

$\therefore$ R is not transitive.

55. Here, $R = \{(x, y) \in W \times W$: the words x and y have at least one letter in common$\}$

R is reflexive as the words x and x have all letters in common.

Hence, R is reflexive.

Also, if $(x, y) \in R$ i.e., x and y have a common letter, then y and x also have a letter in common

$\therefore$ R is symmetric.

R is not transitive as $(x, y) \in R$ and $(y, z) \in R$ need not imply $(x, z) \in R$

For example, let x = CANE, y = NEST and z = WITH

then $(x, y) \in R$ and $(y, z) \in R$, but $(x, z) \notin R$

$\therefore$ R is reflexive and symmetric but not transitive.

56. For reflexive, $\theta = \phi$, so $\sec^2\theta - \tan^2\theta = 1$,

$\therefore$ R is reflexive.

For symmetric, $\sec^2\theta - \tan^2\phi = 1$

so, $(1 + \tan^2\theta) - (\sec^2\phi - 1) \Rightarrow \sec^2\phi - \tan^2\theta = 1$

$\therefore$ R is symmetric

For transitive, Let $\sec^2\theta - \tan^2\phi = 1$ ....(i)

and $\sec^2\phi - \tan^2\gamma = 1$

$\therefore$ $1 + \tan^2\phi - \tan^2\gamma = 1$

$\Rightarrow \sec^2\theta - \tan^2\gamma = 1$ ....[From (i)]

$\therefore$ R is transitive.

57. Given, $f(x) = \cos(\log x) \Rightarrow f(y) = \cos(\log y)$

Then, $f(x).f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$

$= \cos(\log x)\cos(\log y) - \frac{1}{2}\left[\cos\left(\log\frac{x}{y}\right) + \cos(\log xy)\right]$

$= \cos(\log x)\cos(\log y) - \frac{1}{2}[2\cos(\log x)\cos(\log y)] = 0$

58. $f[f(\cos 2\theta)] = f\left[\frac{1-\cos 2\theta}{1+\cos 2\theta}\right] = f(\tan^2\theta)$

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

59. $f(x+1) - f(x) = 8x + 3$

$\Rightarrow [b(x+1)^2 + c(x+1) + d] - (bx^2 + cx + d) = 8x + 3$

$\Rightarrow (2b)x + (b + c) = 8x + 3$

$\Rightarrow 2b = 8,\ b + c = 3$

$\Rightarrow b = 4,\ c = -1$

60. $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$

$f(x) = \cos(9x) + \cos(-10x)$

$[\because \pi = 3.14,$

$\therefore$ $[9.85] = 9$ and $[-9.85] = -10]$

$= \cos(9x) + \cos(10x)$

$= 2\cos\left(\frac{19x}{2}\right)\cos\left(\frac{x}{2}\right)$

$\therefore$ $f\left(\frac{\pi}{2}\right) = 2\cos\left(\frac{19\pi}{4}\right)\cos\left(\frac{\pi}{4}\right);$

$\therefore$ $f\left(\frac{\pi}{2}\right) = 2\times\frac{-1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = -1$

61. $f(x) = f(-x) \Rightarrow f(0+x) = f(0-x)$ is symmetrical about $x = 0$.

$\therefore$ $f(2+x) = f(2-x)$ is symmetrical about $x = 2$.

62. Let $f(x) = \frac{x^2-1}{x^2+1} = \frac{x^2+1-2}{x^2+1} = 1 - \frac{2}{x^2+1}$

$\because$ $x^2 + 1 > 1;$

$\therefore$ $\frac{2}{x^2+1} \le 2$

So $1 - \frac{2}{x^2+1} \ge 1 - 2;$

$\therefore$ $-1 \le f(x) < 1$

Thus, $f(x)$ has the minimum value equal to -1.

63. We have $f(x) = (x-1)(x-2)(x-3)$ and $f(1) = f(2) = f(3) = 0 \Rightarrow f(x)$ is not one-one. For each $y \in R$, there exists $x \in R$ such that $f(x) = y$. Therefore f is onto. Hence, $f : R \to R$ is onto but not one-one.

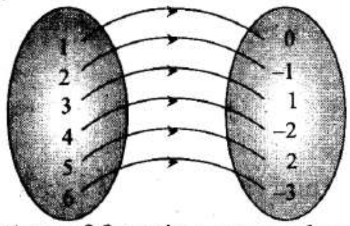
64. The total number of injective functions from a set A containing 3 elements to a set B containing 4 elements is equal to the total number of arrangements of 4 by taking 3 at a time i.e., ${}^4P_3 = 24$.

65. Function $f : R \to R$ is defined by $f(x) = e^x$. Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$ or $e^{x_1} = e^{x_2}$ or $x_1 = x_2$. Therefore f is one-one. Let $f(x) = e^x = y$. Taking log on both sides, we get $x = \log y$. We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number. Therefore function f is into.

66. $f'(x) = 2 + \cos x > 0$. So, $f(x)$ is strictly monotonic increasing so, $f(x)$ is one-to-one and onto.

67. $f : N \to I$

$f(1) = 0,\ f(2) = -1,\ f(3) = 1,\ f(4) = -2,\ f(5) = 2$ and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

68. Let $f(x)$ be periodic with period T.

Then, $f(x + T) = f(x)$ for all $x \in R$

$\Rightarrow x + T - [x + T] = x - [x]$. for all $x \in R$

$\Rightarrow x + T - x = [x + T] - [x]$

$\Rightarrow [x + T] - [x] = T$ for all $x \in R$

$\Rightarrow T = 1, 2, 3, 4, \ldots\ldots.$

The smallest value of T satisfying $f(x + T) = f(x)$ for all $x \in R$ is 1.

Hence, $f(x) = x - [x]$ has period 1.

69. Here, $x + 3 > 0$ and $x^2 + 3x + 2 \ne 0$

$\therefore$ $x > -3$ and $(x+1)(x+2) \ne 0$, i.e., $x \ne -1, -2$.

$\therefore$ Domain $= (-3, \infty) - \{-1, -2\}$.

70. $f(x)$ is to be defined when $x^2 - 1 > 0$

$\Rightarrow x^2 > 1, \Rightarrow x < -1$ or $x > 1$ and $3 + x > 0$

$\therefore$ $x > -3$ and $x \ne -2$

$\therefore$ $D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$.

71. The quantity under root is positive, when $-1-\sqrt{3} \le x \le -1+\sqrt{3}$.

72. The function $f(x) = \sqrt{\log(x^2 - 6x + 6)}$ is defined, when $\log(x^2 - 6x + 6) \ge 0$

$\Rightarrow x^2 - 6x + x \ge 1 \ \Rightarrow (x-5)(x-1) \ge 0$

This inequality holds, if $x \le 1$ or $x \ge 5$. Hence, the domain of the function will be $(-\infty, 1] \cup [5, \infty)$.

73. $-1 \le 1 + 3x + 2x^2 \le 1$

Case I : $2x^2 + 3x + 1 \ge -1$; $2x^2 + 3x + 2 \ge 0$

$x = \dfrac{-3 \pm \sqrt{9-16}}{6} = \dfrac{-3 \pm i\sqrt{7}}{6}$ (imaginary).

Case II : $2x^2 + 3x + 1 \le 1$

$\Rightarrow 2x^2 + 3x \le 0 \Rightarrow 2x\left(x + \dfrac{3}{2}\right) \le 0$

$\Rightarrow \dfrac{-3}{2} \le x \le 0 \Rightarrow x \in \left[-\dfrac{3}{2}, 0\right]$

In case I, we get imaginary value hence, rejected

$\therefore$ Domain of function $= \left[\dfrac{-3}{2}, 0\right]$.

74. To define f(x), $9 - x^2 > 0 \Rightarrow |x| < 3$

$\Rightarrow -3 < x < 3$, ...(i)

and $-1 \le (x - 3) \le 1$

$\Rightarrow 2 \le x \le 4$ .....(ii)

From (i) and (ii), $2 \le x < 3$ i.e., [2, 3).

75. $f(x) = \dfrac{x+2}{|x+2|}$

$f(x) = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$

$\therefore$ Range of f(x) is $\{-1, 1\}$.

76. Since maximum and minimum values of cos – sin x are $\sqrt{2}$ and $-\sqrt{2}$ respectively, therefore range of f(x) is $[-\sqrt{2}, \sqrt{2}]$.

77. Let $\dfrac{x^2 + 34x - 71}{x^2 + 2x - 7} = y$

$\Rightarrow x^2(1 - y) + 2(17 - y)x + (7y - 71) = 0$

For real value of x, $B^2 - 4AC \ge 0$

$\Rightarrow y^2 - 14y + 45 \ge 0 \Rightarrow y \ge 9, y \le 5$.

78. Here, $f(x) = \log\dfrac{1+x}{1-x}$

and $f(-x) = \log\left(\dfrac{1-x}{1+x}\right) = \log\left(\dfrac{1+x}{1-x}\right)^{-1}$

$= -\log\left(\dfrac{1+x}{1-x}\right) = -f(x) = f(-x)$

$\Rightarrow$ f(x) is an odd function.

79. $y = \dfrac{x+2}{x-1} \Rightarrow x = \dfrac{3}{y-1} + 1 = \dfrac{y+2}{y-1} = f(y)$.

80. Given, $f(x) = 2^{x(x-1)}$

$\Rightarrow x(x - 1) = \log_2 f(x)$

$\Rightarrow x^2 - x - \log_2 f(x) = 0$

$\Rightarrow x = \dfrac{1 \pm \sqrt{1 + 4\log_2 f(x)}}{2}$

Only $x = \dfrac{1 + \sqrt{1 + 4\log_2 f(x)}}{2}$ lies in the domain

$\therefore$ $f^{-1}(x) = \dfrac{1}{2}[1 + \sqrt{1 + 4\log_2 x}]$.

81. Let $f(x) = y \Rightarrow x = f^{-1}(y)$

Hence, $f(x) = y = 3x - 5$

$\Rightarrow x = \dfrac{y+5}{3} \Rightarrow f^{-1}(y) = x = \dfrac{y+5}{3}$

$\therefore$ $f^{-1}(x) = \dfrac{x+5}{3}$

Also f is one-one and onto, so f^{-1} exists and is given by $f^{-1}(x) = \dfrac{x+5}{3}$.

82. $f(x) = \dfrac{\alpha x}{x+1}$;

$f(f(x)) = f\left(\dfrac{\alpha x}{x+1}\right) = \dfrac{\alpha\left(\dfrac{\alpha x}{x+1}\right)}{\dfrac{\alpha x}{x+1} + 1}$

But $f(f(x)) = x$

$\therefore$ $\dfrac{\alpha^2 x}{\alpha x + x + 1} = x$

L.H.S, Put $\alpha = -1$,

$\therefore$ $\dfrac{(-1)^2 x}{(-1)x + x + 1} = \dfrac{x}{-x + x + 1} = x$;

$\therefore$ $\alpha = -1$

83. $f(x) = \log\left[\dfrac{1+x}{1-x}\right]$

$\therefore$ $f\left(\dfrac{2x}{1+x^2}\right) = \log\left[\dfrac{1 + \dfrac{2x}{1+x^2}}{1 - \dfrac{2x}{1+x^2}}\right] = \log\left[\dfrac{x^2 + 1 + 2x}{x^2 + 1 - 2x}\right]$

$= \log\left[\dfrac{1+x}{1-x}\right]^2 = 2\log\left[\dfrac{1+x}{1-x}\right] = 2f(x)$

84. Let $x, y \in N$ such that $f(x) = f(y)$

Then, $f(x) = f(y) \Rightarrow x^2 + x + 1 = y^2 + y + 1$

$\Rightarrow (x - y)(x + y + 1) = 0$

$\Rightarrow x = y$ or $x = (-y - 1) \notin N$

$\therefore$ f is one-one.

Again, since for each $y \in N$, there exist $x \in N$

$\therefore$ f is onto.

85. $|x|$ is not one-one; x^2 is not one-one; $x^2 + 1$ is not one-one. But $2x - 5$ is one-one because $f(x) = f(y) \Rightarrow 2x - 5 = 2y - 5 \Rightarrow x = y$
Now, $f(x) = 2x - 5$ is onto.
$\therefore$ $f(x) = 2x - 5$ is bijective.

86. $-\sqrt{1+(-\sqrt{3})^2} \le (\sin x - \sqrt{3}\cos x) \le \sqrt{1+(-\sqrt{3})^2}$
$\therefore$ $-2 \le (\sin x - \sqrt{3}\cos x) \le 2$
$\therefore$ $-2 + 1 \le (\sin x - \sqrt{3}\cos x + 1) \le 2 + 1$
$\therefore$ $-1 \le (\sin x - \sqrt{3}\cos x + 1) \le 3$
i.e., range = $[-1, 3]$
$\therefore$ For f to be onto S = $[-1, 3]$.

88. $f(x) = \log|\log x|$, $f(x)$ is defined if $|\log x| > 0$ and $x > 0$ i.e., if $x > 0$ and $x \ne 1$
($\because$ $|\log x| > 0$ if $x \ne 1$)
$\Rightarrow x \in (0, 1) \cup (1, \infty)$.

89. $f(x) = \sqrt{\log\dfrac{1}{|\sin x|}}$
$\Rightarrow \sin x \ne 0 \Rightarrow x \ne n\pi + (-1)^n 0$
$\Rightarrow x \ne n\pi$. Domain of $f(x) = R - \{n\pi, n \in I\}$.

90. $f(x) = \sin\left(\log\left(x + \sqrt{1+x^2}\right)\right)$

$\Rightarrow f(-x) = \sin[\log(-x + \sqrt{1+x^2})]$

$\Rightarrow f(-x) = \sin \log\left((\sqrt{1+x^2} - x)\dfrac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}\right)$

$\Rightarrow f(-x) = \sin \log\left[\dfrac{1}{(x+\sqrt{1+(x^2)})}\right]$

$\Rightarrow f(-x) = \sin\left[-\log(x+\sqrt{1+x^2})\right]$

$\Rightarrow f(-x) = -\sin\left[\log(x+\sqrt{1+x^2})\right]$

$\Rightarrow f(-x) = -f(x)$
$\therefore$ $f(x)$ is odd function.

91. Let $f(x) = y \Rightarrow x = f^{-1}(y)$. Now,
$y = \dfrac{2x-1}{x+5}, (x \ne -5)$
$xy + 5y = 2x - 1 \Rightarrow 5y + 1 = 2x - xy$
$\Rightarrow x(2 - y) = 5y + 1 \Rightarrow x = \dfrac{5y+1}{2-y}$

$\Rightarrow f^{-1}(y) = \dfrac{5y+1}{2-y}$

$\therefore$ $f^{-1}(x) = \dfrac{5x+1}{2-x}, x \ne 2$

92. Let $y = f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 2$

$\therefore$ $y - 2 = \dfrac{e^{2x}-1}{e^{2x}+1}$
$\Rightarrow (y-2)\,e^{2x} + y - 2 = e^{2x} - 1$
$\Rightarrow e^{2x} = \dfrac{1-y}{y-3} = \dfrac{y-1}{3-y}$

$\Rightarrow 2x = \log_e\left(\dfrac{y-1}{3-y}\right) \Rightarrow x = \dfrac{1}{2}\log_e\left(\dfrac{y-1}{3-y}\right)$

$\Rightarrow f^{-1}(y) = \dfrac{1}{2}\log_e\left(\dfrac{y-1}{3-y}\right)$

$\Rightarrow f^{-1}(x) = \log_e\left(\dfrac{x-1}{3-x}\right)^{\frac{1}{2}}$

93. $e^{f(x)} = \dfrac{10+x}{10-x}, x \in (-10, 10)$

$\Rightarrow f(x) = \log\left(\dfrac{10+x}{10-x}\right)$

$\Rightarrow f\left(\dfrac{200x}{100+x^2}\right) = \log\left[\dfrac{10+\dfrac{200x}{100+x^2}}{10-\dfrac{200x}{100+x^2}}\right]$

$= \log\left[\dfrac{10(10+x)}{10(10-x)}\right]^2$

$= 2\log\left(\dfrac{10+x}{10-x}\right) = 2f(x)$

$\therefore$ $f(x) = \dfrac{1}{2}f\left(\dfrac{200x}{100+x^2}\right)$

$\Rightarrow k = \dfrac{1}{2} = 0.5$

94. Since, $f(x)$ is even.
$\therefore$ $f(-x) = f(x)$

$\therefore$ $\dfrac{a^{-x}-1}{(-x)^n\left(a^{-x}+1\right)} = \dfrac{a^x-1}{x^n\left(a^x+1\right)}$

$\Rightarrow \dfrac{1-a^x}{(-1)^n x^n\left(1+a^x\right)} = \dfrac{a^x-1}{x^n\left(a^x+1\right)}$

$\Rightarrow \dfrac{-1}{(-1)^n} = 1 \Rightarrow -1 = (-1)^n$

$\therefore$ $n = -\dfrac{1}{3}$ can satisfy the equation.

95. Given expression $= \sum_{i=0}^{98}\left[\frac{2}{3}+\frac{i}{99}\right]$

$$= \sum_{i=0}^{32}\left[\frac{2}{3}+\frac{i}{99}\right] + \sum_{i=33}^{98}\left[\frac{2}{3}+\frac{i}{99}\right]$$

$$= 0 + \sum_{i=33}^{98}\left[\frac{2}{3}+\frac{i}{99}\right]$$

$\left[\because \frac{2}{3} \le \frac{2}{3}+\frac{i}{99} < 1 \text{ for } i = 0, 1, 2, \ldots, 32\right]$

$= 66$

[$\because$ each term in the summation is one or more but less than 2 when $i = 33, 34, 35, \ldots, 98$]

96. $f(x) = \frac{1}{2}(1+\cos 2x) + \frac{1}{2}\left[1+\cos\left(\frac{2\pi}{3}+2x\right)\right] - \frac{2\cos x\cos\left(\frac{\pi}{3}+x\right)}{2}$

$$= 1 + \frac{1}{2}\left[\cos 2x + \cos\left(\frac{2\pi}{3}+2x\right) - \cos\left(2x+\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right)\right]$$

$$= \frac{3}{4}+\frac{1}{2}\left[\cos 2x + \cos\left(2x+\frac{2\pi}{3}\right) - \cos\left(2x+\frac{\pi}{3}\right)\right]$$

$$= \frac{3}{4}+\frac{1}{2}\left[\cos 2x - 2\sin\left(2x+\frac{\pi}{2}\right)\sin\left(\frac{\pi}{6}\right)\right]$$

$$= \frac{3}{4}+\frac{1}{2}\left[\cos 2x - 2\sin\left(\frac{\pi}{2}+2x\right).\frac{1}{2}\right]$$

$$= \frac{3}{4}+\frac{1}{2}[\cos 2x - \cos 2x] = \frac{3}{4}$$

97. $D_f = D_g \cap D_h$

where $g(x) = \frac{1}{\log_{10}(1-x)}$ and $h(x) = \sqrt{2+x}$

Now, $D_g = \{x \in R : 1-x > 0, \log_{10}(1-x) \neq 0\}$
$= \{x \in R : x < 1, 1-x \neq 1\}$
$= \{x \in R : x < 1, x \neq 0\}$

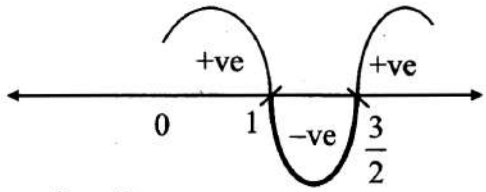
and $D_h = \{x \in R : x+2 \ge 0\}$
$= \{x \in R : x \ge -2\}$

$\therefore$ $D_f = [(-\infty, 1) - \{0\}] \cap [-2, \infty)$
$= [-2, 1) - \{0\}$

98. $f(x) = e^{\sqrt{5x-3-2x^2}}$

$\Rightarrow 5x - 3 - 2x^2 \ge 0$

$\Rightarrow (x-1)\left(x-\frac{3}{2}\right) \le 0$



$\therefore$ $D_f = \left[1, \frac{3}{2}\right]$

99. $(fog)(x) = f(g(x)) = f(x^2) = \sin x^2$

100. Here, $f\left(\frac{1}{2}\right) = \left(25-\frac{1}{16}\right)^{\frac{1}{4}} = \left(\frac{399}{16}\right)^{\frac{1}{4}}$

$$\Rightarrow f\left[f\left(\frac{1}{2}\right)\right] = f\left(\left(\frac{399}{16}\right)^{\frac{1}{4}}\right)$$

$$= \left(25-\frac{399}{16}\right)^{\frac{1}{4}}$$

$$= \left(\frac{1}{16}\right)^{\frac{1}{4}}$$

$$= \frac{1}{2}$$

101. $(gof)(e) + (fog)(\pi) = g(f(e)) + f(g(\pi))$
$= g(1) + f(0)$
$= -1 + 0$
$= -1$

102. $(hofog)(x) = (hof)(g(x))$

$$= (hof)\left(\sqrt{x^2+1}\right)$$

$$= h\left(f\left(\sqrt{x^2+1}\right)\right)$$

$$= h\left[\left(\sqrt{x^2+1}\right)^2 - 1\right]$$

$$= h(x^2+1-1)$$

$$= h(x^2) = \begin{cases} 0 & , \text{if } x = 0 \\ x^2 & , \text{if } x \neq 0 \end{cases}$$

103. $g(f(x)) = g(|x|) = [|x|]$

and $f(g(x)) = f([x]) = |[x]|$

When $x \geq 0$, $[|x|] = [x] = |[x]|$

$\Rightarrow f(g(x)) = g(f(x))$

When $x < 0$, $[x] \leq x < 0$

$\Rightarrow |[x]| \geq |x|$

$\Rightarrow |[x]| \geq |x| \geq [|x|]$

....[$\because$ $[t] \leq t$ for all t]

$\Rightarrow f(g(x)) \geq g(f(x))$

$\therefore$ $g(f(x)) \leq f(g(x))$ for all $x \in R$

104. Given, $f(x) = ax + b$, $g(x) = cx + d$

and $f(g(x)) = g(f(x))$

$\Rightarrow f(cx + d) = g(ax + b)$

$\Rightarrow a(cx + d) + b = c(ax + b) + d$

$\Rightarrow ad + b = cb + d$

$\Rightarrow f(d) = g(b)$

105. Given,

$$f(x) = \sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos\left(x+\frac{\pi}{3}\right)\cos x$$

$$= \frac{1}{2}\left\{1-\cos 2x+1-\cos\left(2x+\frac{2\pi}{3}\right) + \cos\left(2x+\frac{2\pi}{3}\right)+\cos\frac{\pi}{3}\right\}$$

$$=\frac{1}{2}\left[\frac{5}{2}-\left\{\cos 2x+\cos\left(2x+\frac{2\pi}{3}\right)\right\}+\cos\left(2x+\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[\frac{5}{2}-2\cos\left(2x+\frac{\pi}{3}\right)\cos\frac{\pi}{3}+\cos\left(2x+\frac{\pi}{3}\right)\right]$$

$$= \frac{5}{4}$$

$\therefore$ $gof(x) = g[f(x)] = g\left(\frac{5}{4}\right)$

$= 1$ ....$\left[\because g\left(\frac{5}{4}\right)=1\right]$

Hence, $gof(x)$ is a constant function.

106. Let $x, y \in R$ be such that

$f(x) = f(y)$

$\Rightarrow x^3 + 5x + 1 = y^3 + 5y + 1$

$\Rightarrow (x^3 - y^3) + 5(x - y) = 0$

$\Rightarrow (x - y)(x^2 + xy + y^2 + 5) = 0$

$\Rightarrow (x - y)\left[\left(x+\frac{y}{2}\right)^2+\frac{3y^2}{4}+5\right] = 0$

$\Rightarrow x = y$ and $\left(x+\frac{y}{2}\right)^2 + \frac{3y^2}{4} + 5 \neq 0$

$\therefore$ $f : R \to R$ is one-one

Let y be an arbitrary element in R (co-domain).

Then, $f(x) = y$ i.e., $x^3 + 5x + 1 = y$ has at least one real root, say β in R

$\therefore$ $\beta^3 + 5\beta + 1 = y$

$\Rightarrow f(\beta) = y$

Thus, for each $y \in R$ there exists $\beta \in R$ such that $f(\beta) = y$

$\therefore$ $f : R \to R$ is onto

Hence, $f: R \to R$ is one-one onto.

107. As $x - [x] \in [0, 1)$, $\forall x \in R$

$\therefore$ $0 \leq x - [x] < 1$, $\forall x \in R$

$\Rightarrow 1 \leq 1 + x - [x] < 2$, $\forall x \in R$

$\Rightarrow 1 \leq g(x) < 2$, $\forall x \in R$

Hence, $f(g(x)) = 1$ $\forall x \in R$

108. Here, $(f - g)(x) = f(x) - g(x)$

$\therefore$ $(f-g)(x) = \begin{cases} x-0=x, & \text{if } x \text{ is rational} \\ 0-x=-x, & \text{if } x \text{ is irrational} \end{cases}$

Let $k = f - g$

Let x, y be any two distinct real numbers.

Then, $x \neq y$

$\Rightarrow -x \neq -y$

Now, $x \neq y$

$\Rightarrow k(x) \neq k(y) \Rightarrow (f-g)(x) \neq (f-g)(y)$

$\Rightarrow f - g$ is one-one.

Let y be any real number

If y is a rational number, then

$k(y) = y$

$\Rightarrow (f - g)(y) = y$

If y is an irrational number , then

$k(-y) = y$

$\Rightarrow (f - g)(-y) = y$

Thus, every $y \in R$ (co-domain) has its pre-image in R (domain)

$\therefore$ $f - g : R \to R$ is onto.

Hence, $f - g$ is one-one and onto.

109. $(fog)(x) = f(g(x)) = f\left(\frac{x-1}{2}\right)$

$$= 2\left(\frac{x-1}{2}\right)+1 = x$$

$\Rightarrow (fog)(x) = x \Rightarrow x = (fog)^{-1}(x)$

Hence, $(fog)^{-1}\left(\frac{1}{x}\right) = \frac{1}{x}$

110. Here, $f(2) = \frac{2+1}{2-1} = 3$

$\therefore$ $f(f(2)) = f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$

$\therefore$ $f(f(f(2))) = f(2) = \frac{2+1}{2-1} = 3$

111. Given, $f(x) = \sin x$

$\therefore$ $f: R \to R$ is neither one-one nor onto as $R_f = [-1, 1]$.

$f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$

is both one-one and onto.

$f: [0, \pi] \to [-1, 1]$

is neither one-one nor onto as

$R_f = [0, 1]$.

$f: \left[0, \frac{\pi}{2}\right] \to [-1, 1]$ is one-one but not onto as

$R_f = [0, 1]$.

112. $f(x) = \sqrt{x} \Rightarrow \frac{f(25)}{f(16)+f(1)} = \frac{\sqrt{25}}{\sqrt{16}+\sqrt{1}} = \frac{5}{5} = 1$

113. $(gof)(x) = \sin x^2 \Rightarrow (gogof)(x) = \sin(\sin x^2)$

$\Rightarrow$ (fogogof) $(x) = (\sin(\sin x^2))^2 = \sin^2(\sin x^2)$

Now, $\sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \sin(\sin x^2) = 0, 1$

$\Rightarrow \sin x^2 = n\pi, (4n + 1)\frac{\pi}{2}$ $n \in I$

$\Rightarrow \sin x^2 = 0 \Rightarrow x^2 = n\pi$

$\Rightarrow x = \pm\sqrt{n\pi}$ $n \in W$

114. $-1 \le \log_2(x^2 + 5x + 8) \le 1$

$\Rightarrow \frac{1}{2} \le (x^2 + 5x + 8) \le 2$

$\Rightarrow x^2 + 5x + \frac{15}{2} \ge 0$

$\Rightarrow x^2 + 2\left(\frac{5}{2}\right)x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{15}{2} \ge 0$

$\Rightarrow \left(x+\frac{5}{2}\right)^2 + \frac{5}{4} \ge 0$ and $x^2 + 5x + 6 \le 0$

$\Rightarrow (x + 3)(x + 2) \le 0 \Rightarrow x \in [-3, -2]$

115. $f(x) = |x|$

$$f(x) = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Therefore, the function $f^{-1}(x)$ does not exist.

116. Let $y = \log_e\sqrt{4-x^2} \Rightarrow e^y = \sqrt{4-x^2}$

$\Rightarrow e^{2y} = 4 - x^2 \Rightarrow x^2 = 4 - e^{2y} \Rightarrow x = \sqrt{4-e^{2y}}$

$\therefore$ $4 - e^{2y} \ge 0$

$\Rightarrow e^{2y} \le 4 \Rightarrow 2y \le \log_e 4$

$\Rightarrow y \le \frac{1}{2}\log_e 4 \Rightarrow y \le \log_e 2$

$\therefore$ $y \in (-\infty, \log_e 2]$

117. $f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$

$f(x)$ is real valued function when $\frac{\pi^2}{9} - x^2 \ge 0$

$\Rightarrow x^2 \le \frac{\pi^2}{9} \Rightarrow x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ = Domain of $f(x)$

When domain is in closed interval, we use differentiation method.

$f'(x) = \sec^2\sqrt{\frac{\pi^2}{9} - x^2} \cdot \frac{1}{2\sqrt{\frac{\pi^2}{9} - x^2}}(-2x)$

When $f'(x) = 0$, $x = 0$

Finding values of $f(x)$ when $x = 0, -\frac{\pi}{3}, \frac{\pi}{3}$

[End points of domain]

$\therefore$ $f(0) = \tan\sqrt{\frac{\pi^2}{9}} = \sqrt{3}$ and $f\left(-\frac{\pi}{3}\right) = f\left(\frac{\pi}{3}\right) = 0$

$\therefore$ Range of function $= \left[0, \sqrt{3}\right]$

[Taking least value and greatest value for range]

118. $\cos 2x + 7 = a(2 - \sin x) \Rightarrow a = \frac{\cos 2x + 7}{2 - \sin x}$

$\Rightarrow a = \frac{1 - 2\sin^2 x + 7}{2 - \sin x} = \frac{2(4 - \sin^2 x)}{2 - \sin x}$

$\Rightarrow a = 2(2 + \sin x)$

$\therefore$ $a \in [2, 6]$ ....$[\because -1 \le \sin x \le 1]$

119. Let $y = \log_e(3x^2 + 4)$

$\Rightarrow 3x^2 + 4 = e^y \Rightarrow x^2 = \frac{e^y - 4}{3}$

Since, $x^2 \ge 0$

$\therefore$ $\frac{e^y - 4}{3} \ge 0 \Rightarrow e^y - 4 \ge 0 \Rightarrow y \ge \log_e 4$

$\Rightarrow y \ge 2\log_e 2$

So, range $= [2\log_e 2, \infty)$

120. $f(x) = \sin x + \cos x$, $g(x) = x^2$

$\therefore$ $fog(x) = \sin x^2 + \cos x^2$

121. Number of bijective function from a set of 10 elements to itself is ${}^{10}P_{10}$.
So, required number = 10!

122. Function given by $f(x) = ax + b$

$$f^{-1}(x) = \frac{x-b}{a}$$

So, $g(y) = y - 3$

123. $f(g(-1)) = f(-3 -4) = f(-7) = 5 - 49 = -44$

124. $f(x) = f(y)$

$\Rightarrow x + 2 = y + 2 \ \Rightarrow x = y$

$\therefore$ Function f is one-one

125. $$f(-x) = \sec\left[\log\left(-x + \sqrt{1+(-x)^2}\right)\right]$$
$$= \sec\left[\log\left(-x + \sqrt{1+x^2}\right)\right]$$
$$= \sec\left[\log\left(\sqrt{1+x^2} - x\right)\right]$$
$$= \sec\left[\log\left(\frac{1+x^2-x^2}{\sqrt{1+x^2}+x}\right)\right]$$
$$= \sec\left[\log\left(\frac{1}{\sqrt{1+x^2}+x}\right)\right]$$
$$= \sec\left[-\log\left(\sqrt{1+x^2}+x\right)\right]$$
$$= \sec\left[\log\left(\sqrt{1+x^2}+x\right)\right]$$

$\therefore$ f(x) is an even function.

126. $$f(x) = \sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$$

$\therefore \quad -1 \le \frac{1-|x|}{2} \le 1$

$\Rightarrow -2 - 1 \le -|x| \le 2 - 1$

$\Rightarrow -3 \le |x| \le 1$

$\Rightarrow -1 \le |x| \le 3$

$\Rightarrow x \in [-3, 3]$

127. $f = \{(1, 1), (2, 4), (0, -2), (-1, -5)\}$ be a linear function from Z to Z. The function satisfies the above points, if $f(x) = 3x - 2$

128. We have, $f(x) = \frac{5x}{4x+5}, x \in R - \left\{\frac{5}{4}\right\}$

Let $f(x) = y$

$\Rightarrow x = f^{-1}(y)$

$$y = \frac{5x}{4x+5}$$

$\Rightarrow 4xy + 5y = 5x$

$\Rightarrow 5y = 5x - 4xy = x(5 - 4y)$

$$\Rightarrow x = \frac{5y}{5-4y}$$

$$g(y) = f^{-1}(y) = \frac{5y}{5-4y}, \quad x \in R - \left\{\frac{5}{4}\right\}$$

129. Since, f(x) and g(x) has same domain and co-domain A and B and $f(1) = (1)^2 - 1 = 0$

$$g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0$$

$f(1) = 0 = g(1),\ f(0) = 0 = g(0)$

$f(-1) = 2 = g(-1),\ f(2) = 2 = g(2)$

$A = \{-1, 0, 1, 2\},\ B = \{-4, -2, 0, 2\}$

$\therefore$ By definition, the two function are equal f = g

130. $$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{f(x)+1}{f(x)-1} = \frac{2x}{-2}$$

$$\Rightarrow x = \frac{f(x)+1}{1-f(x)}$$

$$\therefore \quad f(2x) = \frac{2x-1}{2x+1} = \frac{2\left[\frac{f(x)+1}{1-f(x)}\right]-1}{2\left[\frac{f(x)+1}{1-f(x)}\right]+1} = \frac{3f(x)+1}{f(x)+3}$$

131. $f : N \to N$

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

Now for n = 1, $f(1) = \frac{1+1}{2} = 1$

and if n = 2, $f(2) = \frac{2}{2} = 1$

$\therefore$ f(1) = f(2), But $1 \neq 2$.

$\therefore$ f(x) is not one-one.

$f(x) = \frac{n+1}{2}$ if n is odd

if $y = \frac{n+1}{2}$ then $n = 2y - 1, \forall y$

Also, $f(x) = \frac{n}{2}$ if n is even i.e., $y = \frac{n}{2}$

or $n = 2y\ \forall y$

$\therefore$ f(x) is onto.