15 Sequence and Series

Formulae

1. Arithmetic Progression (A.P.)

i. A sequence (t_n) is said to be arthmetic progression (A.P.) if $t_{n+1} - t_n = d$ (common difference) for all $n \in N$.

ii. If a is the first term and d is the common difference, then A.P. can be written as

$a + (a + d) + (a + 2d) + ...$

iii. $t_n = S_n - S_{n-1}$

2. General term of an A.P.

i. General term (n^{th} term) of an A.P. is

$t_n = a + (n - 1) d$

We can denote it by T_n also.

ii. If the last term of an A.P. is l, then

$l = a + (n - 1) d$

3. Sum of n terras of an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } S_n = \frac{n}{2}(a + l) \text{ **or** }$$

$$S_n = \frac{n}{2}(a + t_n)$$

4. Arithmetic mean (A.M.)

i. If A is the A.M. between two numbers a and b, then $A = \frac{a+b}{2} \Rightarrow 2A = a + b$

ii. **Sum of n Arithmetic means between two numbers a and b**

If A_1, A_2, ..., A_n are n A.M.'s between a and b, then

$A_1 + A_2 + ... + A_n = nA$

where $A_n = a + nd$

$$= a + n\left(\frac{b-a}{n+1}\right) = \frac{a+nb}{n+1}$$

and $d = \frac{b-a}{n+1}$

5. Selection of terms in an A.P.

i. When the sum is given, the following way is adopted in selecting certain number of terms:

Number of terms	Terms to be taken
3	$a - d, a, a + d$
4	$a - 3d, a - d, a + d, a + 3d$
5	$a - 2d, a - d, a, a + d, a + 2d$

ii. When the sum is not given, then the following way is adopted in selection of terms.

Number of terms	Terms to be taken
3	$a, a + d, a + 2d$
4	$a, a + d, a + 2d, a + 3d$
5	$a, a + d, a + 2d, a + 3d, a + 4d$

6. Geometric Progression (G.P.)

i. A sequence (t_n) is said to be geometric progression if $\frac{t_{n+1}}{t_n} = r$ (common ratio) for all $n \in N$.

ii. If a is the first term and r is the common ratio, then G.P. can be written as

$a + ar + ar^2 + ...$

7. General term of a G.P.

General term (n^{th} term) of an G.P. is

$t_n = ar^{n-1}$ or $Tn = ar^{n-1}$

8. Sum of first n terms of a G.P.

Sum of first n terms of an G.P. is

$$S_n = \frac{a(1-r^n)}{1-r} \text{ or } S_n = \frac{a - t_n r}{1-r} \ (r < 1)$$

$$S_n = \frac{a(r^n - 1)}{1-r} \text{ or } S_n = \frac{t_n r - a}{r-1} \ (r > 1)$$

$S_n = na \ (r = 1)$

9. Sum of infinite terms of a G.P.

$$S_\infty = \frac{a}{1-r}, \text{ if } |r| < 1$$

If $|r| \geq 1$, then S_∞ does not exist.

10. Geometric mean (G.M.)

i. If G is the G.M. between two numbers a and b, then $G^2 = ab \Rightarrow G = \sqrt{ab}$

ii. Product of n Geometric means between a and b

If $G_1, G_2, \ldots, G_n$ are n G.M.'s between a and b, then

$G_1.G_2,\ldots.G_n = G^n$

$\Rightarrow \sqrt[n]{G_1.G_2......G_n} = G$

where $G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

11. Selection of terms in a G.P.

i. When the product is given, the following way is adopted in selecting certain number of terms:

Number of term	Terms to be taken
3	$\frac{a}{r}$, a, ar
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^2$
5	$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2

ii. When the product is not given, then the following way is adopted in selection of terms

Number of term	Terms to be taken
3	a, ar, ar^2
4	a, ar, ar^2, ar^3
5	a, ar, ar^2, ar^3, ar^4

12. Harmonic Progression (H.P.)

A progression is called a harmonic progression (H.P.) if the reciprocals of its terms are in A.P.

13. General term of a H.P.

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots\ldots$ is $t_0 = \frac{1}{a+(n-1)d}$ or

$t_n = \frac{1}{t_n \text{ of A.P.}}$

14. Sum of H.P. does not exist.

15. Harmonic mean (H.M.)

If H is the H.M. between two numbers a and b, then $H = \frac{2ab}{a+b}$

16. Relation between A.M. G.M. and H.M.

If A, G, H are the A.M., G.M. and H.M. of two numbers a and b, then

a. $A \geq G \geq H$

b. $G^2 = AH$

c. A, G, H are in G.P.

17. Arithmetico-geometric progression (A.G.P.)

i. General term of an A.G.P.

a, (a + d)r, (a + 2d)r^2,..... is

$t_n = [a + (n - 1)d]\, r^{n-1}$

ii. Sum of an A.G.P.

Sum of n terms of an A.G-P.

a,(a + d)r, (a + 2d)r^2,.... is

$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$,

where $r \neq 1$

iii. Sum of infinite terms of an A.G.P. is

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} \quad (|r| < 1)$

18. Special Series:

i. Sum of first n natural numbers

$= \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

ii. Sum of squares of first n natural numbers

$= \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

iii. Sum of cubes of first n natural number

$= \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$

19. Exponential Series:

i. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

ii. $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

iii. $e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$.......

iv. $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$.......

v. $e^x + e^{-x} = 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ...\right)$

vi. $\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + ...$

vii. $\frac{e - e^{-1}}{2} = 1 + \frac{1}{3!} + \frac{1}{5!} + ...$

viii. $e^{e^x} = 1 + \frac{e^x}{1!} + \frac{e^{2x}}{2!} + \frac{e^{3x}}{3!} + ...$

ix. If $a > 0$ then

$$a^x = 1 + x(\log_e a) + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + ...$$

20. Logarithmic series

If $x < 1$, then

i. $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$

ii. $-\log(1 - x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$

iii. $\log\left(\frac{1 + x}{1 - x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + ...\right]$

iv. $\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$

Shortcuts

1. If p^{th} term of an A.P. is q and q^{th} term = p, then $tp + q = 0$, $tn = p + q - n$
2. If $T_p = \frac{1}{q}$ and $T_q = \frac{1}{p}$, then $T_{pq} = 1$
3. If $p\,t_p = q\,t_q$ of an A.P., then $t_{p+q} = 0$
4. If $S_p = q$ for an A.P., $S_q = p$, then $S_{p+q} = -(p + q)$
5. If $S_p = S_q$ for an A.P., then $S_{p+q} = 0$
6. If for a G. P., $t_p = P$; $t_q = Q$, then $t_n = \left[\frac{P^{n-q}}{Q^{n-q}}\right]^{\frac{1}{p-q}}$
7. If for a G.P., $t_{m+n} = p$; $t_{m-n} = q$, then $t_m = \sqrt{pq}$; $t_n = p\left(\frac{q}{p}\right)^{\frac{m}{2n}}$
8. If a, b, c are the p^{th}, q^{th}, r^{th} terms of a G.P., then $a^{q-r}.b^{r-p}.c^{p-q} = 1$
9. If a, b, c $\in$ A.P., then 2^{ax+1}, 2^{bx+1}, 2^{cx+1}, $x \neq 0$ are in G.P.
10. If the m^{th} term of a H.P. = n and n^{th} term = m, then $t_{m+n} = \frac{mn}{m+n}$, $t_{mn} = 1$, $t_p = \frac{mn}{p}$
11. In a H.P., t_p – qr, t_q = pr, then t_r = pq
12. If H is H.M. between a and b, then
    i. $(H - 2a)(H - 2b) = H^2$
    ii. $\frac{1}{H - a} + \frac{1}{H - b} = \frac{1}{a} + \frac{1}{b}$
    iii. $\frac{H + a}{H - a} + \frac{H + b}{H - b} = 2$
13. If A_1, A_2 be two A.M.'s, G_1 G_2 be two G.M.'s and H_1, H_2 be two H.M.'s between two numbers a and b, then $\frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2}$
14. If A, G, H be A.M., G.M., H.M. between a and b, then
$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \begin{cases} A \text{ when } n = 0 \\ G \text{ when } n = -\frac{1}{2} \\ H \text{ when } n = -1 \end{cases}$$
15. If A & G be the A.M. and G.M. between two numbers a, b, then a, b are given by $A \pm \sqrt{(A + G)(A - G)}$
16. If the A.M. between two positive numbers a and b (a > b > 0) is n times the geometric mean between them, then $\frac{a}{b} = \frac{n + \sqrt{n^2 - 1}}{n - \sqrt{n^2 - 1}}$, $n > 1$

17. Sum of n arithmetic means between a and b is $n\left(\frac{a+b}{2}\right)$

18. Product of n geometric means between a and b is $(\sqrt{ab})^n$

19. n^{th} Harmonic means between two numbers a and b is $\frac{(n+1)\,ab}{na+b}$

20. a^2, b^2, c^2 are in A.P.

$\Leftrightarrow \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

21. If a_1 a_2,..... a_n are the non-zero terms of a non-constant A.P., then

$$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\frac{1}{a_3a_4}.....+\frac{1}{a_{n-1}\,a_n}=\frac{n-1}{a_1a_n}$$

22. If S_n is the sum of first n terms of the A.P. a + (a + d) + (a + 2d) + ... + l, then

i. $S_n=\frac{l-a+d}{2d}(a+l)$

ii. $S_n=\frac{n}{2}[2l-(n-1)\,d]$

23. 2 + 6 + 12 + 20 ..... n terms

$=\frac{n(n+1)(n+2)}{3}$

24. 1 + 3 + 7 + 13 .....n terms

$=\frac{n(n^2 2)}{3}$

25. 1 + 5 + 14 + 30..... upto n terms

$=\frac{n(n+1)^2(n+2)}{12}$

26. Sum of first n odd natural numbers

$$=1+3+5+.....+(2n-1)=\sum_{r=1}^{n}(2r-1)=n^2$$

27. Sum of first n even natural numbers

$$=2+4+6+.....+2n=\sum_{r=1}^{n}2r=n\,(n+1)$$

28. If a_1, a_2, a_3, ... a_n are in A.P., then

i. $$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+.......+\frac{1}{\sqrt{a_{n.1}}+\sqrt{a_n}}$$

ii. $$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+.....+\frac{1}{a_na_{n+1}}=\frac{1}{(a_2-a_1)}\left[\frac{1}{a_1}-\frac{1}{a_{n+1}}\right]$$

iii. $$\frac{1}{a_1a_2a_3}+\frac{1}{a_2a_3a_4}+\frac{1}{a_3a_4a_5}+.....+\frac{1}{a_na_{n+1}a_{n+2}}=\frac{1}{2(a_2-a_1)}\left[\frac{1}{a_1a_2}-\frac{1}{a_{n+1}a_{n+2}}\right]$$

29. 1 + 2x + 3x² + 4x³ + ... to ∞

$=\frac{1}{(1-x)^2}$ for $|x|<1$

30. 1 + 3x + 5x² + 7x³ + ... to ∞

$=\frac{1+x}{(1-x)^2}$ for $|x|<1$

31. Short cut methods for recurring decimals:

i. $.6\dot{2}\dot{5}=\frac{625-6}{990}=\frac{619}{990}$

ii. $.4\dot{2}\dot{3}=\frac{423-4}{990}=\frac{419}{990}$

iii. $1.2\dot{4}\dot{5}=1+\frac{245-2}{990}=1+\frac{243}{990}=\frac{1233}{990}$

MULTIPLE CHOICE QUESTIONS

Classical Thinking

15.1 Arithmetic Progression (A.P.) and Arithmetic Mean (A.M.)

1. Which term of A.P. 72, 70, 68, 66, ... is 40 ?
a) 16 b) 17
c) 18 d) 20

2. The 10^{th} term of the sequence
$\sqrt{3}, \sqrt{12}, \sqrt{27}, ...$ is
a) $\sqrt{243}$ b) $\sqrt{300}$
c) $\sqrt{363}$ d) $\sqrt{432}$

3. How many terms are there in sequence 3, 6, 9, 12,...., 111 ?
a) 30 b) 37
c) 40 d) 47

4. If a, b, c, d, e, fare in A.P., then e – c =
a) 2(c – a) b) 2(f – d)
c) 2(d – c) d) d – c

5. If a, b, c are in A.P., then $\frac{a}{bc}, \frac{1}{c}, \frac{1}{b}$ are in
a) A.P. b) G.P.
c) H.P. d) none of these

6. If for a sequence (t_n), $S_n = 3(4^n - 1)$, then t_n =
a) $3(4^{n-1})$ b) $9(4^{n-1})$
c) $3(4^n)$ d) $9(4^{n-1})$

7. 15^{th} term of an A.P. 21, 16, 11,6, ...
a) – 25 b) – 29
c) – 49 d) – 39

8. If for an A.P, $S_{16} = 784$, a - 4, then d =
a) 5 b) 6
c) 7 d) 8

9. 7^{th} term of an A.P. is 40. Then the sum of first 13 terms is
a) 520 b) 53
c) 2080 d) 1040

10. The fourth term of an A.P. is 4. Then the sum of the first 7 terms is
a) 4 b) 28
c) 16 d) 40

11. The sum of first n odd natural numbers is
a) n^2 b) 2n
c) $\frac{n(n-1)}{2}$ d) $\frac{n(n+1)}{2}$

12. Sum of first 5 terms of an A.P. is one fourth of the sum of next five terms. If the first term = 2, then the common difference of the A.P. is
a) 6 b) – 6
c) 3 d) 2

13. If 1 + 6 + 11 + 16 + ... + x = 148, then x is equal to
a) 24 b) 36
c) 42 d) 46

14. If the sum of first n terms of an A.P. be $3n^2 - n$ and its common difference is 6, then its first term is
a) 2 b) 3
c) 1 d) 4

15. p^{th} term of the series
$\left(3-\frac{1}{n}\right)+\left(3-\frac{2}{n}\right)+\left(3-\frac{3}{n}\right)+.....$ will be
a) $\left(3+\frac{p}{n}\right)$ b) $\left(3-\frac{p}{n}\right)$
c) $\left(3+\frac{n}{p}\right)$ d) $\left(3-\frac{n}{p}\right)$

16. If the 9^{th} term of an A.P. be zero, then the ratio of its 29^{th} and 19^{th} term is
a) 1 : 2 b) 2 : 1
c) 1 : 3 d) 3 : 1

17. If the p^{th}, q^{th} and r^{th} term of an arithmetic sequence are a , b and c respectively, then the value of [a(q – r) + b(r – p) + c(p – q)] =
a) 1 b) – 1
c) 0 d) 1

18. If the first term of an A.P. is 10, last term is 50 and the sum of all the terms is 300, then the number of terms are
a) 5 b) 8
c) 10 d) 15

19. The solution of the equation $(x+1)+(x+4)+(x+7)+...+(x+28)=155$ is

a) 1 b) 2

c) 3 d) 4

20. The number of terms of the A.P. 3,7,11,15...to be taken so that the sum is 406, is

a) 5 b) 10

c) 12 d) 14

21. If A_1, A_2 are two arithmetic means between $\frac{1}{3}$ and $\frac{1}{24}$, then their values are

a) $\frac{7}{72}, \frac{5}{36}$ b) $\frac{17}{72}, \frac{5}{36}$

c) $\frac{7}{36}, \frac{5}{72}$ d) $\frac{5}{72}, \frac{17}{72}$

22. If A be an arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then

a) $S = nA$ b) $A - nS$

c) $A = S$ d) $A = n^2S$

23. Three numbers are in A.P. whose sum is 33 and product is 792, then the smallest number from these numbers is

a) 4 b) 8

c) 11 d) 14

24. Which term of the sequence $8-6i, 7-4i, 6-2i, ...$ is a real number ?

a) 7^{th} b) 6^{th}

c) 5^{th} d) 4^{th}

15.2 Geometric Progression (G.P.) and Geometric Mean (G.M.)

25. The n^{th} term for a G.P. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$....... is

a) $\left(\frac{1}{2}\right)^n$ b) $\left(\frac{1}{4}\right)^{n-1}$

c) $\left(\frac{1}{2}\right)^{n-1}$ d) $\left(\frac{1}{4}\right)^n$

26. For a G.P. 5, 15, 45,135,....., S_n =

a) $\frac{5(2^n-1)}{3}$ b) $\frac{3(3^n+1)}{2}$

c) $\frac{5(3^n-1)}{2}$ d) $\frac{5(2^n+1)}{3}$

27. For a G.P. $4, -4, 4, -4, ..., t_n$ =

a) $4(-1)^{n-1}$ b) $4(-1)^n$

c) $2(-1)^{n-1}$ d) $2(-1)^n$

28. How many terms of the G.P. 1, 3, 9, 27...... have to be taken to get the sum equal to 3280?

a) 5 b) 8

c) 7 d) 6

29. For a G.P. $1, \frac{-3}{2}, \frac{9}{4}, \frac{-27}{8}, ..., S_n$ =

a) $\left(\frac{3}{5}\right)\left[1-\left(\frac{-3}{2}\right)^n\right]$ b) $\left(\frac{2}{5}\right)\left[1-\left(\frac{3}{2}\right)^n\right]$

c) $\left(\frac{3}{5}\right)\left[1-\left(\frac{3}{2}\right)^n\right]$ d) $\left(\frac{2}{5}\right)\left[1-\left(\frac{-3}{2}\right)^n\right]$

30. All the terms of a G.P. are squared. The new series thus formed is in

a) G.P. b) A.P.

c) H.P d) none of these

31. The n^{th} term of a G.P, 1, 2, 4, 8, ... is

a) $2n$ b) $2^n - 1$

c) 2^{n-1} d) $1 - 2^n$

32. The sum to n terms of 2 + 22 + 222 + ... is

a) $\frac{1}{81}[10(10^n-1)-9n]$

b) $\frac{2}{81}[10(10^n-1)-9n]$

c) $\frac{2}{81}[10(10^n-1)]$

d) $\frac{i}{81}[10(10^n-1)-n]$

33. The sum to n terms of 0.9 + 0.99 + 0.999 is

a) $\frac{9n-[1-(0.1)^n]}{9}$ b) $\frac{5n-[1-(0.1)^n]}{9}$

c) $\frac{9n+[1+(0.1)^n]}{9}$ d) $\frac{6n-[1-(0.1)^n]}{9}$

34. If the first term of a G.P. is 2 and sum to infinity is 6, then r =

a) $\frac{1}{3}$ b) $\frac{2}{3}$

c) $\frac{3}{2}$ d) $\frac{4}{3}$

35. t_n for a G.P. $3, \frac{-3}{2}, \frac{3}{4}, \frac{-3}{8}, \frac{3}{16}$...... is

a) $3\left(-\frac{1}{2}\right)^{n-1}$ b) $3\left(\frac{1}{2}\right)^{n-1}$

c) $3\left(\frac{-1}{2}\right)^{n}$ d) $3^n\left(\frac{1}{2}\right)^{n-1}$

36. The sum of first eight terms of G.P. is 82 times the sum of first four terms. The common ratio of G.P is

a) 4 b) 5

c) 3 d) 2

37. For a G.P. 3, 12, 48, 192,...., S_n =

a) $3\left[\frac{4^n-1}{5}\right]$ b) $3\left[\frac{4^n-1}{4}\right]$

c) $4^n - 1$ d) $3^n - 1$

38. If for a G.P. ,r = 2, S_8 = 510, then t_3 =

a) 8 b) 4

c) 16 d) 12

39. How many terms of G.P. 2, 22, 23 needed to get the sum 30?

a) 4 b) 8

c) 5 d) 6

40. If three numbers a, 8, b, (a ≠ b) are in G.P and a, b, – 8 are in A.P. then the values of a and b respectively are

a) – 4, – 16 b) – 4, 16

c) 16, – 4 d) 16, 4

41. Three numbers are in G. P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.

a) 10, 20, 40 or 40, 20, 10

b) 5, 15, 45 or 45, 15, 5

c) 8, 16, 32 or 32, 16, 8

d) – 1, – 3, – 6 or – 6, – 3, – 1

42. If g_1 and g_2 be the two geometric means between two positive numbers p and q, then g_1g_2 is equal to

a) p + q b) pq

c) $\frac{p+q}{pq}$ d) $\frac{p}{q}$

43. If for a G.P., t_3 = 20 and t_7 = 320, then the first term and common ratio respectively are

a) – 5, 2 b) 5, – 2

c) – 5, – 2 d) 5, 2

44. The first term of G.P. is 1. The sum of third and fifth terms is 90. Find common ratio of G.P.

a) ± 4 b) ± 5

c) ± 3 d) ± 6

45. If for a G.P., t_3 = 20, t_7 = 320, then t_{10} =

a) 2500 b) 2560

c) 2650 d) 2600

46. If the 10th term of a geometric progression is 9 and 4th term is 4, then its 7th term is

a) 6 b) 36

c) $\frac{4}{9}$ d) $\frac{9}{4}$

47. 7th term of the sequence $\sqrt{2}, \sqrt{10}, 5\sqrt{2}$,........ is

a) $125\sqrt{10}$ b) $25\sqrt{2}$

c) 125 d) $125\sqrt{2}$

48. The two geometric means between the number 1 and 64 are

a) 1 and 64 b) 4 and 16

c) 2 and 16 d) 8 and 16

49. If the product of three consecutive terms of a G.P. is 216 and the sum of product of pair-wise is 15, 6, then the numbers will be

a) 1, 3, 9

b) 2, 6, 18

c) 3, 9, 27

d) 2, 4, 8

50. The sum of infinity of a geometric progression is $\frac{4}{3}$ and the first term is $\frac{3}{4}$. The common ratio is

a) 7/16 b) 9/16

c) 1/9 d) 7/9

15.3 Harmonic Progression, Harmonic Mean and Relation between A.M., G.M. and H.M.

51. For two positive numbers, if A.M. = 25 and G.M=12, then H.M. =

a) 6.75 b) 5.86
c) 5.76 d) 6.85

52. If A, G, H denote respectively the A.M., G.M. and H.M. between two unequal positive quantities, then

a) $A < G < H$ b) $A < H < G$
c) $G < H < A$ d) $H < G < A$

53. The A.M. between two numbers is 32 and their H.M. is 8. Thus, G.M. =

a) 16 b) 4
c) 256 d) 64

54. The arithmetic, harmonic and geometric means between two positive numbers are $\frac{144}{15}$, 15 and 12 but not necessarily in this order, then the H.M., G.M and A.M. respectively are

a) $15, 12, \frac{144}{15}$ b) $12, 15, \frac{144}{15}$
c) $\frac{144}{15}, 12, 15$ d) $12, \frac{144}{15}, 15$

55. If A is the A.M. between two distinct positive numbers and H is the H.M. between them, then the G.M. between the numbers is

a) $2A - H$ b) $\frac{A^2}{H}$
c) $\sqrt{AH}$ d) $\sqrt{\frac{A}{H}}$

56. If the 7th term of a H.P. is $\frac{1}{10}$ and the 12th term is $\frac{2}{25}$, then the 20th term is

a) $\frac{1}{37}$ b) $\frac{1}{41}$
c) $\frac{1}{45}$ d) $\frac{1}{49}$

57. If A.M. of two terms is 9 and H.M. is 36, then G.M. will be

a) 18 b) 12
c) 16 d) 17

15.4 Arithmetico Geometric Progression (A.G.P.)

58. If $3 + 5r + 7r^2 + ... \text{ to } \infty$ is $\frac{44}{9}$, then r is equal to

a) $\frac{17}{11}$ b) $\frac{1}{4}$
c) $\frac{11}{17}$ d) 4

59. If $|x| < 1$, then $1 + 3x + 5x^2 + 7x^3 + ...$ upto ∞ is equal to

a) $\frac{1+x}{(1-x)^2}$ b) $\frac{2+x}{(1-x)^2}$
c) $\frac{x}{(1-x)^2}$ d) $\frac{1+x}{(1-x)}$

60. The sum to, infinity of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......$ is

a) 2 b) 3
c) 4 d) 6

15.5 Special series, Exponential series, Logarithmic series

61. $\sum_{r=1}^{n}(2r+5) =$

a) $n(5n + 6)$ b) $2n(n + 3)$
c) $n(n + 3)$ d) $n(n + 6)$

62. The sum of the series $x^2+\frac{x^4}{2}+\frac{x^6}{3}+\frac{x^8}{4}+......$ $|x| < 1$, is

a) $(1 - x^2)^{-1}$ b) $\frac{x^2}{1-x^2}$
c) $-\log_e(1 - x^2)$ d) $\log(1 + x^2)$

63. The sum of $1^3 + 2^3 + 3^3 + ... + 25^3 =$

a) 106525 b) 105525
c) 104525 d) 105625

64. The sum of $(31)^2 + (32)^2 + (33)^2 + ... + (60)^2$ is
a) 64355 b) 64533
c) 64535 d) 64553

65. $(2^2 - 1^2) + (4^2 - 3^2) + (6^2 - 5^2) + ..., ...=$
a) $n(3n+1)$ b) $n^2(2n+1)$
c) $n^2(3n+1)$ d) $n(2n+1)$

66. $\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + ...$ is
a) $(\log_e 3)(\log_e 2)$ b) $\log_e 3$
c) $\log_e 2$ d) $\frac{\log_e 5}{\log_e 3}$

67. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...$, then x is equal to
a) e^y b) $e^y + 1$
c) $e^y - 1$ d) $\log(1+y)$

68. $2.1^2 + 3.2^2 + 4.3^2 + ...$ upto 10 terms =
a) 4310 b) 3640
c) 3410 d) 4230

69. If $1^2 + 2^2 + 3^2 + ... + n^2 = 1015$, then the value of n is
a) 13 b) 14
c) 15 d) 16

70. The sum of n terms of the following series $1.2 + 2.3 + 3.4 + 4.5 + ...$ will be
a) n^3 b) $\frac{1}{3}n(n+1)(n+2)$
c) $\frac{1}{6}n(n+1)(n+2)$ d) $\frac{1}{3}n(n+1)(2n+1)$

Critical Thinking

15.1 Arithmetic Progression (A.P.) and Arithmetic Mean (A.M.)

1. Which term of the sequence $8 - 6i, 7 - 4i, 6 - 2i$ is purely imaginary ?
a) 6th b) 8th
c) 9th d) 10th

2. The fourth term of the sequence
$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}, ...$ is
a) $\frac{1}{1-2\sqrt{x}}$ b) $\frac{1}{2-\sqrt{x}}$
c) $\frac{1+2\sqrt{x}}{1-x}$ d) $\frac{1}{1-x}$

3. Let S_n denote the sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then the ratio $\frac{S_{3n}}{S_n}$ is equal to
a) 4 b) 6
c) 8 d) 10

4. The number of terms in the A.P. $a + b + . . . + c$ is
a) c b) $\frac{c-a}{b-a}$
c) $\frac{b+c-2a}{b-a}$ d) $\frac{a}{a+b}$

5. If a, b, c, are in A.P., then $\frac{a-b}{b-c}$ equals
a) $\frac{b}{a}$ b) $\frac{a}{b}$
c) $\frac{a}{c}$ d) 1

6. If the numbers a, b, c, d, e form an A.P. then the value of $a - 4b + 6c - 4d + e$ is
a) 1 b) 2
c) 0 d) – 1

7. In an A.P., $S_1 = 6$, $S_7 = 105$, then $S_n : S_{n-3}$ is same as
a) $(n+3) : (n-3)$ b) $(n+3) : n$
c) $n : (n-3)$ d) 1

8. The sum ol the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+.....$ upto 9 terms is

a) $\frac{-5}{6}$ b) $-\frac{1}{2}$

c) 1 d) $-\frac{3}{2}$

9. The first, second and last terms of an A.P. are a, b and 2a. The number of terms in the A.P. is

a) $\frac{b}{b-a}$ b) $\frac{b}{b+a}$

c) $\frac{a}{b-a}$ d) $\frac{a}{b+a}$

10. The sum of all 2-digit numbers which leave remainder 1 when divided by 3 is

a) 1616 b) 1602

c) 1605 d) 1606

11. If $S_n = nP + \frac{n(n-1)}{2}Q$, where S_n denotes the sum of the first n terms of an A.P., then the common difference is

a) P + Q b) 2P + 3Q

c) 2Q d) Q

12. Find the sum of three digit numbers, which are divisible by 7 ?

a) 70770 b) 70330

c) 70336 d) 70777

13. 150 workers were engaged to finish a certain piece of work in a certain number of days. Four workers dropped the second day, four more dropped third day and so on. It takes 8 more days to finish work now. Find the number of days in which the work was completed ?

a) 25 b) 30

c) 15 d) 10

14. The sum of first n terms of an A.P. whose last term is l and common difference is d is

a) $\frac{n}{2}[2l+(n-1)d]$ b) $\frac{n}{2}[2l-(n-1)d]$

c) $\frac{n}{2}[l+(n-1)d]$ d) $\frac{n}{2}[l-(n-1)d]$

15. A student read common difference of an A.P. as – 2 instead of 2 and got the sum of first 5 terms as – 5. Actual sum of first five terms is

a) 25 b) – 25

c) – 35 d) 35

16. The sum of first four terms of an A.P. is 56 and sum of last four terms is 112. If the first term is 11, then the number of terms is

a) 10 b) 12

c) 11 d) none of these

17. n^{th} term of 3.8 + 6.11 + 9.14 + 12.17 + ... will be

a) 3n(3n + 5) b) 3n(n + 5)

c) n(3n + 5) d) n(n + 5)

18. If the angles of a quadrilateral are in A.P. whose common difference is 10^0, then the angles of the quadrilateral are

a) $65^0, 85^0, 95^0, 105^0$ b) $75^0, 85^0, 95^0, 105^0$

c) $65^0, 75^0, 85^0, 95^0$ d) $65^0, 95^0, 105^0, 115^0$

19. If a, b, c, are in A.P. and also $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., then

a) $a = b \neq c$ b) $a \neq b = c$

c) $a = b = c$ d) $a \neq b \neq c$

20. If the ratio of the sum of a terms of two A.P.'s be (7n + 1) : (4n + 27), then the ratio of their 11^{th} term will be

a) 2 : 3 b) 3 : 4

c) 4 : 3 d) 5 : 6

21. The number of numbers between 105 and 1000, which are divisible by 7 is

a) 142 b) 128

c) 127 d) 126

22. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P., then x equals

a) $\log_3 4$ b) $1-\log_3 4$

c) $1-\log_4 3$ d) $\log_4 3$

23. If the sum of n terms of an A.P. is $2n^2 + 5n$, then the n^{th} term will be

a) 4n + 3 b) 4n + 5

c) 4n + 6 d) 4n + 7

24. The n^{th} term of an A.P. is 3n – 1 . Choose from the following, the sum of its first five terms

a) 14 b) 35

c) 80 d) 40

25. There are 15 terms in an arithmetic progression. Its first term is 5 and their sum is 390. The middle term is

a) 23 b) 26
c) 29 d) 32

26. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the A.M. of a and b, then n =

a) 1 b) – 1
c) 0 d) 2

27. The sum of n arithmetic mean between a and b, is

a) $\frac{n(a+b)}{2}$ b) $n(a+b)$
c) $\frac{(n+1)(a+b)}{2}$ d) $(n+1)(a+b)$

28. After inserting n A.M.'s between 2 and 38, the sum of the resulting progression is 200. The value of n is

a) 10 b) 8
c) 9 d) 7

29. If log 2, $\log(2^n-1)$ and $\log(2^n+3)$ are in A.P., then n =

a) 5/2 b) $\log_2 5$
c) $\log_3 5$ d) 3/2

30. If the sum of three numbers of a arithmetic sequence is 15 and the sum of their squares is 83, then the numbers are

a) 4, 5, 6 b) 3, 5, 7
c) 1, 5, 9 d) 2, 5, 8

31. If a, b, c are in A.P., then
(a + 2b – c) (2b + c – a) (c + a – b) equals

a) – abc b) abc
c) 2 abc d) 4 abc

15.2 Geometric Progression (G.P.) and Geometric Mean (G.M.)

32. If the third term of a G.P. is 4, then product of first five terms is

a) 4^3 b) 4^5
c) 4^4 d) 4^2

33. S_n of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \frac{5}{16}$,...... is

a) $5\left[\frac{2^n-1}{2^n}\right]$ b) $5\left[\frac{2^{n-1}-1}{2^n}\right]$
c) $5\left[\frac{2^n-1}{2^{n-1}}\right]$ d) $5\left[\frac{2^{n-1}}{2^n}\right]$

34. If for a G.P., $t_3 = 36$ and $t_6 = 972$, then t_8 =

a) 8748 b) 84
c) 4784 d) 88

35. If a, b, c are in G.P., then

a) $a(b^2 + a^2) = c(b^2 + c$
b) $a(b^2 + c^2) = c(a^2 + b$
c) $a^2(b + c) = c^2(a + b)$
d) none of these

36. If for a G.P., r = 2, t_9 = 128, then a =

a) $\frac{1}{4}$ b) $\frac{1}{2}$
c) $\frac{3}{4}$ d) $\frac{3}{2}$

37. The sum to n terms of 4 + 44 + 444 +. ... is

a) $\frac{1}{9}\left\{\frac{10}{9}(10^n-1)-n\right\}$
b) $\frac{4}{9}\left\{\frac{10}{9}(10^n-1)\right\}$
c) $\frac{4}{9}\left\{\frac{10}{9}(10^n-1)-n\right\}$
d) $\frac{4}{9}\left\{\frac{1}{9}(10^n-1)\right\}$

38. The n^{th} term of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$...... is

a) $\frac{1+(n-1)2}{2^n}$ b) $1-\frac{1}{2^n}$
c) $2^{-n} - 1$ d) $\frac{1}{2^n}$

39. The sum of the first 10 terms of the series $\sqrt{2}+\sqrt{6}+\sqrt{18}+$..... is

a) $121(\sqrt{6}+\sqrt{2})$ b) $\frac{121}{\sqrt{3}-1}$
c) $243(\sqrt{3}+1)$ d) $242(\sqrt{3}-1)$

40. The first and second terms of a G.P. are x^{-4} and x^n respectively. If x^{52} is the eighth term of the same progression, then n is equal to

a) 13 b) 4

c) 5 d) 3

41. For a G.P. whose 4^{th} term is 24 and 9^{th} term is 768, S_{10} =

a) $3(2^{10-1} - 1)$ b) $3(2^{10} - 1)$

c) $2(3^{10} - 1)$ d) $2(3^{10-1} - 1)$

42. The sum to n terms of $1 + (1 + x) + (1 + x + x^2) + \ldots$ upton terms is

a) $\frac{n}{1-x} - \frac{x(1-x)^n}{(1-x)^2}$ b) $\frac{n}{1-x} - \frac{x(1-x^n)}{(1-x)^2}$

c) $\frac{n}{1-x} + \frac{x(1-x^n)}{(1-x)^2}$ d) $\frac{n}{1-x} + \frac{x(1-x)^n}{(1-x)^2}$

43. Find a G.P. for which the sum of first two terms is – 4 and the 5^{th} term is 4 times the 3^{rd} term

a) $\frac{-8}{3}, \frac{-14}{3}, \frac{-16}{3}, \ldots\ldots$ or $4, -8, 12, 16, \ldots\ldots$

b) $\frac{-8}{3}, \frac{-11}{3}, \frac{-16}{3}, \ldots\ldots$ or $2, 4, 8, \ldots\ldots$

c) $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$ or $4, -8, 16, -32, \ldots\ldots$

d) $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$ or $4, -8, 12, 24, \ldots\ldots$

44. The product of first three terms of a G.P. is 1000. If 6 is added to its 2^{nd} term and 7 is added to its 3^{rd} term, the terms become in A.P. Find the G.P.

a) 5, 10, 20 b) 3, 6, 12

c) 7, 14, 28 d) 1, 3, 9

45. In a G.P., the first term is 'a', second term is 'b' and the last term is 'c', then sum of the series is

a) $\frac{a^2 - b}{b}$ b) $\frac{a^2 - bc}{a - b}$

c) $\frac{a^3 - bc}{a + b}$ d) $\frac{a^3 - b}{a + b}$

46. The sum of first 10 terms of G.P. is equal to 244 times to sum of its first five terms. Then the common ratio is

a) 7 b) 3

c) 4 d) 5

47. $2.3\overline{45}$ in $\frac{p}{q}$ form is

a) $\frac{129}{55}$ b) $\frac{129}{56}$

c) $\frac{55}{17}$ d) $\frac{155}{74}$

48. If second term of G.P. is 2 and the sum of its infinite terms is 8, then its first term is

a) $\frac{1}{4}$ b) $\frac{1}{2}$

c) 2 d) 4

49. Sum of n terms of the series

$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$ is equal to

a) $2^n - n - 1$ b) $1 - 2^{-n}$

c) $2^n - 1$ d) $n + 2^{-n} - 1$

50. The sum to infinity of

$\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \ldots$ is

a) $\frac{3}{16}$ b) $\frac{1}{5}$

c) $\frac{1}{24}$ d) $\frac{1}{16}$

51. If $A = 1 + r^2 + r^{22} + r^{32} + \ldots \infty$, then the of r will be

a) $A(1 - A)^z$ b) $\left(\frac{A-1}{A}\right)^{\frac{1}{z}}$

c) $\left(\frac{1}{A} - \right)^{\frac{1}{z}}$ d) $A(1-A)^{\frac{1}{z}}$

52. If b is the G.M of a and c, then $\frac{1}{b-a} + \frac{1}{b-c} =$

a) $\frac{1}{a}$ b) $\frac{1}{c}$

c) 1 d) $\frac{1}{b}$

53. If g_1 and g_2 be the two geometric means between two positive numbers a and b then $\frac{g_1^2}{g_2}+\frac{g_2^2}{g_1}$ is equal to

a) $a+b$ b) ab

c) $\frac{a+b}{ab}$ d) $\frac{ab}{a+b}$

54. If the p^{th}, q^{th} and r^{th} term of a G.P. are a, b, c respectively, then $a^{q-r}.b^{r-p}.c^{p-q}$ is equal to

a) 0 b) 1

c) abc d) pqr

55. If $x=\sum_{n=0}^{\infty}a^n, y=\sum_{n=0}^{\infty}b^n, z=\sum_{n=0}^{\infty}(ab)^n$, where a, $b<1$,then

a) $xyz=x+y+z$ b) $xz+yz=xy+z$

c) $xy+yz=xz+y$ d) $xy+xz=yz+x$

56. The sum of first three terms of a G.P. to the sum of next three terms is 125 : 27. The common ratio of the G.P. is

a) $\frac{1}{2}$ b) $\frac{3}{5}$

c) $\frac{5}{3}$ d) $\frac{4}{5}$

57. Fifth term of a G.P. is 2, then the product of its 9 terms is

a) 256 b) 512

c) 1024 d) 128

58. If the sum of n terms of a G.P. is 255 and n^{th} term is 128 and common ratio is 2, then first term will be

a) 1 b) 3

c) 7 d) 2

59. The solution of the equation $1+a+a^2+a^3+....+ax=(1+a)(1+a^2)(1+a^4)$ is given by x is equal to

a) 3 b) 5

c) 7 d) – 5

60. If in a geometric progression $\{a_n\}$, $a_1=3$, $a_n=96$ and $S_n=189$ then the value of n is

a) 5 b) 6

c) 7 d) 8

61. The first term of a G.P. is 7, the last term is 448 and sum of all terms is 889, then the common ratio is

a) 5 b) 4

c) 3 d) 2

62. If G be the geometric mean of x and y, then

$$\frac{1}{G^2-x^2}+\frac{1}{G^2-y^2}=$$

a) G^2 b) $\frac{1}{G^2}$

c) $\frac{2}{G^2}$ d) $3G^2$

63. The G.M. of the numbers 3, 3^2, 3^3,.. is

a) $3^{\frac{2}{n}}$ b) $3^{\frac{n+1}{2}}$

c) $3^{\frac{n}{2}}$ d) $3^{\frac{n-1}{2}}$

64. The value of $4^{1/3}.4^{1/9}.4^{1/27}.......\infty$ is

a) 2 b) 3

c) 4 d) 9

65. The sum to infinity of the progression

$$9-3+1-\frac{1}{3}+.....\text{ is}$$

a) 9 b) $\frac{9}{2}$

c) $\frac{27}{4}$ d) $\frac{15}{2}$

66. The first term of an infinite geometric progression is x and its sum is 5. Then

a) $0\le x\le 10$ b) $0<x<10$

c) $-10<x<0$ d) $x>10$

15.3 Harmonic Progression, Harmonic Mean and Relation between A.M., G.M. and H.M.

67. If a, b, c are in H.P., then $\frac{a-b}{b-c}$ equals

a) $\frac{a}{a}$ b) $\frac{a}{b}$

c) $\frac{a}{c}$ d) $\frac{1}{a}$

68. If c is the harmonic mean between a and b, then $\frac{c}{a}+\frac{c}{b}$ is equal to

a) 2 b) $\frac{a+b}{ab}$

c) $\frac{ab}{a+b}$ d) 1

69. If H is the Harmonic mean between a and b, then $\frac{1}{H-a}+\frac{1}{H-b}=$

a) $\frac{1}{a}-\frac{1}{b}$ b) $\frac{1}{a}+\frac{1}{b}$

c) $\frac{1}{a}$ d) $\frac{1}{b}$

70. If H is the harmonic mean between a and b, then $\frac{H+a}{H-a}+\frac{H+b}{H-b}$ is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{2}$

c) 2 d) 1

71. If the 7^{th} term of a harmonic progression is 8 and the 8 term is 7, then its 1 5^{th} term is

a) 16 b) 14

c) $\frac{27}{14}$ d) $\frac{56}{15}$

72. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is

a) $\frac{a}{\sqrt{1-a^2b^2}}$ b) $\frac{a}{1-a^2b^2}$

c) a d) $\frac{1}{1-a^2b^2}$

73. If a, b, c, and b, c, dare in H.P., then ab + bc + cd is

a) 3ad b) (a + b)(c + d)

c) 3ac d) 3bd

74. H.M. between two numbers is 4. The A.M. ‘A’ and the G.M. ‘G’ between them satisfy the relation $2A + G^2 = 27$. The numbers are

a) 6, 3 b) 4, 2

c) 6, 9 d) 3, 5

75. Which number should be added to the numbers 13, 15, 19 so that the resulting numbers be the consecutive terms of a H.P.

a) 7 b) 6

c) – 6 d) – 7

76. In a H.P., p^{th} term is q and the q^{th} term is p. Then pq^{th} term is

a) 0 b) 1

c) pq d) pq(p + q)

77. If $\log_a x, \log_b x, \log_c x$ be in H.P., then a, b, c are in

a) A.P. b) H.P.

c) G.P. d) None of these

78. If a and b are two different positive real numbers, then which of the following relations is true

a) $2\sqrt{ab} > (a+b)$ b) $2\sqrt{ab} < (a+b)$

c) $2\sqrt{ab} = (a+b)$ d) None of these

79. If a, b, c are in A.P. and |a|, |b|, |c| < 1 and

$x = 1 + a + a^2 + ... \infty$

$y = 1 + b + b + ... \infty$

$z = 1 + c + c^2 ...... \infty$

Then x, y, z shall be in

a) A.P. b) G.P.

c) H.P. d) None of these

80. If a, b, c are in A.P., then $3^a, 3^b, 3^c$ shall be in

a) A.P. b) G.P.

c) H.P. d) None of these

81. If the ratio of H.M. and G.M. of two quantities is 12 : 13, then the ratio of the numbers is

a) 1 : 2 b) 2 : 3

c) 3 : 4 d) 9 : 4

82. If the ratio of two numbers be 9 : 1, then the ratio of geometric and harmonic means between them will be

a) 1 : 9 b) 5 : 3

c) 3 : 5 d) 2 : 5

83. If a, b, c are in A.P., then $10^{ax+10}, 10^{bx+10}, 10^{cx+10}$ will be

a) A.P.

b) G.P. only when x > 0

c) G.P. for all values of x

d) G.P. for x < 0

84. If $a^x = b^y = c^2$ and a, b, c are in G.P., then x, y, z are in

a) A.P. b) G.P.

c) H.P. d) None of these

85. If a, b, c are in A.P., then $2^{ax+1}, 2^{bx+1}, 2^{cx+1}$, $x \neq 0$ are in

a) A.P. b) G.P. only when $x > 0$

c) G.P. if $x < 0$ d) G.P. for all $x \neq 0$

15.4 Arithmetico Geometric Progression (A.G.P.)

86. The sum of the series $1 + 2x + 3x^2 + 4x^3 + ...+ ......$ upto n terms is

a) $\frac{1-(n+1)x^n + nx^{n+1}}{(1-x)^2}$

b) $\frac{1-x^n}{1-x}$

c) x^{n+1}

d) $\frac{x^{n+1}}{(1-x)^2}$

87. The sum of the numbers

$1 + 2.2 + 3.2^2 + 4.2^3 ... + 100.2^{99}$ is

a) 99×2^{100} b) $1 + 99 \times 2^{100}$

c) $99 \times 2^{99} - 1$ d) none of these

88. The sum of $1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + ...$ upto n terms is

a) $\frac{25}{16} - \frac{4n+5}{16 \times 5^{n-1}}$ b) $\frac{3}{4} - \frac{2n+5}{16 \times 5^{n+1}}$

c) $\frac{3}{7} - \frac{3n+5}{16 \times 5^{n-1}}$ d) $\frac{1}{2} - \frac{5n+1}{3 \times 5^{n+2}}$

15.5 Special series, Exponential series, Logarithmic series

89. $\frac{1^3 + 2^3 + 3^3 + ...+ 12^3}{1^2 + 2^2 + 3^2 + ...+ 12^2}$ is equal to

a) $\frac{243}{25}$ b) $\frac{234}{25}$

c) $\frac{263}{27}$ d) $\frac{263}{25}$

90. $\sum_{r=1}^{n} r(r+3) =$

a) $\frac{n(n+1)(n+5)}{3}$ b) $\frac{n(n+3)(n+5)}{3}$

c) $\frac{n(n+1)(n+7)}{3}$ d) $\frac{n(n+1)(n+2)}{3}$

91. The sum of the series

$\frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + ...\ \infty$ is

a) $\log 2$ b) e^{-1}

c) $\sqrt{e}$ d) $\log 3$

92. $1.3 + 5.7 + 9.11 + ...$ upto n terms is equal to

a) $\frac{n(n^2-7)}{3}$ b) $\frac{n(16n-7)}{3}$

c) $\frac{n(16n^2+7)}{4}$ d) $\frac{n(16n^2-7)}{3}$

93. If S_1, S_2 and S_3 are the sum of the sum of first n natural numbers, their squares and their cubes respectively, then $S_3(1 + 8S_1) =$

a) $9S_2$ b) $9S_1$

c) $9S_2^3$ d) $9S_2^2$

94. The sum of the series from $n = 1$ to $n = \infty$ whose n^{th} term is $\frac{1}{(n+1)!}$ is

a) $e - 1$ b) $e - 2$

c) e d) 0

95. The sum of n terms of the sequence whose r^{th} term is $\frac{1^3 + 2^3 + ... + r^3}{(r+1)^2}$ is

a) $\frac{n(n+1)(2n+1)}{24}$ b) $\frac{n(n+1)(2n+1)}{12}$

c) $\frac{n(n+2)(2n+3)}{24}$ d) $\frac{n(n+2)(2n+3)}{12}$

96. The sum of $1^3 + 3^3 + 5^3 + ... + 21^3$ is

a) 26191 b) 29161

c) 21619 d) 29611

97. The sum to n terms of the sequence whose r^{th} term is $\frac{1+2+3+....+r}{r}$ is

a) $\frac{n(n+3)}{2}$ b) $\frac{n(n+5)}{4}$

c) $\frac{n(n+5)}{2}$ d) $\frac{n(n+3)}{4}$

98. $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+....}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+......}=$

a) $\frac{e^2-1}{e^2+1}$ b) $\frac{e^2+1}{e^2-1}$

c) $\frac{e-1}{e+1}$ d) $\frac{e+1}{e-1}$

99. The sum of 24 terms of the following series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+......$ is

a) 300 b) $300\sqrt{2}$

c) $200\sqrt{2}$ d) 200

100. n^{th} term of the series 2 + 4 + 7+ 11 + ... will be

a) $\frac{n^2+n+1}{2}$ b) $n^2 + n + 2$

c) $\frac{n^2+n+2}{2}$ d) $\frac{n^2+2n+2}{2}$

101. The sum of i – 2 – 3i + 4 + ... upto 100 terms, where $i=\sqrt{-1}$ is

a) $50(1 – i)$ b) 25i

c) $25(1+i)$ d) $100(1 – i)$

102. $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+......+\frac{1}{n(n+1)}$ equals

a) $\frac{n+1}{n}$ b) $\frac{n(n+1)}{6}$

c) $\frac{n}{n+1}$ d) $\frac{n^2}{n+1}$

103. $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+......$ equals

a) log 2 b) log e

c) e d) e^{-1}

104. The sum of the series $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+.......$ will be

a) e^{-1} b) e^{-2}

c) $2e^{-1}$ d) $2e^{-2}$

105. Sum of the squares of first n natural numbers exceeds their sum by 330, then n =

a) 8 b) 10

c) 15 d) 20

106. Sum of the series $\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+.......$ to n terms is

a) $n-\frac{1}{2}(3^n-1)$ b) $n+\frac{1}{2}(3^n-1)$

c) $n+\frac{1}{2}(1-3^n)$ d) $n+\frac{1}{2}(3^{-n}-1)$

107. $11^3 + 12^3 + ... + 20^3$

a) Is divisible by 3

b) Is an odd integer, divisible by 5

c) Is an even integer, which is not divisible by 5

d) Is an odd integer, which is not divisible by 5

108. The n^{th} term of the series

$\frac{2}{1!}+\frac{7}{2!}+\frac{15}{3!}+\frac{26}{4!}+.....$ is

a) $\frac{n(3n-1)}{2(n)!}$ b) $\frac{n(3n+1)}{2(n)!}$

c) $\frac{n}{2}\frac{3n}{(n)!}$ d) $\frac{2n!}{n(3n+1)}$

109. n^{th} term of the series

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.....$ will be

a) $n^2 + 2n + 1$ b) $\frac{n^2+2n+1}{8}$

c) $\frac{n^2+2n+1}{4}$ d) $\frac{n^2-2n+1}{4}$

Competitive Thinking

15.1 Arithmetic Progression (A. P.) and Arithmetic Mean (A.M.)

1. If the p^{th} term of an A.P. be q and q^{th} term be p, then its r^{th} term will be

a) $p+q+r$ b) $p+q-r$

c) $p+r-q$ d) $p-q-r$

2. If $\tan n\theta = \tan m\theta$, then the different values of θ will be in

a) A.P. b) G.P.

c) H.P. d) None of these

3 . If m^{th} terms of the series 63 + 65 + 67 + 69 + ... and 3 + 10 + 17 + 24 + ... be equal, then m =

a) 11 b) 12

c) 13 d) 15

4. The 9^{th} term of the series

$27+9+5\frac{2}{5}+3\frac{6}{7}+$....... will be

a) $1\frac{10}{17}$ b) $\frac{10}{17}$

c) $\frac{16}{27}$ d) $\frac{17}{27}$

5. If a, b, c are in A.P., then $\frac{(a-c)^2}{(b^2-ac)}=$

a) 1 b) 2

c) 3 d) 4

6. If $\log_3 2$, $\log_3(2x-5)$ and $\log_3\left(2^x-\frac{7}{2}\right)$ are in A.P., then x is equal to

a) $1, \frac{1}{2}$ b) $1, \frac{1}{3}$

c) $1, \frac{3}{2}$ d) None of these

7. If n^{th} term of two A.P.'s are 3n + 8 and 7n + 15, then the ratio of their 12^{th} term will be

a) $\frac{4}{9}$ b) $\frac{7}{16}$

c) $\frac{3}{7}$ d) $\frac{8}{15}$

8. If p times the pth term of an A.P. is equal to q times the q^{th} term of an A.P., then $(p+q)^{th}$ term is

a) 0 b) 1

c) 2 d) 3

9. The sum of n terms of two arithmetic series are in the ratio 2n + 3 : 6n + 5, then the ratio of their 13^{th} term is

a) 53 : 155 b) 27 : 77

c) 29 : 83 d) 31 : 89

10. Let t_r be the r^{th} term of an A.P. for r = 1, 2, 3, ...

If for some positive integers m, n we have $t_m=\frac{1}{n}$ and $t_n=\frac{1}{m}$ then t_{mn} equals

a) $\frac{1}{mn}$ b) $\frac{1}{m}+\frac{1}{n}$

c) 1 d) 0

11. If the sum of the series 2 + 5 + 8 + 11 + ... is 60100, then the number of terms is

a) 100 b) 200

c) 150 d) 250

12. The sum of all natural numbers between 1 and 100 which are multiples of 3 is

a) 1680 b) 1683

c) 1681 d) 1682

13. If the sum of first n terms of an A.P. be equal to the sum of its first m terms, $(m \neq n)$, then the sum of its first (m + n) terms will be

a) 0 b) n

c) m d) m + n

14. If a_1, a_2, a_3 ..... a_n are in A.P., where $a_i > 0$ for all i, then the value of

$$\frac{1}{\sqrt{a_1}+\sqrt{a_n}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+.....+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=$$

a) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ b) $\frac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$

c) $\frac{n-1}{\sqrt{a_1}-\sqrt{a_n}}$ d) $\frac{n+1}{\sqrt{a_1}-\sqrt{a_n}}$

15. If the sum of n terms of an A.P. is $nA + n^2B$, where A, B are constants, then its common difference will be

a) A – B b) A + B
c) 2A d) 2B

16. If a, b, c, d, e are in A.P., then the value of a + b + 4c – 4d + e in terms of a, if possible is

a) 4a b) 2a
c) 3 d) None of these

17. The interior angles of a polygon are hi A.P. If the smallest angle be 120^0 and the common difference be 5^0, then the number of sides is

a) 8 b) 10
c) 9 d) 6

18. If a_1, a_2 .... a_n are in A.P., with common difference, d, then, the sum of the following series is

sin d(cosec a_1.cosec a_2 + cosec a_2. cosec a_3 + ... + cosec a_{n-1} cosec a_n)

a) sec a_1 – sec a_n
b) cot a_1 – cot a_n
c) tan a_1 – tan a_n
d) cosec a_1 – cosec a_n

19. If $a_1, a_2, ..., a_{n+1}$ are in A.P., then

$$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+.....+\frac{1}{a_na_{n+1}} \text{ is}$$

a) $\frac{n-1}{a_1a_{n+1}}$ b) $\frac{1}{a_1a_{n+1}}$
c) $\frac{n+1}{a_1a_{n+1}}$ d) $\frac{n}{a_1a_{n+1}}$

20. If sum of n terms of an A.P. is $3n^2 + 5n$ and T_m = 1 64 then m =

a) 26 b) 27
c) 28 d) – 26

21. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then

a) x = y = z b) x = y = – z
c) x = 1, y = 2, z = 3 d) x = 2, y = 4, z = 6

22. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are m A.P., then their common difference will be

a) ± 1 b) ± 2
c) ± 3 d) ± 4

23. Let a_1, a_2, a_3, ... be terms of an A.P. If

$$\frac{a_1+a_2+.....a_p}{a_1+a_2+....+a_q}=\frac{p^2}{q^2},\ p \neq q, \text{ then } \frac{a_6}{a_{21}} \text{ equals}$$

a) $\frac{41}{11}$ b) $\frac{7}{2}$
c) $\frac{2}{7}$ d) $\frac{11}{41}$

24. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

a) 18 months b) 19 months
c) 20 months d) 21 months

25. Let S_1, S_2,...., S_{101} be consecutive terms of an A.P. If $\frac{1}{S_1S_2}+\frac{1}{S_2S_3}+....+\frac{1}{S_{100}S_{101}}=\frac{1}{6}$ and $S_1 + S_{101} = 50$, then $|S_1 - S_{101}|$ is equal to

a) 10 b) 20
c) 30 d) 40

26. If $a_1, a_2, a_3, ..., a_n$ are in A.P. and $a_1 = 0$, then the value of

$$\left(\frac{a_3}{a_2}+\frac{a_4}{a_3}+\frac{a_n}{a_{n-1}}\right)-a_2\left(\frac{1}{a_2}+\frac{1}{a_3}+.....+\frac{1}{a_{n-2}}\right)$$

is equal to

a) $(n-2)+\frac{1}{(n-2)}$ b) $\frac{1}{(n-2)}$
c) n – 2 d) n – 1

27. The first four terms of an A.P. are a, 9, 3a – b, 3a + b. The 2011th term of the A.P. is

a) 2015 b) 4025
c) 5030 d) 8045

28. The sequence $\log a$, $\log\frac{a^2}{b}$, $\log\frac{a^3}{b^2}$,...... is

a) A.G.P.
b) an A.P.
c) a H.P.
d) both a G.P. and a H.P.

29. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an A.P. with common difference – 2, then the time taken by him to count all notes is

a) 24 minutes b) 34 minutes
c) 125 minutes d) 135 minutes

30. The sum of all two digit natural numbers which leave a remainder 5 when they are divided by 7 is equal to

a) 715 b) 702
c) 615 d) 602

31. An A.P. consists of 23 terms. If the sum of the three terms in the middle is 141 and the sum of the last three terms is 261, then the first term is

a) 6 b) 5
c) 4 d) 3

32. If $S_1 = a_2 + a_4 + a_6 + ...$ upto 100 terms and $S_2 = a_1 + a_3 + a_5 + ...$ upto 100 terms of a certain A.P., then its common difference is

a) $S_1 - S_2$ b) $S_2 - S_1$
c) $\frac{S_1 - S_2}{2}$ d) None of these

33. If 100 times the 100^{th} term of an A.P. with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is

a) – 150
b) 150 times its 50^{th} term
c) 150
d) zero

15.2 Geometric Progression (G.P.) and Geometric Mean (G.M.)

34. If the 4^{th}, 7^{th} and 10^{th} terms of a G.P. be a, b, c respectively, then the relation between a, b, c is

a) $b = \frac{a+c}{2}$ b) $a^2 = bc$
c) $b^2 = ac$ d) $c^2 = ab$

35. If x, 2x + 2, 3x + 3 are in G.P., then the fourth term is

a) 27 b) – 27
c) 13.5 d) – 13.5

36. If x, y, z are in G.P. and $a^x = r^y = c^z$, then

a) $\log_a c = \log_b a$ b) $\log_b a = \log_c b$
c) $\log_c b = \log_a c$ d) ab = bc

37. If the 5^{th} term of a G.P. is $\frac{1}{3}$ and 9^{th} term is $\frac{16}{243}$, then the 4^{th} term will be

a) $\frac{3}{4}$ b) $\frac{1}{2}$
c) $\frac{1}{3}$ d) $\frac{2}{5}$

38. If every term of a G.P. with positive term is the sum of its two previous terms, then the common ratio of the series is

a) 1 b) $\frac{2}{\sqrt{5}}$
c) $\frac{\sqrt{5}-1}{2}$ d) $\frac{\sqrt{5}+1}{2}$

39. The sum of few terms of any ratio series is 728, if common ratio is 3 and last term is 486, then first term of series will be

a) 2 b) 1
c) 3 d) 4

40. If five G.M.'s are inserted between 486 and $\frac{2}{3}$, then fourth G.M. will be

a) 4 b) 6
c) 12 d) – 6

41. Let (n > 1) be a positive integer, then the largest integer m such that $(n^m + 1)$ divides $(1 + n + n^2 + ... + n^{127})$, is

a) 32 b) 63
c) 64 d) 127

42. The value of $0.2\dot{3}\dot{4}$ is

a) $\frac{232}{990}$ b) $\frac{232}{9990}$
c) $\frac{232}{909}$ d) $\frac{232}{999}$

43. If $3 + 3\alpha + 3\alpha^2 + ...\ \infty = \frac{45}{8}$, then the value of α will be

a) $\frac{15}{23}$ b) $\frac{7}{15}$
c) $\frac{7}{8}$ d) $\frac{15}{7}$

44. $x = 1 + a + a^2 + \ldots\ldots\ \infty\ (a < 1)$

$y = 1 + b + b^2\ \ldots\ldots\ \infty\ (b < 1)$

Then, the value of $1 + ab + a^2b^2 + \ldots\ldots\infty$ is

a) $\frac{xy}{x+y-1}$ b) $\frac{xy}{x+y+1}$

c) $\frac{xy}{x-y-1}$ d) $\frac{xy}{x-y+1}$

45. $0.4\dot{2}\dot{3} =$

a) $\frac{419}{990}$ b) $\frac{419}{999}$

c) $\frac{417}{990}$ d) $\frac{417}{999}$

46. If $y = x - x^2 + x^3 - x^4 + \ldots\ldots \infty$, then value of x will be

a) $y+\frac{1}{y}$ b) $\frac{y}{1+y}$

c) $y-\frac{1}{y}$ d) $\frac{y}{1-y}$

47. The sum of infinite terms of a G.P. is x and on squaring the each term of it, the sum will be y, then the common ratio of this series is

a) $\frac{x^2-y^2}{x^2+y^2}$ b) $\frac{x^2+y^2}{x^2-y^2}$

c) $\frac{x^2-y}{x^2+y}$ d) $\frac{x^2+y}{x^2-y}$

48. If S is the sum to infinity of a G.P., whose first term is a, then the sum of the first n terms is

a) $S\left(1-\frac{a}{S}\right)^n$ b) $S\left[1-\left(1-\frac{a}{S}\right)^n\right]$

c) $a\left[1-\left(1-\frac{a}{S}\right)^n\right]$ d) $\left(1-\frac{a}{S}\right)^n$

49. 0.14189189189., can be expressed as a rational number

a) $\frac{7}{3700}$ b) $\frac{7}{50}$

c) $\frac{525}{111}$ d) $\frac{21}{148}$

50. The sum of the series

$5.05 + 1.212 + 0.29088 + \ldots.\ \infty$ is

a) 6.93378 b) 6.87342

c) 6.74384 d) 6.64474

51. The sum of the series 3 + 33 + 333 + ... + n terms is

a) $\frac{1}{27}(10^{n+1}+9n-28)$

b) $\frac{1}{27}(10^{n+1}-9n-10)$

c) $\frac{1}{27}(10^{n+1}+10n-9)$

d) 27

52. The sum of a G.P. with common ratio 3 is 364, and last term is 243, then the number of terms is

a) 6 b) 5

c) 4 d) 10

53. The G.M. of roots of the equation $x^2 - 18x + 9 = 0$ is

a) 3 b) 4

c) 2 d) 1

54. Consider an infinite G.P. with first term a and common ratio r, its sum is 4 and the second term is 3/4, then

a) $a=\frac{7}{4}, r=\frac{3}{7}$ b) $a=\frac{3}{2}, r=\frac{1}{2}$

c) $a=2, r=\frac{3}{8}$ d) $a=3, r=\frac{1}{4}$

55. The sum of infinite terms of the geometric progression $\frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{2-\sqrt{2}}, \frac{1}{2}\ldots\ldots$ is

a) $\sqrt{2}\,(\sqrt{2}+1)^2$ b) $(\sqrt{2}+1)^2$

c) $5\sqrt{2}$ d) $3\sqrt{2}+\sqrt{5}$

56. If the sum of the series

$1+\frac{2}{x}+\frac{4}{x^2}+\frac{8}{x^3}+\ldots.\infty$ is a finite number, then

a) $x > 2$ b) $x > -2$

c) $x > \frac{1}{2}$ d) $x < \frac{1}{2}$

57. If $1 + \cos\alpha + \cos^2\alpha + ... \infty = 2 - \sqrt{2}$, then α, $(0 < \alpha < \pi)$ is

a) $\pi/8$ b) $\pi/6$

c) $\pi/4$ d) $3\pi/4$

58. If $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P., then x, y, z will be in

a) A.P. b) G.P.

c) H.P. d) None of these

59. The 5^{th} term of the series $\frac{10}{9}, \frac{1}{3}, \sqrt{\frac{20}{3}}, \frac{2}{3}, ...$ is

a) $\frac{1}{3}$ b) 1

c) $\frac{2}{5}$ d) $\sqrt{\frac{2}{3}}$

60. The value of $i - i^2 + i^3 - i^4 + ... - i^{100}$ is equal to

a) i b) – i

c) 1 – i d) 0

61. If $a_1, a_2, ..., a_{50}$ are in G.P., then

$$\frac{a_1 - a_3 + a_5 ...... + a_{49}}{a_2 - a_4 + a_6 - ...... + a_{50}}$$

a) 0 b) 1

c) $\frac{a_1}{a_2}$ d) $\frac{a_{25}}{a_{24}}$

62. The product $(32)(32)^{1/6}(32)^{1/36}$ .... to ∞ is

a) 16 b) 32

c) 64 d) 0

63. If $1 + \sin x + \sin^2 x + ...$ upto $\infty = 4 + 2\sqrt{3}$, $0 < x < \pi$ and $x \neq \frac{\pi}{2}$, then x =

a) $\frac{\pi}{3}, \frac{2\pi}{3}$ b) $\frac{\pi}{6}, \frac{\pi}{3}$

c) $\frac{\pi}{3}, \frac{5\pi}{6}$ d) $\frac{2\pi}{3}, \frac{\pi}{6}$

64. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

a) – 12 b) 12

c) 4 d) – 4

65. Three numbers whose sum is 15 are in A.P. If they are added by 1, 4 and 19 respectively they are in G.P. The numbers are

a) 2,5,8

b) 26, 5, – 16

c) 2, 5, 8 and 26, 5, – 16

d) None of these

66. If sum of an infinite geometric series is $\frac{4}{3}$ and its 1^{st} term is $\frac{3}{4}$, then its common ratio is

a) $\frac{7}{16}$ b) $\frac{9}{16}$

c) $\frac{1}{9}$ d) $\frac{7}{9}$

67. In a G.P., $t_2 + 1_5 = 216$ and $t_4 : t_6 = 1 : 4$ and all terms are integers, then its first term is

a) 16 b) 14

c) 12 d) None of these

68. Consider an infinite geometric series with first term 'a' and common ratio 'r'. If the sum is 4 and the second term is $\frac{3}{4}$, then

a) $a = 2, r = \frac{3}{8}$ b) $a = \frac{4}{7}, r = \frac{3}{7}$

c) $a = \frac{3}{2}, r = \frac{1}{2}$ d) $a = 3, r = \frac{1}{4}$

69. If a, b and c are positive numbers in a G.P., then the roots of the quadratic equation $(\log_e a)x^2 - (2\log_e b)x + (\log_e c) = 0$ are

a) -1 and $\frac{\log_e c}{\log_e a}$

b) 1 and $\frac{\log_e c}{\log_e a}$

c) 1 and $\log_a c$

d) – 1 and $\log_e a$

15.3 Harmonic Progression, Harmonic Mean and Relation between A.M., G.M. and H.M.

70. The fifth term of the H.P. $2, 2\frac{1}{2}, 3\frac{1}{3}, \ldots\ldots$ will be

a) $5\frac{1}{5}$ b) $3\frac{1}{5}$

c) $\frac{1}{10}$ d) 10

71. If $a_1, a_2, a_3, \ldots\ldots\ a_n$ are in H.P., then $a_1a_2 + a_2a_3 + \ldots\ldots\ a_{n-1}a_n$ will be equal to

a) a_1a_n b) na_1a_n

c) $(n-1)a_1a_n$ d) $n-1$

72. If 5^{th} term of a H.P. is $\frac{1}{45}$ and 11^{th} term is $\frac{1}{69}$, then its 16^{th} term will be

a) $\frac{1}{89}$ b) $\frac{5}{21}$

c) $\frac{1}{80}$ d) $\frac{1}{79}$

73. H.M between the roots of the equation $x^2 - 10x + 11 = 0$ is

a) $\frac{1}{5}$ b) $\frac{5}{11}$

c) $\frac{21}{20}$ d) $\frac{11}{5}$

74. The sixth H.M. between 3 and $\frac{6}{13}$ is

a) $\frac{63}{120}$ b) $\frac{63}{12}$

c) $\frac{126}{105}$ d) $\frac{120}{63}$

75. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the harmonic mean between a and b, then the value of n is

a) 1 b) – 1

c) 0 d) 2

76. If the A.M. and G.M. of roots of a quadratic equations are 8 and 5 respectively, then the quadratic equation will be

a) $x^2 - 16x - 25 = 0$ b) $x^2 - 8x + 5 = 0$

c) $x^2 - 16x + 25 = 0$ d) $x^2 + 16x - 25 = 0$

77. If a, b, c are in H.P., then which one of the following is true?

a) $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{b}$ b) $\frac{ac}{a+c} = b$

c) $\frac{b+a}{b-a} + \frac{b+c}{b-c} = 1$ d) None of these

78. The product of n positive numbers is unity. Their sum is

a) A positive integer b) Equal to $n + \frac{1}{n}$

c) Divisible by n d) Never less than n

79. If H is the harmonic mean between p and q, then the value of $\frac{H}{p} + \frac{H}{q}$ is

a) 2 b) $\frac{pq}{p+q}$

c) $\frac{p+q}{pq}$ d) None of these

80. If three numbers be in G.P., then their logarithms will be in

a) A.P. b) G.P.

c) H.P. d) None of these

81. If x, 1, z are in A.P. and x, 2, z are in G.P., then x, 4, z will be in

a) A.P b) G.P.

c) H.P. d) None of these

82. If A_1, A_2 are the two A.M.'s between two numbers a and b and G_1, G_2 be two G.M.'s between same two numbers, then $\frac{A_1 + A_2}{G_1.G_2} =$

a) $\frac{a+b}{ab}$ b) $\frac{a+b}{2ab}$

c) $\frac{2ab}{a+b}$ d) $\frac{ab}{a+b}$

83. If the A.M. is twice the G.M. of the numbers a and b, then a : b will be

a) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$ b) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

c) $\frac{\sqrt{3}-2}{\sqrt{3}+2}$ d) $\frac{\sqrt{3}+2}{\sqrt{3}-2}$

84. x + y + z = 15 if 9, x, y, z, a are in A.P.; while $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$ if 9, x, y, z, a are in H.P., then the value of a will be

a) 1 b) 2

c) 3 d) 9

85. If $\frac{x+y}{2}$, y, $\frac{y+z}{2}$ are in H.P., then x, y, z are in

a) A.P. b) G.P.

c) H.P. d) None of these

86. If the ratio of H.M. and G.M. between two numbers a and b is 4 : 5, then the ratio of the two numbers will be

a) 1 : 2 b) 1 : 4

c) 4 : 1 d) (b) and (c)

87. If A is the A.M. of the roots of the equation $x^2-2ax+b=0$ and G is the G.M. of the roots of the equation $x^2-2bx+a^2=0$, then

a) A > G b) $A \neq G$

c) A = G d) A < G

88. If the altitudes of a triangle are in A.P., then the sides of the triangle are in

a) A.P. b) H.P.

c) G.P. d) A.G.P.

89. A boy goes to school from his home at a speed of x km/hour and comes back at a speed of y km/hour, then the average speed is given by

a) A.M. b) G.M.

c) H.M. d) None of these

90. If x > 1, y > 1, z > 1 are in G.P., then

$\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in

a) A.P. b) H.P.

c) G.P. d) None of these

91. If x, y, z are in H.P., then the value of expression log(x + z) + log(x – 2y + z) will be

a) log(x – z) b) 21og(x – z)

c) 31og (x – z) d) 41og(x – z)

92. If b^2 ,a^2, c^2 are in A.P., then a + b, b + c, c + a will be in

a) A.P. b) G.P.

c) H.P. d) None of these

93. If H_1, H_2 are two harmonic means between two positive numbers a and b($a \neq b$), A and G are the arithmetic and geometric means between a and b, then

a) $\frac{2A}{G}$ b) $\frac{A}{2G^2}$

c) $\frac{A}{G^2}$ d) $\frac{2A}{G^2}$

94. If a, b, c are in G.P. and x, y are the arithmetic means between a, b and b, c respectively, then $\frac{a}{x}+\frac{c}{y}$ is equal to

a) 0 b) 1

c) 2 d) i

95. If A.M. and G.M. of x an.dy are in the ratio p : q, then x: y is

a) $p-\sqrt{p^2+q^2} : p+\sqrt{p^2+q^2}$

b) $p+\sqrt{p^2-q^2} : p-\sqrt{p^2-q^2}$

c) p + q

d) $p+\sqrt{p^2+q^2} : p-\sqrt{p^2+q^2}$

96. If p, q, r are in G.P. and $\tan^{-1} p$, $\tan^{-1} q$, $\tan^{-1} r$ are in A.P., then p, q, r satisfies the relation

a) p = q = r b) $p \neq q \neq r$

c) p + q = r d) None of these

97. Let a be a positive number such that the arithmetic mean of a and 2 exceeds their geometric mean by 1. Then, the value of a is

a) 3 b) 5

c) 9 d) 8

98. If A_1 A_2 ; G_1, G_2 and H_1, H_2 be two A.M.s, G.M.s and H.M.s. between two numbers respectively, then $\frac{G_1G_2}{H_1H_2}\times\frac{H_1+H_2}{A_1+A_2}=$

a) 1 b) 0

c) 2 d) 3

99. If a, b, c are in G.P., then $\log_a x$., $\log_b x$, $\log_c x$ are in
a) A.P. b) G.P.
c) H.P. d) None of these

100. If (y – x), 2(y – z) and (y – z) are in H.P., then x – a, y – a, z – a are in
a) A.P. b) G.P.
c) H.P. d) None of these

101. If a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then
a) $a \neq b \neq c$
b) $a^2 = b^2 = \frac{c^2}{2}$
c) a, b, c are in G.P.
d) $\frac{-a}{2}$, b, c are in G.P.

102. If $a_1, a_2, ..., a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + ... + a_{n-1} + 2a_n$ is
a) $n(2c)^{\frac{1}{n}}$ b) $(n+1)c^{\frac{1}{n}}$
c) $2nc^{\frac{1}{n}}$ d) $(n+1)(2c)^{\frac{1}{n}}$

103. If arithmetic mean of two positive numbers is A, their geometric mean is G and harmonic mean is H, then H is equal to
a) $\frac{G^2}{A}$ b) $\frac{G}{A^2}$
c) $\frac{A^2}{G^2}$ d) $\frac{A}{G^2}$

104. The difference between two numbers is 48 and the difference between their arithmetic mean and their geometric mean is 18. Then, the greater of two numbers is
a) 96 b) 60
c) 54 d) 49

105. If three real numbers a, b, c are in harmonic progression, then which of the following is true?
a) $\frac{1}{a}, b, \frac{1}{c}$ are in A.P.
b) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in H.P.
c) ab, bc, ca are in H.P.
d) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in H.P.

106. The geometric mean of $1, 2, 2^2, ..., 2n$ is
a) $2^{\frac{n}{2}}$ b) $2^{\frac{(n+1)}{2}}$
c) $2^{\frac{n(n+1)}{2}}$ d) $2^{\frac{(n-1)}{2}}$

107. The value of n for which $\frac{x^{n+1} + y^{n+1}}{x^n + y^n}$ is the geometric mean of x and y is
a) $n = -\frac{1}{2}$ b) $n = \frac{1}{2}$
c) $n = 1$ d) $n = -1$

108. G.M. and H.M. of two numbers are 10 and 8 respectively. The numbers are
a) 5, 20 b) 4, 25
c) 2, 50 d) 1, 100

109. The value of $1^2 - 2^2 + 3^2 - 4^2 + ... + 11^2$ is equal to
a) 55 b) 66
c) 77 d) 88

110. Let f(x) = x + 1/2. Then, the number of real values of x for which the three unequal terms f(x), f(2x), f(4x) are in H.P. is
a) 1 b) 0
c) 3 d) 2

15.4 Arithmetico Geometric Progression (A.G.P.)

111. $1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + ...\infty$ is equal to
a) 3 b) 6
c) 9 d) 12

112. The sum of infinite terms of the following series
$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...$ will be
a) $\frac{3}{16}$ b) $\frac{35}{8}$
c) $\frac{35}{4}$ d) $\frac{35}{16}$

113. The sum to infinity of the series ,
$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ is
a) 2 b) 3
c) 4 d) 6

114. If $t_n = \frac{1}{4}(n+2)(n+3)$ for n = 1, 2, 3, ..., then

$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + ... + \frac{1}{t_{2003}} =$

a) $\frac{4006}{3006}$ b) $\frac{4003}{3007}$

c) $\frac{4006}{3008}$ d) $\frac{4006}{3009}$

115. If $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} ......\infty = \frac{\pi^4}{90}$, then the value of

$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{3^4} ......\infty$ is

a) $\frac{\pi^4}{96}$ b) $\frac{\pi^4}{45}$

c) $\frac{89}{90}\pi^4$ d) None of these

15.5 Special series, Exponential series, Logarithmic series

116. The sum of the series 1 + (1+ 2) + (1 + 2 + 3) + ... upto n terms, will be

a) $n^2 - 2n + 6$ b) $\frac{n(n+1)(2n-1)}{6}$

c) $n^2 + 2n + 6$ d) $\frac{n(n+1)(n+2)}{6}$

117. The sum 1(1!) + 2(2!) + 3(3!) + ...+ n(n!) equals

a) 3(n!) + n – 3 b) (n + 1)! – (n – 1)!

c) (n + 1)! – !! d) 2(n!) – 2n – 1

118. The sum of the series

3.6 + 4.7 + 5.8 + ... upto (n – 2) terms

a) $n^3 + n^2 + n + 2$

b) $\frac{1}{6}(2n^3 + 12n^2 + 10n - 84)$

c) $n^3 + n^2 + n$

d) $2n^3 + 12n^2 + 10n$

119. The sum of the series

1.2.3 + 2.3.4 + 3.4.5 + ... to n terms is

a) n(n + l)(n + 2)

b) (n + l)(n + 2)(n + 3)

c) $\frac{1}{4}$ n(n + l)(n + 2)(n + 3)

D) $\frac{1}{4}$ (n + l)(n + 2)(n + 3)

120. The sum of $1^3 + 2^3 + 3^3 + 4^3 + ... + 15^3$, is

a) 22,000 b) 10,000

c) 14,400 d) 15,000

121. The n^{th} term of series

$\frac{1}{1} + \frac{1+2}{2} + \frac{1+2+3}{3} + ...$ will be

a) $\frac{n+1}{2}$ b) $\frac{n-1}{2}$

c) $\frac{n^2+1}{2}$ d) $\frac{n^2-1}{2}$

122. The sum of the series $1.3^2 + 2.5^2 + 3.7^2 + ...$ upto 20 terms is

a) 188090 b) 189080

c) 199080 d) 188809

123. Sum of the n terms of the series

$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + ...$ is

a) $\frac{2n}{n+1}$ b) $\frac{4n}{n+1}$

c) $\frac{6n}{n+1}$ d) $\frac{9n}{n+1}$

124. 1 + 3 + 7 + 15 + 31 + ... ton terms =

a) $2^{n+1} - n$ b) $2^{n+1} - n - 2$

c) $2^n - n - 2$ d) $2^n - n$

125. 2 + 4 + 7 + 11 + 16 + ... to n terms =

a) $\frac{1}{6}(n^2+3n+8)$ b) $\frac{n}{6}(n^2+3n+8)$

c) $\frac{1}{6}(n^2-3n+8)$ d) $\frac{n}{6}(n^2-3n+8)$

126. Sum of n terms of series

12 +16 + 24 + 40 +.....will be

a) $2(2^n - 1) + 8n$

b) $2(2^n - 1) + 6n$

c) $3(2^n - 1) + 8n$

d) $4(2^n - 1) + 8n$

127. 99^{th} term of the series 2 + 7+ 14 + 23 + 34 + ...is

a) 9998 b) 9999

c) 10000 d) 9988

128. If the sum of first n terms of an A. P. is cn^2, then the sum of squares of these n terms is

a) $\frac{n(4n^2-1)c^2}{6}$ b) $\frac{n(4n^2+1)c^2}{3}$

c) $\frac{n(4n^2-1)c^2}{3}$ d) $\frac{n(4n^2+1)c^2}{6}$

129. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1$ is equal to

a) $\frac{n(n+1)(2n+1)}{6}$ b) $\left[\frac{n(n+1)}{2}\right]^2$

c) $\frac{n(n+1)}{2}$ d) $\frac{n(n+1)(n+2)}{6}$

130. The sum of the series $(1+2)+(1+2+2^2)+(1+2+2^2+2^3)+....$ upto n terms is

a) $2^{n+2}-n-4$ b) $2(2^n-1)-n$

c) $2^{n+1}-n$ d) $2^{n+1}-1$

131. For any integer $n \geq 1$, the sum $\sum_{k=1}^{n}k\,(k+2)$ is equal to

a) $\frac{n(n+1)(n+2)}{6}$ b) $\frac{n(n+1)(2n+1)}{6}$

c) $\frac{n(n+1)(2n+7)}{6}$ d) $\frac{n(n+1)(2n+9)}{6}$

132. The sum of the first n terms of

$\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+......$ is

a) $\frac{n^2-2n}{3}$ b) $\frac{2n^2+n}{3}$

c) $\frac{n(n+2)}{3}$ d) $\frac{2n^2-n}{3}$

133. Sum of n terms of the following series $1^3+3^3+5^3+7^3+......$ is

a) $n^2(2n^2-1)$ b) $n^3(n-1)$

c) n^3+8n+4 d) $2n^4+3n^2$

Evaluation Test

1. If $a_1, a_2, a_3, ..., a_n$ are in A.P. with common difference 5 and if $a_i a_j \neq -1$ for i, j = 1, 2, ..., n, then $\tan^{-1}\left(\frac{5}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{5}{1+a_2a_3}\right)+....+\tan^{-1}\left(\frac{5}{1+a_{n-1}a_n}\right)$ is equal to

a) $\tan^{-1}\left(\frac{5}{1+a_na_{n-1}}\right)$ b) $\tan^{-1}\left(\frac{5a_1}{1+a_na_1}\right)$

c) $\tan^{-1}\left(\frac{5n-5}{1+a_na_1}\right)$ d) $\tan^{-1}\left(\frac{5n-5}{1+a_1a_{n+1}}\right)$

2. Let $a_1, a_2, a_3, ...$ be in harmonic progression with ai = 5 and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

a) 22 b) 23

c) 24 d) 25

3. If p, q, r are in A.P. and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for

a) $\left|\frac{r}{p}-7\right| \geq 4\sqrt{3}$ b) $\left|\frac{p}{r}-7\right| < 4\sqrt{3}$

c) all p and r d) no p and r

4. If a, b, c be respectively the p^{th}, q^{th} and r^{th} terms of a G.P., then $\Delta = \begin{vmatrix} \log a & \log b & \log c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

a) 1

b) 0

c) – 1

d) None of these

5. If x_1, x_2, x_3, as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

a) lie on a straight line

b) lie on an ellipse

c) lie on a circle

d) are vertices of a triangle

6. Let S_1, S_2, ... be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1sq. cm?

a) 7 b) 8

c) 5 d) 6

7. If the p^{th} term of an A.P. be $\frac{1}{q}$ and q^{th} term be $\frac{1}{p}$, then the sum of its $(pq)^{th}$ terms will be

a) $\frac{pq-1}{2}$ b) $\frac{1-pq}{2}$

c) $\frac{pq+1}{2}$ d) $-\frac{pq+1}{2}$

8. Given that a, b, c are in A.P. The determinant $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ in its simpliest form is equal to

a) $x^3 + 3ax + 7c$ b) 0

c) 15 d) $10x^2 + 5x + 2c$

9. The sum of the series $1 + \frac{1.3}{6} + \frac{1.3.5}{6.8} + ... \infty$ is

a) 1 b) 0

c) ∞ d) 4

10. A man arranges to pay off a debt of % 3600 by 40 annual installments which are in A.P. When 30 of the installments are paid he dies leaving one third of the debt unpaid. The value of the 8^{th} installment is

a) 735 b) 750

c) 765 d) None of these

Classical Thinking

1. (B)	2. (B)	3. (B)	4. (C)	5. (A)	6. (D)	7. (C)	8. (B)	9. (A)	10. (B)
11. (A)	12. (B)	13. (B)	14. (A)	15. (B)	16. (B)	17. (C)	18. (C)	19. (A)	20. (D)
21. (B)	22. (A)	23. (A)	24. (D)	25. (C)	26. (C)	27. (A)	28. (B)	29. (D)	30. (A)
31. (C)	32. (B)	33. (A)	34. (B)	35. (A)	36. (C)	37. (C)	38. (A)	39. (A)	40. (D)
41. (A)	42. (B)	43. (D)	44. (C)	45. (B)	46. (A)	47. (D)	48. (B)	49. (B)	50. (A)
51. (C)	52. (D)	53. (A)	54. (C)	55. (C)	56. (D)	57. (A)	58. (B)	59. (A)	60. (B)
61. (D)	62. (C)	63. (D)	64. (A)	65. (D)	66. (A)	67. (C)	68. (C)	69. (B)	70. (B)

Critical Thinking

1. (C)	2. (C)	3. (B)	4. (C)	5. (D)	6. (C)	7. (A)	8. (D)	9. (A)	10. (C)
11. (D)	12. (C)	13. (A)	14. (B)	15. (D)	16. (C)	17. (A)	18. (B)	19. (C)	20. (C)
21. (B)	22. (B)	23. (A)	24. (D)	25. (B)	26. (C)	27. (A)	28. (B)	29. (B)	30. (B)
31. (D)	32. (B)	33. (A)	34. (A)	35. (B)	36. (B)	37. (C)	38. (B)	39. (A)	40. (B)
41. (B)	42. (B)	43. (C)	44. (A)	45. (B)	46. (B)	47. (A)	48. (D)	49. (D)	50. (A)
51. (B)	52. (D)	53. (A)	54. (B)	55. (B)	56. (B)	57. (B)	58. (A)	59. (C)	60. (B)
61. (D)	62. (B)	63. (B)	64. (A)	65. (C)	66. (B)	67. (C)	68. (A)	69. (B)	70. (C)
71. (D)	72. (C)	73. (A)	74. (A)	75. (D)	76. (B)	77. (C)	78. (B)	79. (C)	80. (B)
81. (D)	82. (B)	83. (C)	84. (C)	85. (D)	86. (A)	87. (B)	88. (A)	89. (B)	90. (A)
91. (A)	92. (D)	93. (D)	94. (B)	95. (A)	96. (B)	97. (D)	98. (C)	99. (B)	100. (C)
101. (A)	102. (C)	103. (D)	104. (A)	105. (B)	106. (D)	107. (B)	108. (B)	109. (C)	

Competitive Thinking

1. (B)	2. (A)	3. (C)	4. (A)	5. (D)	6. (D)	7. (A)	8. (A)	9. (A)	10. (C)
11. (B)	12. (B)	13. (A)	14. (A)	15. (D)	16. (D)	17. (C)	18. (B)	19. (D)	20. (B)
21. (A)	22. (C)	23. (D)	24. (D)	25. (A)	26. (A)	27. (D)	28. (B)	29. (B)	30. (B)
31. (D)	32. (D)	33. (D)	34. (C)	35. (D)	36. (B)	37. (B)	38. (D)	39. (A)	40. (B)
41. (C)	42. (A)	43. (B)	44. (A)	45. (A)	46. (D)	47. (C)	48. (B)	49. (D)	50. (D)
51. (B)	52. (A)	53. (A)	54. (D)	55. (A)	56. (A)	57. (D)	58. (A)	59. (C)	60. (D)
61. (C)	62. (C)	63. (A)	64. (A)	65. (C)	66. (A)	67. (C)	68. (D)	69. (C)	70. (D)
71. (C)	72. (A)	73. (D)	74. (A)	75. (B)	76. (C)	77. (D)	78. (D)	79. (A)	80. (A)
81. (C)	82. (A)	83. (B)	84. (A)	85. (B)	86. (D)	87. (C)	88. (B)	89. (C)	90. (B)
91. (B)	92. (C)	93. (D)	94. (C)	95. (B)	96. (A)	97. (D)	98. (A)	99. (C)	100. (B)
101. (D)	102. (A)	103. (A)	104. (D)	105. (B)	106. (A)	107. (A)	108. (A)	109. (B)	110. (A)
111. (B)	112. (D)	113. (B)	114. (D)	115. (A)	116. (D)	117. (C)	118. (B)	119. (C)	120. (C)
121. (A)	122. (A)	123. (C)	124. (B)	125. (B)	126. (D)	127. (A)	128. (C)	129. (D)	130. (A)
131. (C)	132. (C)	133. (A)							

Answers to Evaluation Test

1. (C)	2. (D)	3. (A)	4.(B)	5. (A)	6. (B)	7. (C)	8.(B)	9. (D)	10.(C)

Answer Key

Classical Thinking

1. (B)	2. (B)	3. (B)	4. (C)	5. (A)	6. (D)	7. (C)	8. (B)	9. (A)	10. (B)
11. (A)	12. (B)	13. (B)	14. (A)	15. (B)	16. (B)	17. (C)	18. (C)	19. (A)	20. (D)
21. (B)	22. (A)	23. (A)	24. (D)	25. (C)	26. (C)	27. (A)	28. (B)	29. (D)	30. (A)
31. (C)	32. (B)	33. (A)	34. (B)	35. (A)	36. (C)	37. (C)	38. (A)	39. (A)	40. (D)
41. (A)	42. (B)	43. (D)	44. (C)	45. (B)	46. (A)	47. (D)	48. (B)	49. (B)	50. (A)
51. (C)	52. (D)	53. (A)	54. (C)	55. (C)	56. (D)	57. (A)	58. (B)	59. (A)	60. (B)
61. (D)	62. (C)	63. (D)	64. (A)	65. (D)	66. (A)	67. (C)	68. (C)	69. (B)	70. (B)

Critical Thinking

1. (C)	2. (C)	3. (B)	4. (C)	5. (D)	6. (C)	7. (A)	8. (D)	9. (A)	10. (C)
11. (D)	12. (C)	13. (A)	14. (B)	15. (D)	16. (C)	17. (A)	18. (B)	19. (C)	20. (C)
21. (B)	22. (B)	23. (A)	24. (D)	25. (B)	26. (C)	27. (A)	28. (B)	29. (B)	30. (B)
31. (D)	32. (B)	33. (A)	34. (A)	35. (B)	36. (B)	37. (C)	38. (B)	39. (A)	40. (B)
41. (B)	42. (B)	43. (C)	44. (A)	45. (B)	46. (B)	47. (A)	48. (D)	49. (D)	50. (A)
51. (B)	52. (D)	53. (A)	54. (B)	55. (B)	56. (B)	57. (B)	58. (A)	59. (C)	60. (B)
61. (D)	62. (B)	63. (B)	64. (A)	65. (C)	66. (B)	67. (C)	68. (A)	69. (B)	70. (C)
71. (D)	72. (C)	73. (A)	74. (A)	75. (D)	76. (B)	77. (C)	78. (B)	79. (C)	80. (B)
81. (D)	82. (B)	83. (C)	84. (C)	85. (D)	86. (A)	87. (B)	88. (A)	89. (B)	90. (A)
91. (A)	92. (D)	93. (D)	94. (B)	95. (A)	96. (B)	97. (D)	98. (C)	99. (B)	100. (C)
101. (A)	102. (C)	103. (D)	104. (A)	105. (B)	106. (D)	107. (B)	108. (B)	109. (C)	

Competitive Thinking

1. (B)	2. (A)	3. (C)	4. (A)	5. (D)	6. (D)	7. (A)	8. (A)	9. (A)	10. (C)
11. (B)	12. (B)	13. (A)	14. (A)	15. (D)	16. (D)	17. (C)	18. (B)	19. (D)	20. (B)
21. (A)	22. (C)	23. (D)	24. (D)	25. (A)	26. (A)	27. (D)	28. (B)	29. (B)	30. (B)
31. (D)	32. (D)	33. (D)	34. (C)	35. (D)	36. (B)	37. (B)	38. (D)	39. (A)	40. (B)
41. (C)	42. (A)	43. (B)	44. (A)	45. (A)	46. (D)	47. (C)	48. (B)	49. (D)	50. (D)
51. (B)	52. (A)	53. (A)	54. (D)	55. (A)	56. (A)	57. (D)	58. (A)	59. (C)	60. (D)
61. (C)	62. (C)	63. (A)	64. (A)	65. (C)	66. (A)	67. (C)	68. (D)	69. (C)	70. (D)
71. (C)	72. (A)	73. (D)	74. (A)	75. (B)	76. (C)	77. (D)	78. (D)	79. (A)	80. (A)
81. (C)	82. (A)	83. (B)	84. (A)	85. (B)	86. (D)	87. (C)	88. (B)	89. (C)	90. (B)
91. (B)	92. (C)	93. (D)	94. (C)	95. (B)	96. (A)	97. (D)	98. (A)	99. (C)	100. (B)
101. (D)	102. (A)	103. (A)	104. (D)	105. (B)	106. (A)	107. (A)	108. (A)	109. (B)	110. (A)
111. (B)	112. (D)	113. (B)	114. (D)	115. (A)	116. (D)	117. (C)	118. (B)	119. (C)	120. (C)
121. (A)	122. (A)	123. (C)	124. (B)	125. (B)	126. (D)	127. (A)	128. (C)	129. (D)	130. (A)
131. (C)	132. (C)	133. (A)							

Hints

Classical Thinking

1. $a = 72, d = -2$

Let n^{th} term be 40.

$\therefore$ $t_n = a + (n-1)d$

$\therefore$ $40 = 72 + (n-1)(-2)$

$\Rightarrow n = 17$

2. $a = \sqrt{3}$, $d = \sqrt{12} - \sqrt{3} = \sqrt{3}$

$\therefore$ $t_{10} = \sqrt{3} + 9\sqrt{3} = 10\sqrt{3} = \sqrt{300}$

3. $a = 3, d = 3$

Let there be n terms.

$\therefore$ $3 + (n-1)3 = 111$

$\Rightarrow n = 37$

4. $d - c = e - d$
$\Rightarrow 2d = e + c$
$\Rightarrow 2d - 2c = e - c$
$\Rightarrow 2(d - c) = e - c$

5. a, b, c are in A.P., dividing by bc we get
$\frac{a}{bc}, \frac{1}{c}, \frac{1}{b}$ are in A.P.

6. $S_n = 3(4^n - 1)$
$\therefore$ $S_{n-1} = 3(4^{n-1} - 1)$
$\therefore$ $t_n = S_n - S_{n-1} = 3(4^n - 1) - 3(4^{n-1} - 1) = 9(4^{n-1})$

7. $a = 21, d = 16 - 21 = -5$
$t_n = a + (n - 1)d$
$\therefore$ $t_{15} = 21 + (15 - 1)(-5) = 21 - 70 = -49$

8. $S_n = \frac{n}{2}[2a + (n - 1)d]$
$\therefore$ $S_{16} = \frac{16}{2}[2(4) + (15)d]$
$\therefore$ $784 = 8(8 + 15d)$
$\therefore$ $8 + 15d = \frac{784}{8}$
$\therefore$ $15d = 90$
$\therefore$ $d = 6$

9. $t_7 = 40 \Rightarrow a + 6d = 40$
$S_{13} = \frac{13}{2}[2a + (13 - 1)d] = 13(a + 6d) = 520$

10. $t_4 = a + 3d = 4$ and
$S_7 = \frac{7}{2}[2a + (7 - 1)d] = \frac{7}{2}[2a + 6d]$
$= 7(a + 3d)$
$= 7(4) = 28$

11. Required sum $= 1 + 3 + 5 + \ldots$ upto n terms
$= \frac{n}{2}[2 \times 1 + (n - 1)2]$
$= n^2$

12. $S_5 = \frac{1}{4}(S_{10} - S_5) \Rightarrow 5S_5 = S_{10}$
$\therefore$ $5 \times \frac{5}{2}(2 \times 2 + 4d) = \frac{10}{2}(2 \times 2 + 9d)$
$\Rightarrow d = -6$

13. The terms of given sequence are in A.P. with $a = 1$, $d = 5$ and $S_n = 148$
$\therefore$ $\frac{n}{2}[2a + (n - 1)d] = 148 \Rightarrow n = 8$
Now, $x = n^{th}$ term $\Rightarrow x = a + (n - 1)d = 36$

14. $S_n = 3n^2 - n$
$\Rightarrow 3n^2 - n = \frac{n}{2}[2a + (n - 1)6]$
$\Rightarrow a = 2$

15. Given series
$\left(3 - \frac{1}{n}\right) + \left(3 - \frac{2}{n}\right) + \left(3 - \frac{3}{n}\right) + \ldots\ldots$ (A.P.)
Therefore, common difference
$d = \left(3 - \frac{2}{n}\right) - \left(3 - \frac{1}{n}\right) = -\frac{1}{n}$ and first term
$a = \left(3 - \frac{1}{n}\right)$
Now, p^{th} term of the series $= a + (p - 1)d$
$= \left(3 - \frac{1}{n}\right) + (p - 1)\left(-\frac{1}{n}\right)$
$= 3 - \frac{1}{n} + \frac{1}{n} - \frac{p}{n} = \left(3 - \frac{p}{n}\right)$

16. Given that, 9^{th} term $= a + (9 - 1)d = 0$
$\Rightarrow a + 8d = 0$
Now, ratio of 29^{th} and 19^{th} terms
$= \frac{a + 28d}{a + 18d} = \frac{(a + 8d) + 20d}{(a + 8d) + 10d} = \frac{20d}{10d} = \frac{2}{1}$

17. Let the first term and common difference of an A.P. be A and D respectively.
Now, p^{th} term $= A + (p - 1)D = a$
q^{th} term $= A + (q - 1)D = b$
and r^{th} term $= A + (r - 1)D = c$
$\therefore$ $a(q - r) + b(r - p) + c(p - q)$
$= a\left\{\frac{b - c}{D}\right\} + b\left\{\frac{c - a}{D}\right\} + c\left\{\frac{a - b}{D}\right\}$
$= \frac{1}{D}(ab - ac + bc - ab + ca - bc) = 0$

18. Given that first term $a = 10$, last term $l = 50$ and sum $S = 300$
$\therefore$ $S = \frac{n}{2}(a + l) \Rightarrow 300 = \frac{n}{2}(10 + 50) \Rightarrow n = 10$

19. $(x + 1) + (x + 4) + \ldots + (x + 28) = 155$
Let n be the number of terms in the A.P. on L.H.S. Then,
$x + 28 = (x + 1) + (n - 1)3 \Rightarrow n = 10$
$\therefore$ $(x + 1) + (x + 4) + \ldots + (x + 28) = 155$
$\Rightarrow \frac{10}{2}[(x + 1) + (x + 28)] = 155$
$\Rightarrow x = 1$

20. $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 406 = \frac{n}{2}[6 + (n-1)4]$

$\Rightarrow 812 = n[6 + 4n - 4] \Rightarrow 812 = 2n + 4n^2$

$\Rightarrow 406 = 2n^2 + n \Rightarrow 2n^2 + n - 406 = 0$

$\Rightarrow n = \frac{-1 \pm \sqrt{1 + 4.2.406}}{2.2} = \frac{-1 \pm \sqrt{3249}}{4}$

$= \frac{-1 \pm 57}{4}$

Taking (+) sign, $n = \frac{-1 + 57}{4} = 14$

21. Here, $\frac{1}{3}, A_1, A_2, \frac{1}{24}$ will be in A.P.,

then $A_1 - \frac{1}{3} = \frac{1}{24} - A_2$

$\Rightarrow A_1 + A_2 = \frac{3}{8}$ ......(i)

Now, A_1 is a arithmetic mean of $\frac{1}{3}$ and A_2.

∴ $2A_1 = \frac{1}{3} + A_2 \Rightarrow 2A_1 - A_2 = \frac{1}{3}$ ......(ii)

From (i) and (ii), we get $A_1 = \frac{17}{72}$ and $A_2 = \frac{5}{36}$

22. Let the two numbers be a and b and let $A_1, A_2, \ldots, A_n$ be the n A.M.'s between them. Then a, $A_1, A_2, \ldots, A_n$, b are in A.P. and let d be the common difference.

Now, $T_{n+2} = b = a + (n + 2 - 1)d$

$\Rightarrow d = \frac{b-a}{n+1}$

Also, $A_1 + A_2 + \ldots + A_n = S_{n+1} - a$

$= \frac{1}{2}(n+1)\left[2a + (n+1-1)\frac{(b-a)}{(n+1)}\right] - a$

$= \frac{n}{2}[2a + (b-a)] = \frac{n}{2}(a+b) = n\left(\frac{a+b}{2}\right)$

∴ S = Na

23. Let the three numbers be a + d, a, a − d. therefore, a + d + a + a − d = 33

$\Rightarrow a = 11$

and $a(a+d)(a-d) = 792$

$\Rightarrow 11(121 - d^2) = 792 \Rightarrow d = 7$

The required numbers are 4, 11, 18.

Hence, the smallest number is 4.

24. $d = -1 + 2i$, $t_4 = t_3 + d = 6 - 2i + (-1 + 2i) = 5$ which is purely real.

25. $t_n = ar^{n-1} = 1\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{n-1}$

26. a = 5, r = 3

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{5(3^n - 1)}{2}$

28. a = 1, r = 3

$S_n = \frac{a(r^n - 1)}{r - 1}$

$3280 = \frac{3^n - 1}{2}$

$6561 = 3^n$

$\Rightarrow 3^8 = 3^n \Rightarrow n = 8$

31. $t_n = ar^{n-1} = 1.(2)^{n-1} = 2^{n-1}$

32. $S_n = 2 + 22 + 222 + \ldots$ n terms

$= 2[1 + 11 + 111 + \ldots$ n terms]

$= \frac{2}{9}[(10-1) + (100-1) + (1000-1) + \ldots$ n terms]

$= \frac{2}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right] = \frac{2}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$= \frac{2}{81}[10(10^n - 1) - 9n]$

33. $S_n = 0.9 + 0.99 + 0.999 + \ldots$ n terms

$= 1 - 0.1 + 1 - 0.01 + 1 - 0.001 + \ldots$ n terms

$= 1 + 1 + 1 + \ldots$ n terms $- [0.1 + 0.01 + 0.001 + \ldots$ n terms]

$= n - 0.1\left[\frac{1 - (0.1)^n}{1 - 0.1}\right]$

$= n - \frac{1}{9}[1 - (0.1)^n]$

$= \frac{9n - [1 - (0.1)^n]}{9}$

34. $a = 2, S_\infty = 6$

Now, $S_\infty = \frac{a}{1-r}$

∴ $6 = \frac{2}{1-r}$ ∴ $1 - r = \frac{1}{3}$

∴ $r = 1 - \frac{1}{3}$

∴ $r = \frac{2}{3}$

35. $a = 3, r = \frac{\left(\frac{-3}{2}\right)}{3} = \frac{-1}{2}$

$\Rightarrow t_n = ar^{n-1} = 3\left(\frac{-1}{2}\right)^{n-1}$

36. $S_8 = 82\ (S_4)$

Let the G.P. be $a + ar + ar^2 + \ldots$, then

$\frac{a(1-r^8)}{1-r} = 82\left\{\frac{a(1-r^4)}{1-r}\right\}$

$\therefore$ $(1-r^4)(1+r^4) = 82(1-r^4) \Rightarrow r = 3$

37. $a = 3$ and $r = \frac{12}{3} = 4 > 1$

$\therefore$ $S_n = a\left[\frac{r^n-1}{r-1}\right] = 3\left[\frac{4^n-1}{4-1}\right] = 4^n - 1$

38. $S_n = \frac{a(r^n-1)}{r-1}, r = 2$

$\therefore$ $S_8 = \frac{a(2^8-1)}{2-1} \Rightarrow a(2^8-1) = 510 \Rightarrow a = 2$

$\therefore$ $t_3 = 2(2)^{3-1} = 2(2)^2 = 8$

39. Let n be the number of terms needed.

For G.P. $2, 2^2, 2^3, \ldots$, $a = 2, r = 2$ and $S_n = 30$

$S_n = \frac{a(r^n-1)}{r-1} \Rightarrow 30 = \frac{2(2^n-1)}{2-1} \Rightarrow n = 4$

40. a, 8, b are in G.P. and $a \neq b$

$\Rightarrow \frac{8}{a} = \frac{b}{8}$

$\Rightarrow ab = 64$

and a, b, – 8 are in A.P.

$\Rightarrow b - a = -8 - b$

$\therefore$ $b = \left(\frac{a-8}{2}\right)$

Solving, $a = 16$ and $b = 4$

41. Let the numbers be a, ar, ar^2

Sum $= 70 \Rightarrow a(1 + r + r^2) = 70$

It is given that $4a, 5ar, 4ar^2$ are in A.P.

$\therefore$ $2(5ar) = 4a + 4ar^2 \Rightarrow r = 2$ or $r = \frac{1}{2}$

Substituting values of r, $a = 10$ and $a = 40$

$\therefore$ The numbers are 10, 20, 40 or 40, 20, 10

42. $\frac{g_1}{p} = \frac{q}{g_2} \Rightarrow g_1g_2 = pq$

43. $t_3 = ar^{3-1} = ar^2 = 20$ and

$t_7 = ar^{7-1} = ar^6 = 320$

Solving, $a = 5$ and $r = 2$

44. Let r be common ratio of G.P.

$\Rightarrow t_3 = r^2, t_5 = r^4$

$\therefore$ $t_3 + t_5 = 90 \Rightarrow r^2 + r^4 = 90$

$\Rightarrow r^2 = 9$

$\Rightarrow r = \pm 3$

46. Accordingly, $ar^9 = 9$ and $ar^3 = 4$

$r^3 = \frac{3}{2}$ and $a = \frac{8}{3}$

$\therefore$ 7^{th} term i.e., $ar^6 = \frac{8}{3}\left(\frac{3}{2}\right)^2 = 6$

Trick : 7^{th} term is equidistant from 10^{th} and 4^{th} so it will be $\sqrt{9 \times 4} = 6$.

47. Given sequence is $\sqrt{2}, \sqrt{10}, \sqrt{50}$........

Common ratio $r = \sqrt{5}$, first term $a = \sqrt{2}$, then 7^{th} term

$t_7 = \sqrt{2}(\sqrt{5})^{7-1}$

$= \sqrt{2}(\sqrt{5})^6 = \sqrt{2}(5)^3 = 125\sqrt{2}$

48. Let 1, a, b, 64

$\Rightarrow a^2 = b$ and $b^2 = 64a$

$\Rightarrow a = 4$ and $b = 16$

49. Let numbers are $\frac{a}{r}$, a, ar

According to given conditions,

$\frac{a}{r} . a . ar = 216$

$\Rightarrow a = 6$

And, sum of product pairwise = 156

$\Rightarrow \frac{a}{r} . a + \frac{a}{r} . ar + a . ar = 156$

$\Rightarrow r = 3$

Hence, numbers are 2, 6, 18.

Trick : Since $2 \times 6 \times 18 = 216$ (as given) and no other option gives the value.

50. According to condition, $\frac{3/4}{1-r} = \frac{4}{3}$

$\Rightarrow r = \frac{7}{16}$

51. $G^2 = AH$

$\Rightarrow 144 = 25H$

$\Rightarrow H = 5.76$

54. $\because H < G < A$

56. Considering corresponding A.P.

$a + 6d = 10$ and $a + 11d = 25 \Rightarrow d = 3, a = -8$

$\Rightarrow t_{20} = a + 19d = -8 + 57 = 49$

Hence, 20^{th} term of the corresponding H.P. is $\frac{1}{49}$.

57. $(A.M.)(H.M.) = (G.M)^2$

$\Rightarrow 9.36 = (G.M)^2 \Rightarrow G.M. = 18$

58. Here $a = 3$, $d = 2$ and $r = r$

Now $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} \quad (|r| < 1)$

$\therefore \quad S_\infty = \frac{3}{1-r} + \frac{2r}{(1-r)^2}$

$\therefore \quad \frac{44}{9} = \frac{3-r}{(1-r)^2}$

$\therefore \quad 44r^2 - 79r + 17 = 0$

$\therefore \quad r = \frac{1}{4}$ or $\frac{17}{11}$

But, $r \neq \frac{17}{11}$

$\therefore \quad r = \frac{1}{4}$

59. Let $S = 1 + 3x + 5x^2 + 7x^3 + \ldots$

Then, $xS = 1x + 3x^2 + 5x^3 + \ldots$

$S - xS = 1 + 2x + 2x^2 + 2x^3 + \ldots$ to ∞

$\therefore \quad S(1-x) = 1 + 2x + 2x^2 + 2x^3 + \ldots$ to ∞

$= 1 + \frac{2x}{1-x} = \frac{1-x+2x}{1-x}$

$\therefore \quad S = \frac{1+x}{(1-x)^2}$

60. $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ to ∞

$(S-1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ to ∞

$(S-1) \times \frac{1}{3} = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots$ to ∞

Subtracting,

$\frac{2}{3}(S-1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots$ to ∞

$= \frac{2}{3} + \frac{\frac{4}{3^2}}{1-\frac{1}{3}}$

$\Rightarrow \frac{2}{3}(S-1) = \frac{2}{3} + \frac{2}{3}$

$\therefore \quad S = 3$

61. $\sum_{r=1}^{n}(2r+5) = 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 5 = \frac{2(n)(n+1)}{2} + 5n$

$= n(n+6)$

62. Sum of given series $= y + \frac{y^2}{2} + \frac{y^3}{3} + \ldots,$

where $y = x^2$.

$\therefore$ Sum of given series $= -\log(1-y)$

$= -\log_e(1-x^2)$

63. $1^3 + 2^3 + 3^3 + \ldots + 25^3 = \sum_{r=1}^{25} r^3$

$= \frac{(25)^2(25+1)^2}{4}$

$= 105625$

64. $(31)^2 + (32)^2 + (33)^2 + \ldots + (60)^2$

$= [(1)^2 + (2)^2 + (3)^2 + \ldots + (60)^2]$

$- [(1)^2 + (2)^2 + (3)^2 + \ldots + (30)^2]$

$= \sum_{r=1}^{60} r^2 - \sum_{r=1}^{30} r^2$

$= 64355$

65. $(2^2 - 1^2) + (4^2 - 3^2) + (6^2 - 5^2) + \ldots$

$= (2^2 + 4^2 + 6^2 + \ldots) - (1^2 + 3^2 + 5^2 + \ldots)$

$= \sum_{r=1}^{n}(2r)^2 - \sum_{r=1}^{n}(2r-1)^2 = \sum_{r=1}^{n} 4r - \sum_{r=1}^{n} 1$

$= 4\left[\frac{n(n+1)}{2}\right] - n$

$= n(2n+1)$

66. $\log_e 3 - \frac{\log_e 3^2}{2^2} + \frac{\log_3 3^3}{3^2} - \frac{\log_e 3^4}{4^2} + \ldots$

$= \log_e 3\left\{1 - \frac{2}{2^2} + \frac{3}{3^2} - \frac{4}{4^2} + \ldots\right\}$

$= \log_e 3\left\{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right\}$

$= \log_e 3 \log_e(1+1)$

$= \log_e 3 \log_e 2$

67. $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

$= \log_e(1+x)$

$\therefore \quad 1 + x = e^y \Rightarrow x = e^y - 1$

68. $2(1)^2 + 3(2)^2 + 4(3)^2 + \ldots$ upto 10 terms

$= \sum_{r=1}^{10}(r+1)r^2 = \sum_{r=1}^{10} r^3 + \sum_{r=1}^{10} r^2$

$= 3410$

69. $\frac{n(n+1)(2n+1)}{6} = 1015$

$\therefore$ $n(n+1)(2n+1) = 6090$

$\Rightarrow n(n+1)(2n+1) = 14 \times 15 \times 29$

$\Rightarrow n = 14$

70. The first factors of the terms of the given series is 1, 2, 3, 4, ..., n and second factors of the terms of the given series is

2, 3, 4, ...(n + 1)

$\therefore$ n^{th} term of the given series

$= n(n+1) = n^2 + n$

Hence, sum =

$$\Sigma n^2 + \Sigma n = \frac{1}{6}n(n+1)(2n+1) + \frac{n}{2}(n+1)$$

$$= \frac{1}{6}n(n+1)(2n+1+3)$$

$$= \frac{1}{3}n(n+1)(n+2)$$

Critical Thinking

1. Given sequence is in A.P.

$\therefore$ $a = 8 - 6i$, $d = -1 + 2i$

$\therefore$ $t_n = a + (n-1)d = (9-n) + i(2n-8)$

For purely imaginary term, $9 - n = 0$

$\Rightarrow n = 9$

2. $\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}, ...$

i.e., $\frac{1-\sqrt{x}}{1-x}, \frac{1}{1-x}, \frac{1+\sqrt{x}}{1-x}, ...,$

which is an A.P. with $d = \frac{\sqrt{x}}{1-x}$

$\therefore$ The fourth term $= t_3 + d = \frac{1+\sqrt{x}}{1-x} + \frac{\sqrt{x}}{1-x}$

$$= \frac{1+2\sqrt{x}}{1-x}$$

3. $S_{2n} = 3S_n$

$\therefore$ $\frac{2n}{2}[2a + (2n-1)d] = \frac{3n}{2}[2a + (n-1)d]$

$\Rightarrow 2a = (n+1)d$

$$\therefore \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = 6$$

4. First term = a, d = b – a and last term = c

If the no. of terms is n, then

$t_n = c = a + (n-1)(b-a) \Rightarrow \frac{c-a}{b-a} = n - 1$

Solving, $n = \frac{b+c-2a}{b-a}$

5. a, b, c are in A.P.

$\Rightarrow b - a = c - b \Rightarrow \frac{b-a}{c-b} = 1$

6. If D is the common difference of the A.P. a, b, c, d, e, then $b = a + D$, $c = a + 2D$, $d = a + 3D$, $e = a + 4D$

$\therefore$ $a - 4b + 6c - 4d + e$

$= a - 4(a+D) + 6(a+2D)$
$- 4(a+3D) + a + 4D = 0$

7. Here $a = S_1 = 6$

$S_7 = 105 \Rightarrow \frac{7}{2}[2 \times 6 + (7-1)d] = 105 \Rightarrow d = 3$

$$\therefore \frac{S_n}{S_{n-3}} = \frac{\frac{n}{2}\{2\times6+(n-1)3\}}{\frac{(n-3)}{2}\{2\times6+(n-4)3\}} = \frac{n+3}{n-3}$$

8. $d = \frac{1}{3} - \frac{1}{2} = \frac{-1}{6}$

$$\therefore S_9 = \frac{9}{2}\left\{2\times\frac{1}{2} + (9-1)\left(\frac{-1}{6}\right)\right\} = -\frac{3}{2}$$

9. $d = b - a$ and if the number of terms is n, then

$2a = a + (n-1)(b-a)$

$\Rightarrow \frac{a}{b-a} + 1 = n \Rightarrow n = \frac{b}{b-a}$

10. Required sum = 10 + 13 + 16 + ... + 97

$= \frac{n}{2}(10 + 97)$ .... (i)

Here, $97 = 10 + (n-1)3 \Rightarrow n = 30$

$\therefore$ From (i), $S_n = \frac{30}{2}(10+97) = 1605$

11. $t_n = S_n - S_{n-1}$

$$= \left\{nP + \frac{n(n-1)}{2}Q\right\} - \left\{(n-1)P + \frac{(n-1)(n-2)}{2}Q\right\}$$

$= P + (n-1)Q$

$\therefore$ Common difference $= t_n - t_{n-1}$

$= [P + (n-1)Q] - [P + (n-2)Q] = Q$

12. The smallest 3 digit no. divisible by 7 is 105 and greatest is 994.
Given sequence is in A.P. with d = 7

$\therefore$ $994 = 105 + (n-1)7 \Rightarrow n = 128$

$\therefore$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{128}{2}[2(105) + (128-1)7] = 70336$

13. Suppose work is completed in n days
$\frac{n}{2}[2 \times 150 + (n-1)(-4)] = n(152 - 2n)$
Had no worker dropped from work, total no. of workers who would have worked all the n days is 150 (n – 8)

$\therefore$ $n(152 - 2n) = 150(n-8) \Rightarrow n = 25$

14. $l = a + (n-1)d$ and
$S_n = \frac{n}{2}(a + l)$
Eliminating a, we get
$S_n = \frac{n}{2}\{l - (n-1)d + l\} = \frac{n}{2}\{2l - (n-1)d\}$

15. d = – 2, sum = – 5

$\therefore$ $-5 = \frac{5}{2}\{2a + 4(-2)\} \Rightarrow a = 3$

Hence, the actual sum (when d = 2) is
$\frac{5}{2}\{2\times3+(5-1)\times2\} = \frac{5}{2}(6+8) = 35$

17. Given series 3.8 + 6.11 + 9.14 + 12.17 + ...
First factors are 3, 6, 9, 12 whose n^{th} term is 3n and second factors are 8, 11, 14, 17
$t_n = [8 + (n-1)3] = (3n + 5)$
Hence n^{th} term of given series = 3n(3n + 5).

18. Suppose that $\angle A = x°$, then $\angle B = x + 10°$, $\angle C = x + 20°$ and $\angle D = x + 30°$
So, we know that $\angle A + \angle B + \angle C + \angle D = 2\pi$
Putting these values, we get
$(x°) + (x° + 10°) + (x° + 20°) + (x° + 30°) = 360°$
$\Rightarrow x = 75°$
Hence, the angles of the quadrilateral are 75°, 85°, 95°, 105°.

19. a, b, c, are in A.P $\Rightarrow 2b = a + c$
Also, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore$ $\frac{2}{\frac{a+c}{2}} = \frac{a+c}{ac} \Rightarrow a = c$ and $b = a$

20. Let S_n and S'_n be the sum of n terms of two A.P.'s and t_{11} and t'_{11} be the respective 11^{th} terms, then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a + \frac{(n-1)}{2}d}{a' + \frac{(n-1)}{2}d'} = \frac{7n+1}{4n+27}$$

Now put n = 21,
we get $\frac{a+10d}{a'+10d'} = \frac{t_{11}}{t'_{11}} = \frac{148}{111} = \frac{4}{3}$

21. Required number n is the number of terms in the series 105 + 112 + 119 + ... + 994

$\therefore$ 994 = n^{th} term of the above A.P.

$\therefore$ $994 = 105 + (n-1) \times 7$

$\therefore$ $n = \frac{994 - 98}{7}$

$\therefore$ $n = 128$

22. The given numbers are in A.P.

$\therefore$ $2\log_9(3^{1-x} + 2) = \log_3(4.3^x - 1) + 1$

$\Rightarrow 2\log_{3^2}(3^{1-x} + 2) = \log_3(4.3^x - 1) + \log_3 3$

$\Rightarrow \frac{2}{2}\log_3(3^{1-x} + 2) = \log_3[3(4.3^x - 1)]$

$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$

$\Rightarrow \frac{3}{y} + 2 = 12y - 3$, where $y = 3^x$

$\Rightarrow 12y^2 - 5y - 3 = 0$

$\therefore$ $y = \frac{-1}{3}$ or $\frac{3}{4} \Rightarrow 3^x = \frac{-1}{3}$ or $3^x = \frac{3}{4}$

$\therefore$ $x = \log_3\left(\frac{3}{4}\right) \Rightarrow x = 1 - \log_3 4$

23. As we know $T_n = S_n - S_{n-1}$
$= (2n^2 + 5n) - \{2(n-1)^2 + 5(n-1)\}$
$= 2n^2 + 5n - 2n^2 + 4n - 2 - 5n + 5$
$= 4n + 3$

24. Here, $T_n = 3n - 1$, putting n = 1, 2, 3, 4, 5 we get first five terms, 2, 5, 8, 11, 14
Hence, sum is 2 + 5 + 8 + 11 + 14 = 40.

25. According to the given condition
$\frac{15}{2}[10 + 14 \times d] = 390 \Rightarrow d = 3$
Hence, middle term i.e., 8^{th} term is given by
$5 + 7 \times 3 = 26$

26. $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}=\dfrac{a+b}{2}$

$\Rightarrow a^{n+1}-ab^n+b^{n+1}-ba^n=0$

$\Rightarrow (a-b)(a^n-b^n)=0$

If $a^n-b^n=0$. Then $\left(\dfrac{a}{b}\right)^n=1=\left(\dfrac{a}{b}\right)^0$

Hence, $n=0$

27. The sum of n arithmetic mean between a and b

$=\dfrac{n}{2}(a+b)$

28. The resulting progression will have n + 2 terms with 2 as the first term and 38 as the last term.

Therefore, the sum of the progression

$=\dfrac{n+2}{2}(2+38)$

$=20(n+2)$

By hypothesis, $20(n+2)=200$

$\Rightarrow n=8.$

29. As, log 2, $\log(2^n-1)$ and $\log(2^n+3)$ are in A.P.

Therefore,

$2\log(2^n-1)=\log 2+\log(2^n+3)$

$2^{2n}-4.2^n-5=0$

$\Rightarrow (2^n-5)(2^n+1)=0$

As 2^n cannot be negative, hence $2^n-5=0$

$\Rightarrow 2^n=5$ or $n=\log_2 5$

30. Let the three numbers be $a-d$, a, $a+d$

We get $a-d+a+a+d=15$

$\Rightarrow a=5$

and $(a-d)^2+a^2+(a+d)^2=83$

$\Rightarrow a^2+d^2-2ad+a^2+a^2+d^2+2ad=83$

$\Rightarrow 2(a^2+d^2)+a^2=83$

Putting $a=5$

$\Rightarrow 2(25+d^2)+25=83$

$\Rightarrow 2d^2=8$

$\Rightarrow d=2$

Thus, numbers are 3, 5, 7.

Trick :

Since $3+5+7=15$ and $3^2+5^2+7^2=83$

31. $(a+2b-c)(2b+c-a)(c+a-b)$

$=(a+a+c-c)(a+c+c-a)(2b-b)$

$=4abc$

($\because$ a, b, c are in A.P., $\therefore$ $2b=a+c$).

32. $t_3=4\Rightarrow ar^2=4$

$\therefore$ $a\times ar\times ar^2\times ar^3\times ar^4=(ar^2)^5=4^5$

33. $a=\dfrac{5}{2}$, $r=\dfrac{1}{2}<1$

$\therefore$ $S_n=\dfrac{a(1-r^n)}{1-r}=5\left[\dfrac{2^n-1}{2^n}\right]$

34. $t_3=ar^{3-1}=ar^2=36$ and $t_6=ar^{6-1}=ar^5=972$

Solving, $a=4$ and $r=3$

$\therefore$ $t_8=ar^7=4(3)^7=8748$

35. $ab^2=a(ac)$ and $cb^2=c(ac)$

$\therefore$ $ab^2-cb^2=a^2c-ac^2$

$\Rightarrow a(b^2+c^2)=c(a^2+b^2)$

36. $t_n=ar^{n-1}$ and $r=2$

$\therefore$ $t_n=a(2)^{n-1}\Rightarrow t_9=a(2)^8$

$\therefore$ $a(2)^8=128\Rightarrow a=\dfrac{128}{256}=\dfrac{1}{2}$

37. $S_n=4+44+444+\ldots\ldots$ to n terms

$=\dfrac{4}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\ldots\ldots+(10^n-1)\right]$

$=\dfrac{4}{9}\left\{(10+10^2+10^3+\ldots+10^n)-(1+1+1+\ldots\ldots n\text{ times})\right\}$

$=\dfrac{4}{9}\left\{\dfrac{10(10^n-1)}{10-1}-n\right\}$

$=\dfrac{4}{9}\left\{\dfrac{10}{9}(10^n-1)-n\right\}$

38. $\left(1-\dfrac{1}{2}\right)+\left(1-\dfrac{1}{4}\right)+\left(1-\dfrac{1}{8}\right)+\ldots.$

$\therefore$ $t_n=1-\left(n^{th}\text{ term of G.P. }\dfrac{1}{2},\dfrac{1}{4},\dfrac{1}{8}\right)$

$=1-\dfrac{1}{2}\left(\dfrac{1}{2}\right)^{n-1}$

$=1-\dfrac{1}{2^n}$

39. Given series is a G.P. with $a=\sqrt{2}$ and $r=\sqrt{3}$

$\therefore$ $S_{10}=\dfrac{\sqrt{2}\left((\sqrt{3})^{10}-1\right)}{\sqrt{3}-1}=\dfrac{\sqrt{2}(243-1)}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$=121\sqrt{6}+121\sqrt{2}$

$=121(\sqrt{6}+\sqrt{2})$

40. The common ratio of the G.P. is x^{n+4}

$\therefore$ 8^{th} term $=x^{52}=x^{-4}(x^{n+4})^7$

$\Rightarrow 7n=28$

$\Rightarrow n=4$

41. $t_4 = 24$ and $t_9 = 768$

$\therefore$ $t_4 = ar^3 \Rightarrow ar^3 = 24$

and $t_9 = ar^8 \Rightarrow ar^8 = 768$

Solving, a = 3 and r = 2 > 1

$\therefore$ $S_{10} = \dfrac{a\left[r^{10}-1\right]}{r-1} = \dfrac{3\left[2^{10}-1\right]}{2-1} = 3(2^{10}-1)$

42. $1 + (1+x) + (1+x+x^2) + \ldots + (1+x+x^2+\ldots+x^{n-1})$

$= \dfrac{1-x}{1-x} + \dfrac{1-x^2}{1-x} + \dfrac{1-x^3}{1-x} + \ldots\ldots + \dfrac{1-x^n}{1-x}$

$= \dfrac{1}{1-x}[(1+1+\ldots\text{ n times}) - (x+x^2+\ldots+x^n)]$

$= \dfrac{1}{1-x}\left[n - \dfrac{x(1-x^n)}{1-x}\right]$

$= \dfrac{n}{1-x} - \dfrac{x(1-x^n)}{(1-x)^2}$

43. $a + ar = -4$ and $ar^4 = 4ar^2 \Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

Substituting $r = \pm 2$, we get $a = \dfrac{-4}{3}$ and $a = 4$

$\therefore$ Required G.P. is $\dfrac{-4}{3}, \dfrac{-8}{3}, \dfrac{-16}{3}, \ldots$

or $4, -8, 16, -32, \ldots$

44. Let the terms of given G.P. be $\dfrac{a}{r}$, a, ar

then product $= \dfrac{a}{r} \times a \times ar = 1000$

$\dfrac{a}{r}$, a + 6, ar + 7 are in A.P.

$\therefore$ $2(a+6) = \dfrac{a}{r} + ar + 7$

$\therefore$ $25 = \dfrac{10}{r} + 10r$

$\therefore$ $2r^2 - 5r + 2 = 0$

$\therefore$ $(2r-1)(r-2) = 0$

$\therefore$ $r = 2, \dfrac{1}{2}$

Hence, the G.P is 5, 10, 20,.... or 20, 10, 5....

45. $r = \dfrac{t_2}{t_1} = \dfrac{b}{a}$; last term = c

$\Rightarrow ar^{n-1} = c \Rightarrow \dfrac{ar^n}{r} = c$

$\Rightarrow ar^n = cr$

$\therefore$ $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a-ar^n}{1-r} = \dfrac{a-cr}{1-r} = \dfrac{a-c\left(\frac{b}{a}\right)}{1-\frac{b}{a}}$

47. $2.3\overline{45} = 2.3 + 0.045 + 0.00045 + \ldots$

$= \dfrac{23}{10} + \dfrac{45}{1000} + \dfrac{45}{100000} + \ldots$

From 2^{nd} term onwards, the terms are in G.P.

$\therefore$ $S_\infty = \dfrac{a}{1-r} = \dfrac{\frac{45}{1000}}{1-\frac{1}{100}} = \dfrac{1}{22}$

$\therefore$ $2.3\overline{45} = \dfrac{23}{10} + \dfrac{1}{22} = \dfrac{129}{55}$

Alternate Method:

$2.3\overline{45} = 2 + \dfrac{345-3}{990} = 2 + \dfrac{342}{990} = \dfrac{129}{55}$

48. Let the G.P. be $a + ar + ar^2 + \ldots$, $|r| < 1$,

then ar = 2 and $\dfrac{a}{1-r} = 8$

$\therefore$ $\dfrac{ar(1-r)}{a} = \dfrac{2}{8}$

$\Rightarrow r = \dfrac{1}{2}$ and a = 4

49. $\left(1-\dfrac{1}{2}\right) + \left(1-\dfrac{1}{4}\right) + \left(1-\dfrac{1}{8}\right) + \ldots$

$\therefore$ $S_n = n - \left(\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots\text{upto n terms}\right)$

$= n - \dfrac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = n - \left(1 - \dfrac{1}{2^n}\right)$

$= n - 1 + 2^{-n}$

50. $\dfrac{1}{7} + \dfrac{2}{7^2} + \dfrac{1}{7^3} + \dfrac{2}{7^4} + \ldots$ upto ∞

$= \left(\dfrac{1}{7} + \dfrac{1}{7^3} + \dfrac{1}{7^5} + \ldots\right) + 2\left(\dfrac{1}{7^2} + \dfrac{1}{7^4} + \dfrac{1}{7^6} + \ldots\right)$

$= \dfrac{\frac{1}{7}}{1-\frac{1}{7^2}} + \dfrac{2\left(\frac{1}{7^2}\right)}{1-\frac{1}{7^2}} = \dfrac{3}{16}$

51. $A = 1 + r^z + r^{2z} + r^{3z} + \ldots.. \infty$
$A = 1 + [r^z + r^{2z} + r^{3z} + \ldots.. \infty]$
We know that sum of infinite G.P. is
$S_\infty = \frac{a}{1-r} (-1 < r < 1)$
Therefore, $A = 1 + \left[\frac{r^z}{1-r^z}\right]$
$\Rightarrow A = \frac{1-r^z+r^z}{1-r^z} \Rightarrow A = \frac{1}{1-r^z}$
$\Rightarrow 1 - r^z = \frac{1}{A} \Rightarrow r^z = \frac{A-1}{A}$
Hence, $r = \left[\frac{A-1}{A}\right]^{\frac{1}{z}}$

52. $G.M = b = \sqrt{ac}$
$\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{\sqrt{ac}-a} + \frac{1}{\sqrt{ac}-c}$
$= \frac{1}{\sqrt{a}\left[\sqrt{c}-\sqrt{a}\right]} + \frac{1}{\sqrt{c}\left[\sqrt{a}-\sqrt{c}\right]}$
$= \frac{1}{\sqrt{a}\left[\sqrt{c}-\sqrt{a}\right]} - \frac{1}{\sqrt{c}\left[\sqrt{c}-\sqrt{a}\right]} = \frac{1}{\sqrt{ac}} = \frac{1}{b}$

53. a, g_1, g_2, b are in G.P. $\Rightarrow \frac{g_1}{a} = \frac{g_2}{g_1} = \frac{b}{g_2}$
$\therefore \frac{g_1}{a} = \frac{g_2}{g_1}$ and $\frac{g_2}{g_1} = \frac{b}{g_2} \Rightarrow a = \frac{g_1^2}{g_2}$ and $b = \frac{g_2^2}{g_1}$
$\therefore \frac{g_1^2}{g_2} + \frac{g_2^2}{g_1} = a + b$

54. Let $AR^{p-1} = a$,
$AR^{q-1} = b$,
$AR^{r-1} = c$
So
$a^{q-r} b^{r-p} c^{p-q} = (AR^{p-1})^{q-r} (AR^{q-1})^{r-p} (AR^{r-1})^{p-q}$
$= A^{(q-r+r-p+p-q)} R^{(pq-pr-q+r+qr-pq-r+p+pr-rq-p+q)}$
$= A^0R^0 = 1$

55. We have, $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow a = \frac{x-1}{x}$
$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \Rightarrow b = \frac{y-1}{y}$
$z = \sum_{n=0}^{\infty} a^n b^n = \frac{1}{1-ab} \Rightarrow ab = \frac{z-1}{z}$
$\therefore \frac{x-1}{x} \cdot \frac{y-1}{y} = \frac{z-1}{z}$
$\Rightarrow xy + z = zx + yz$

56. $\frac{S_3}{S_6 - S_3} = \frac{125}{27} \Rightarrow \frac{S_3}{S_6} = \frac{125}{152}$
$\therefore \frac{a(1-r^3)}{a(1-r^6)} = \frac{125}{152} \Rightarrow \frac{1}{1+r^3} = \frac{125}{152}$
$\Rightarrow r^3 = \frac{27}{125} \Rightarrow r = \frac{3}{5}$

57. Let the 9 terms of a G.P. be
$\frac{a}{r^4}, \frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3, ar^4$
Given, fifth term a = 2
Hence, product of 9 terms is $a^9 = (2)^9 = 512$

58. Given that $\frac{a(r^n-1)}{r-1} = 255$ ($\because r > 1$) ....(i)
$ar^{n-1} = 128$ ....(ii)
and common ratio r = 2 ....(iii)
From (i), (ii) and (iii), we get
$a(2)^{n-1} = 128$ ....(iv)
and $\frac{a(2^n-1)}{2-1} = 255$ ....(v)
Dividing (v) by (iv), we get
$\frac{2^n-1}{2^{n-1}} = \frac{255}{128}$
$\Rightarrow 2 - 2^{-n+1} = \frac{255}{128}$
$\Rightarrow 2^{-n} = 2^{-8}$
$\Rightarrow n = 8$
Putting n = 8 in equation (iv),
we get $a.2^7 = 128 = 2^7$ or a = 1

59. We have
$1 + a + a^2 + \ldots.. + a^x = (1 + a)(1 + a^2)(1 + a^4)$
$\Rightarrow \frac{(1-a^{x+1})}{(1-a)} = (1 + a)(1 + a^2) + (1 + a^4)$
$\Rightarrow (1 - a^{x+1}) = (1 - a)(1 + a)(1 + a^2)(1 + a^4)$
$\Rightarrow (1 - a^{x+1}) = (1 - a^8)$
$\Rightarrow x + 1 = 8$
$\Rightarrow x = 7$

60. $a_1 = 3$, $a_n = 96$
$\Rightarrow a_1 r^{n-1} = 96$
$\Rightarrow r^{n-1} = 32$
Now, $S_n = \frac{a_1(r^n-1)}{r-1} = 189$
$\Rightarrow \frac{3(32r-1)}{r-1} = 189$
Hence, r = 2 and n = 6

61. $a = 7$ and $ar^{n-1} = 448$

Now, $S_n = \dfrac{a(r^n - 1)}{r - 1} = 889$

$\Rightarrow \dfrac{(ar^{n-1}.r - a)}{r-1} = 889 \Rightarrow \dfrac{448r - 7}{r-1} = 889$

$\Rightarrow r = 2$

62. As given, $G = \sqrt{xy}$

$\therefore \dfrac{1}{G^2 - x^2} + \dfrac{1}{G^2 - y^2} = \dfrac{1}{xy - x^2} + \dfrac{1}{xy - y^2}$

$= \dfrac{1}{x-y}\left\{-\dfrac{1}{x} + \dfrac{1}{y}\right\} = \dfrac{1}{xy} = \dfrac{1}{G^2}$

63. $a = 3, r = 3$

G.M. $= (3.3^2.3^3.........3^n)^{1/n}$

$= (3^{1+2+3.......+n})^{1/n} = \left(3^{\frac{n(n+1)}{2}}\right)^{1/n} = 3^{\frac{(n+1)}{2}}$

64. $4^{1/3}.4^{1/9}.4^{1/27}.......\infty$

$\therefore$ $S = 4^{1/3 + 1/9 + 1/27......\infty}$

$\Rightarrow S = 4^{\left(\frac{1/3}{1-1/3}\right)} = 4^{\frac{1/3}{2/3}}$

$\Rightarrow S = 4^{1/2}$

$\Rightarrow S = 2$

65. Infinite series $9 - 3 + 1 - \dfrac{1}{3} + ...\infty$ is a G.P. with $a = 9, r = \dfrac{-1}{3}$

$\therefore$ $S_\infty = \dfrac{a}{1-r} = \dfrac{9}{1+\left(\dfrac{1}{3}\right)} = \dfrac{9\times 3}{4} = \dfrac{27}{4}$

66. $5 = \dfrac{x}{1-r} \Rightarrow 5 - 5r = x \Rightarrow r = 1 - \dfrac{x}{5}$

As $|r| < 1$ i.e., $\left|1 - \dfrac{x}{5}\right| < 1$

$\therefore$ $-1 < 1 - \dfrac{x}{5} < 1$

$\therefore$ $-5 < 5 - x < 5$

$\therefore$ $-10 < -x < 0$

$\therefore$ $10 > x > 0$

$\therefore$ $0 < x < 10$

67. a, b, c are in H.P. $\Rightarrow \dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P.

$\therefore$ $\dfrac{1}{b} - \dfrac{1}{a} = \dfrac{1}{c} - \dfrac{1}{b} \Rightarrow \dfrac{a-b}{b-c} = \dfrac{a}{c}$

68. $c = \dfrac{2ab}{a+b} \Rightarrow \dfrac{c}{a} = \dfrac{2b}{a+b}$ and $\dfrac{c}{b} = \dfrac{2a}{a+b}$

$\therefore$ $\dfrac{c}{a} + \dfrac{c}{b} = \dfrac{2b}{a+b} + \dfrac{2a}{a+b} = 2$

69. $H = \dfrac{2ab}{a+b}$

$\Rightarrow H - a = \dfrac{2ab}{a+b} - a = \dfrac{ab - a^2}{a+b}$

and $H - b = \dfrac{2ab}{a+b} - b = \dfrac{ab - b^2}{a+b}$

$\therefore$ $\dfrac{1}{H-a} + \dfrac{1}{H-b} = \dfrac{a+b}{ab - a^2} + \dfrac{a+b}{ab-b^2}$

$= \dfrac{(a+b)}{(b-a)}\left[\dfrac{(b-a)}{ab}\right]$

$= \dfrac{1}{a} + \dfrac{1}{b}$

70. $H = \dfrac{2ab}{a+b} \Rightarrow \dfrac{H}{a} = \dfrac{2b}{a+b}$

$\therefore$ $\dfrac{H+a}{H-a} = \dfrac{3b+a}{b-a}$

Similarly, $\dfrac{H+b}{H-b} = \dfrac{3a+b}{a-b} = -\dfrac{3a+b}{b-a}$

$\therefore$ $\dfrac{H+a}{H-a} + \dfrac{H+b}{H-b} = \dfrac{2b-2a}{b-a} = 2$

71. 7^{th} term of corresponding A.P. is $\dfrac{1}{8}$ and 8^{th} term will be $\dfrac{1}{7}$

$\Rightarrow a + 6d = \dfrac{1}{8}$ and $a + 7d = \dfrac{1}{7}$

Solving these, we get $d = \dfrac{1}{56}$ and $a = \dfrac{1}{56}$

Therefore, 15^{th} term of this A.P.

$= \dfrac{1}{56} + 14 \times \dfrac{1}{56} = \dfrac{15}{56}$

Hence, the required 15^{th} term of the H.P. is $\dfrac{56}{15}$.

72. H.M. $= \dfrac{2\left(\dfrac{a^2}{1-a^2b^2}\right)}{\dfrac{a}{1-ab} + \dfrac{a}{1+ab}} = \dfrac{2a^2}{2a} = a$

73. a, b, c are in H.P.

$\therefore \quad b = \dfrac{2ac}{a+c}$

Also b, c, d are in H.P. $\Rightarrow c = \dfrac{2bd}{b+d}$

Multiplying we get, $bc = \dfrac{4abcd}{(a+c)(b+d)}$

$\therefore \quad ab + bc + cd + ad = 4ad$

$\Rightarrow ab + bc + cd = 3ad$

74. Let the numbers be a and b, then

$4 = \dfrac{2ab}{a+b} \Rightarrow a + b = \dfrac{ab}{2}$

$A = \dfrac{a+b}{2}$ and $G = \sqrt{ab}$

Also, $2A + G^2 = 27$

$\therefore \quad a + b + ab = 27 \Rightarrow \dfrac{ab}{2} + ab = 27 \Rightarrow ab = 18$

and hence $a + b = 9$.

Only option A satisfies this condition.

75. Suppose that x to be added then numbers 13, 15, 19 so that new numbers $x + 13$, $15 + x$, $19 + x$ will be in H.P.

$\Rightarrow (15 + x) = \dfrac{2(x+13)(19+x)}{x+13+x+19}$

$\Rightarrow x^2 + 31x + 240 = x^2 + 32x + 247$

$\Rightarrow x = -7$

76. Let a be the first term and d be the common difference of the corresponding A.P.

p^{th} term of A.P. $(T_p) = a + (p-1)d$

$= \dfrac{1}{q}$ .....(i)

q^{th} term of A.P. $(T_q) = a + (q-1)d$

$= \dfrac{1}{p}$ .....(ii)

From (i) – (ii), $(p-q)d = \dfrac{1}{q} - \dfrac{1}{p} = \dfrac{p-q}{pq}$

$\Rightarrow d = \dfrac{1}{pq}$

From (i),

$a + (p-1)\dfrac{1}{pq} = \dfrac{1}{q} \Rightarrow a = \dfrac{1}{pq}$

$\therefore \quad T_{pq} = a + (pq-1)d$.

$= \dfrac{1}{pq} + (pq-1)\dfrac{1}{pq} = 1$

Therefore, pq^{th} term is 1.

77. Here, $\dfrac{\log x}{\log a}, \dfrac{\log x}{\log b}, \dfrac{\log x}{\log c}$ are in H.P.

$\Rightarrow \dfrac{\log a}{\log x}, \dfrac{\log b}{\log x}, \dfrac{\log c}{\log x}$ are in A.P.

$\Rightarrow \log_x a, \log_x b, \log_x c$ are in A.P.

$\Rightarrow$ a, b, c are in G.P.

78. We know that $A > G > H$

Where A is arithmetic mean, G is geometric mean and H is harmonic mean, then $A > G$

$\Rightarrow \dfrac{a+b}{2} > \sqrt{ab}$ or $(a+b) > 2\sqrt{ab}$

79. Clearly, $x = \dfrac{1}{1-a}, y = \dfrac{1}{1-b}, z = \dfrac{1}{1-c}$

Since a, b, c are in A.P.

$\Rightarrow 1-a, 1-b, 1-c$ are also in A.P.

$\Rightarrow \dfrac{1}{1-a}, \dfrac{1}{1-b}, \dfrac{1}{1-c}$ are in H.P.

$\therefore \quad x, y, z$ are in H.P.

80. As given, $2b = a + c \Rightarrow 3^{2b} = 3^{a+c}$ or $(3^b)^2 = 3^a.3^c$ i.e $3^a, 3^b, 3^c$ are in G.P.

81. Given that $\dfrac{\text{H.M.}}{\text{G.M.}} = \dfrac{12}{13}$

$\Rightarrow \dfrac{\frac{2ab}{a+b}}{\sqrt{ab}} = \dfrac{12}{13}$ or $\dfrac{a+b}{2\sqrt{ab}} = \dfrac{13}{12}$

$\Rightarrow \dfrac{(a+b)+2\sqrt{ab}}{(a+b)-2\sqrt{ab}} = \dfrac{13+12}{13-12} = \dfrac{25}{1}$

$\Rightarrow \dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \dfrac{5}{1}$

$\Rightarrow \dfrac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \dfrac{5+1}{5-1}$

$\Rightarrow \dfrac{2\sqrt{a}}{2\sqrt{b}} = \dfrac{6}{4}$

$\Rightarrow \left(\dfrac{a}{b}\right)^{1/2} = \dfrac{6}{4}$

$\Rightarrow a : b = 9 : 4$

82. Given that $\dfrac{a}{b} = \dfrac{9}{1}$ or $a = 9b$

Here, $H = \dfrac{2ab}{a+b}$ and $G = \sqrt{ab}$

$\Rightarrow H : G = \dfrac{2ab}{a+b} : \sqrt{ab} = \dfrac{2.9b^2}{10b} : 3b = \dfrac{3}{5}$

Hence, $G : H = 5 : 3$

83. a, b, c are in A.P. $\Rightarrow 2b = a + c$

Now,

$(10^{bx+10})^2 = (10^{ax+10}.\ 10^{cx+10})$

$\Rightarrow 10^{2\,(bx+10)} = 10^{ax+cx+20}$

$\Rightarrow 2(bx + 10) = ax + cx + 20, \forall x$

$\Rightarrow 2b = a + c$ i.e. a, b, c are in A.P.

Hence, these are in G.P. $\forall x$

Alternate Method :

As we know if a, b, c are in A.P., then x^{an+r}, x^{bn+r}, x^{cn+r} are in G.P. for every n and r.

84. $\because$ a, b, c are in G.P. $\Rightarrow b^2 = ac$ .....(i)

Let $a^x = b^y = c^z = k$

$\Rightarrow a = k^{1/x}, b = k^{1/y}, c = k^{1/z}$

Putting these values in (i),

$k^{2/y} = k^{1/x}.k^{1/z} = k^{\frac{1}{x}+\frac{1}{z}}$ *i.e.*, $\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\therefore$ $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. or x, y, z are in H.P.

85. $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\Rightarrow 2^{(b-a)x} = 2^{(c-b)x}$

$\Rightarrow (b - a)x = (c - b)x$

$\Rightarrow (b - a) = (c - b) \forall x, x \neq 0$

$\therefore$ $2^{ax+1}, 2^{bx+1}, 2^{cx+1}$ is a G.P., $\forall x \neq 0$

86. Let $S_n = 1 + 2x + 3x^2 + 4x^3 + \ldots + nx^{n-1}$ .... (i)

$xS_n = x + 2x^2 + 3x^3 + \ldots + nx^n$ ...... (ii)

Subtracting (ii) from (i), we get

$(1 - x) S_n = 1 + x + x^2 + x^3 + \ldots$ to n terms $- nx^n$

$= \frac{(1-x^n)}{1-x} - nx^n$

$\Rightarrow S_n = \frac{(1-x^n) - nx^n(1-x)}{(1-x)^2}$

$= \frac{1-(n+1)x^n + nx^{n+1}}{(1-x)^2}$

87. Let $S = 1 + 2.2 + 3.2^2 + 4.2^3 + \ldots + 100.2^{99}$

$2S = 1.2 + 2.2^2 + 3.2^3 + \ldots + 100.2^{100}$

$S - 2S = 1 + (1.2 + 1.2^2 + 1.2^3$

$+ \ldots$ upto 99 terms$) - 100\cdot2^{100}$

$\therefore$ $S = -1 - \frac{2(2^{99}-1)}{2-1} + 100.2^{100}$

$= -1 - 2^{100} + 2 + 100.2^{100}$

$= 1 + 99 \times 2^{100}$

88. Given series, let

$S_n = 1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + \ldots\ldots + \frac{n}{5^{n-1}}$

$\frac{1}{5}S_n = \frac{1}{5} + \frac{2}{5^2} + \frac{3}{5^3} + \ldots\ldots + \frac{n}{5^n}$

Subtracting,

$\left(1-\frac{1}{5}\right)S_n = 1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3}$

$+ \ldots\ldots +$ upto n terms $- \frac{n}{5^n}$

$\Rightarrow \frac{4}{5}S_n = \frac{1-\frac{1}{5^n}}{\frac{4}{5}} - \frac{n}{5^n}$

$\Rightarrow S_n = \frac{25}{16} - \frac{4n+5}{16 \times 5^{n-1}}$

89. Given fraction $\frac{\sum_{n=1}^{12} n^3}{\sum_{n=1}^{12} n^2} = \frac{\left\{\frac{12(12+1)}{2}\right\}^2}{\frac{12(12+1)(2\times12+1)}{6}}$

$= \frac{12\times13}{4} \times \frac{6}{25} = \frac{234}{25}$

91. $\frac{1}{1\times2} + \frac{1}{3\times4} + \frac{1}{5\times6} + \ldots \infty$

$= \left(1-\frac{1}{2}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{5}-\frac{1}{6}\right) + \ldots$ to ∞

$= \log 2$

92. $S_n = \sum_{r=1}^{n}(4r-3)(4r-1)$

$= \sum_{r=1}^{n}(16r^2 - 16r + 3)$

$= \frac{16n(n+1)(2n+1)}{6} - \frac{16n(n+1)}{2} + 3n$

$= n\left(\frac{16n^2-7}{3}\right)$

93. $S_1 = \frac{n(n+1)}{2}, S_2 = \frac{n(n+1)(2n+1)}{6}$

$S_3 = \left(n\left(\frac{n+1}{2}\right)\right)^2$

For, $S_3(1 + 8S_1) = \frac{n^2(n+1)^2}{4}\left(1 + \frac{8n(n+1)}{2}\right)$

$= \left(\frac{n(n+1)(2n+1)}{6}\right)^2 \times 9$

$= 9S_2^2$

94. Let $t_n = \frac{1}{(n+1)!}$

$$S_n = \frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+....$$

$$= \left[1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+....\right]-\left[1+\frac{1}{1!}\right]$$

$$= e-(1+1)$$

$$= e-2$$

95. $t_r = \frac{1^3+2^3+....+r^3}{(r+1)^2} = \frac{r^2}{4}$

$$\therefore \quad S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\frac{r^2}{4} = \frac{1}{4}\sum_{r=1}^{n} r^2$$

$$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{24}$$

96. $1^3+3^3+5^3+....+21^3$

$$= (1^3+2^3+3^3+4^3+....+21^3) - (2^3+4^3+6^3+.....+20^3)$$

$$= \sum_{r=1}^{21} r^3 - 8\sum_{r=1}^{10} r^3$$

$$= \frac{(21)^2(21+1)^2}{4} - \frac{8\times 10^2(10+1)^2}{4}$$

$$= 29161$$

97. $t_r = \frac{1+2+3+.....+r}{r} = \frac{\frac{r(r+1)}{2}}{r} = \frac{r+1}{2}$

$$\therefore \quad S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\frac{r+1}{2} = \frac{1}{2}\left[\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)}{2}+n\right]$$

$$= \frac{1}{4}(n^2+3n)$$

$$= \frac{n(n+3)}{4}$$

98. Given ratio $= \frac{\frac{1}{2}\left(e+\frac{1}{e}\right)-1}{\frac{1}{2}\left(e-\frac{1}{e}\right)} = \frac{(e-1)^2}{(e-1)(e+1)}$

$$= \frac{e-1}{e+1}$$

99. We have $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+.......$

$$= 1\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+.....$$

$$= \sqrt{2}\,[1+2+3+4+....\text{ upto 24 terms}]$$

$$= \sqrt{2}\times\frac{24\times 25}{2}$$

$$= 300\sqrt{2}$$

100. Let $S = 2+4+7+11+16+....+t_n$

$S = 2+4+7+11+16+.....t_{n-1}+t_n$

Subtracting, we get

$0 = 2+\{2+3+4+.....+(t_n-t_{n-1})\}-t_n$

$\Rightarrow t_n = 1+\{1+2+3+4+.....\text{ upto n terms}\}$

$$\Rightarrow t_n = 1+\frac{1}{2}n(n+1)$$

$$= \frac{2+n^2+n}{2} = \frac{n^2+n+2}{2}$$

101. Let $S = i-2-3i+4+5i+.....+100i^{100}$

$\Rightarrow S = i+2i^2+3i^3+4i^4+5i^5+.....+100i^{100}$

$\Rightarrow iS = i^2+2i^3+3i^4+4i^5+...+99i^{100}+100i^{101}$

$\therefore \quad S-iS = [i+i^2+i^3+i^4+....+i^{100}]-100i^{101}$

$\Rightarrow S(1-i) = 0-100i^{101} = -100\,i$

$$\therefore \quad S = \frac{-100i}{1-i} = -50i(1+i) = -50(i-1)$$

$$= 50(1-i)$$

102. Here, $T_r = \frac{1}{r(r+1)}$, $r = 1, 2, ...n$

$$\Rightarrow T_r = \frac{1}{r}-\frac{1}{r+1}$$

$$\therefore \quad \text{Required sum} = \sum_{r=1}^{n} T_r$$

$$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)....+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$= 1-\frac{1}{n+1} = \frac{n}{n+1}$$

103. $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+.... = 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....$

which is the expansion of e^{-1}

104. $e^{-x} = (1-x)+\frac{x^2}{2!}\left(1-\frac{x}{3}\right)+\frac{x^4}{4!}\left(1-\frac{x}{5}\right)+...$

$$\therefore \quad e^{-1} = (1-1)+\frac{1}{2!}\left(1-\frac{1}{3}\right)+\frac{1}{4!}\left(1-\frac{1}{5}\right)+....$$

$$= \frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+....$$

105. $\Sigma n^2 = 330 + \Sigma n$

$\Rightarrow \dfrac{n(n+1)(2n+1)}{6} = 330 + \dfrac{n(n+1)}{2}$

$\Rightarrow \dfrac{n(n+1)}{2}\left[\dfrac{2n+1}{3} - 1\right] = 330$

$\Rightarrow \dfrac{n(n+1)}{2} \cdot \dfrac{2(n-1)}{3} = 330$

$\Rightarrow n(n+1)(n-1) = 990$

$\Rightarrow n = 10$

106. $T_n = \dfrac{3^n - 1}{3^n} = 1 - \left(\dfrac{1}{3}\right)^n$

$$S_n = n - \sum_{n=1}^{n}\left(\frac{1}{3}\right)^n = n - \frac{\frac{1}{3}\left[1 - \left(\frac{1}{3}\right)^n\right]}{\left(1 - \frac{1}{3}\right)}$$

$= n - \dfrac{1}{2}(1 - 3^{-n})$

$= n + \dfrac{1}{2}(3^{-n} - 1)$

107. $\sum_{n=1}^{20}(n^3) - \sum_{n=1}^{10}(n^3) = \left[\dfrac{n(n+1)}{2}\right]^2_{n=20} - \left[\dfrac{n(n+1)}{2}\right]^2_{n=10}$

$\Rightarrow \left[\dfrac{20 \times 21}{2}\right]^2 - \left[\dfrac{10 \times 11}{2}\right]^2$

$= 44100 - 3025$

$= 41075$

108. The series is

$\dfrac{2}{1!} + \dfrac{(2+5)}{2!} + \dfrac{(2+5+8)}{3!} + \dfrac{(2+5+8+11)}{4!} + \ldots\ldots$

Hence, $T_n = \dfrac{(2+5+8+\ldots\ldots n \text{ terms})}{n!}$

$= \dfrac{\frac{n}{2}[2.2 + (n-1)3]}{n!}$

$T_n = \dfrac{n(3n+1)}{2(n)!}$

109. Here $T_n = \dfrac{1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3}{1 + 3 + 5 + \ldots\ldots\ldots \text{upto } n \text{ terms}}$

$= \dfrac{\Sigma n^3}{\frac{n}{2}[2 + (n-1)2]} = \dfrac{1}{4}\,\dfrac{n^2(n+1)^2}{n^2}$

$= \dfrac{1}{4}(n^2 + 2n + 1)$

Competitive Thinking

1. Given that, $t_p = a + (p-1)d = q$ …. (i)

and $t_q = a + (q-1)d = p$ …. (ii)

From (i) and (ii), we get $d = -\dfrac{(p-q)}{(p-q)} = -1$

Putting the value of d in equation (i), we get

$a = p + q - 1$

$t_r = a + (r-1)d = (p+q-1) + (r-1)(-1)$

$= p + q - r$

2. We have, $\tan n\theta = \tan m\theta$

$\Rightarrow n\theta = N\pi + (m\theta)$

$\Rightarrow \theta = \dfrac{N\pi}{n-m}$, putting $N = 1,2,3\ldots\ldots$, we get

$\dfrac{\pi}{n-m}, \dfrac{2\pi}{n-m}, \dfrac{3\pi}{n-m}\ldots\ldots$ which are in A.P.

Since, common difference, $d = \dfrac{\pi}{n-m}$.

3. Given series $63 + 65 + 67 + 69 + \ldots\ldots$ …. (i)

and $3 + 10 + 17 + 24 + \ldots\ldots$ …. (ii)

Now from (i), m^{th} term $= (2m + 61)$ and m^{th} term of (ii) series $= (7m - 4)$

According to the given condition,

$7m - 4 = 2m + 61$

$\Rightarrow 5m = 65 \Rightarrow m = 13$

4. Given series $27 + 9 + 5\dfrac{2}{5} + 3\dfrac{6}{7} + \ldots\ldots$

$= 27 + \dfrac{27}{3} + \dfrac{27}{5} + \dfrac{27}{7} + \ldots\ldots + \dfrac{27}{2n-1} + \ldots\ldots$

Hence, n^{th} term of given series $t_n = \dfrac{27}{2n-1}$

So, $t_9 = \dfrac{27}{2 \times 9 - 1} = \dfrac{27}{17} = 1\dfrac{10}{17}$

5. If a, b, c are in A.P., then $2b = a + c$

So, $\dfrac{(a-c)^2}{(b^2 - ac)} = \dfrac{(a-c)^2}{\left\{\left(\frac{a+c}{2}\right)^2 - ac\right\}}$

$= \dfrac{4(a-c)^2}{[a^2 + c^2 + 2ac - 4ac]}$

$= \dfrac{4(a-c)^2}{(a-c)^2} = 4$

Trick: Put a = 1, b = 2, c = 3, then the required value is $\dfrac{4}{1} = 4$.

6. $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P.

$\Rightarrow 2\log_3(2^x - 5) = \log_3\left[(2)\left(2^x - \frac{7}{2}\right)\right]$

$\Rightarrow (2^x - 5)^2 = 2^{x+1} - 7$

$\Rightarrow 2^{2x} - 12.2^x + 32 = 0$

$\Rightarrow x = 2, 3$

But $x = 2$ does not hold, hence $x = 3$

7. Required ratio is $\frac{44}{99} = \frac{4}{9}$

8. According to the given condition,

$p\{a+(p-1)d\} = q\{a+(q-1)d\}$

$\Rightarrow a(p-q) + (p^2 - q^2)d + (q-p)d = 0$

$\Rightarrow (p-q)\{a + (p+q-1)d\} = 0$

$\Rightarrow a + (p+q-1)d = 0$ .... $[\because p \neq q]$

$\Rightarrow t_{p+q} = 0$

9. We have $\frac{S_{n_1}}{S_{n_2}} = \frac{2n+3}{6n+5}$

$$\Rightarrow \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2n+3}{6n+5}$$

$$\Rightarrow \frac{2\left[a_1 + \left(\frac{n-1}{2}\right)d_1\right]}{2\left[a_2 + \left(\frac{n-1}{2}\right)d_2\right]} = \frac{2n+3}{6n+5}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{2n+3}{6n+5}$$

Put $n = 25$ then $\frac{a_1 + 12d_1}{a_2 + 12d_2} = \frac{2(25)+3}{6(25)+5}$

$$\Rightarrow \frac{t_{13_1}}{t_{13_2}} = \frac{53}{155}$$

10. $t_m = a + (m-1)d = \frac{1}{n}$ and

$t_n = a + (n-1)d = \frac{1}{m}$

On solving, $a = \frac{1}{mn}$ and $d = \frac{1}{mn}$

$\therefore$ $t_{mn} = a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$

11. Series, $2 + 5 + 8 + 11 + \ldots$

$a = 2, d = 3$ and let number of terms be n,

then sum of A.P. $= \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 60100 = \frac{n}{2}\{2 \times 2 + (n-1)3\}$

$\Rightarrow 120200 = n(3n+1)$

$\Rightarrow 3n^2 + n - 120200 = 0$

$\Rightarrow (n-200)(3n+601) = 0$

Hence, $n = 200$

12. The series of all natural numbers is

3, 6, 9, 12, ... 99

Here $n = \frac{99}{3} = 33, a = 3, d = 3$

$\therefore$ $S_{33} = \frac{33}{2}\{2 \times 3 + (33-1)3\}$

$= \frac{33}{2} \times 102$

$= 33 \times 51$

$= 1683$

13. Acording to the given condition,

$\frac{n}{2}\{2a + (n-1)d\} = \frac{m}{2}\{2a + (m-1)d\}$

$\Rightarrow 2a(m-n) + d(m^2 - m - n^2 + n) = 0$

$\Rightarrow (m-n)\{2a + d(m+n-1)\} = 0$

$\Rightarrow 2a + (m+n-1)d = 0$ ....$[\because m \neq n]$

$\therefore$ $S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$

$= \frac{m+n}{2}\{0\}$

$= 0$

14. As given $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$

Where d is the common difference of the given A.P.

Also $a_n = a_1 + (n-1)d$

Then by rationalising each term,

$$\frac{1}{\sqrt{a_2}+\sqrt{a_1}} + \frac{1}{\sqrt{a_3}+\sqrt{a_2}} + \ldots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3}-\sqrt{a_2}}{a_3 - a_2} + \ldots + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n - a_{n-1}}$$

$$= \frac{1}{d}\left(\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \ldots + \sqrt{a_n} - \sqrt{a_{n-1}}\right)$$

$= \frac{1}{d}(\sqrt{a_n} - \sqrt{a_1}) = \frac{1}{d}\left(\frac{a_n - a_1}{\sqrt{a_n} + \sqrt{a_1}}\right)$

$= \frac{1}{d}\left\{\frac{(n-1)d}{\sqrt{a_n} + \sqrt{a_1}}\right\} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}}$

15. Given that $S_n = nA + n^2B$
Putting n = 1, 2, 3, ... we get
$S_1 = A + B$, $S_2 = 2A + 4B$, $S_3 = 3A + 9B$
....
....
Therefore,
$T_1 = S_1 = A + B$,
$T_2 = S_2 - S_1 = A + 3B$,
$T_3 = S_3 - S_2 = A + 5B$,
....
....
Hence, the sequence is
$(A + B), (A + 3B), (A + 5B)$.......
Here, a = A + B and common difference d = 2B

16. It is not possible to express $a + b + 4c - 4d + e$ in terms of a.

17. Let the number of sides of the polygon be n.
Then the sum of interior angles of the polygon
$= (2n - 4)\frac{\pi}{2} = (n - 2)\pi$
Since, the angles are in A.P.and a = 120°, d = 5 therefore,
$\frac{n}{2}[2 \times 120 + (n - 1)5] = (n - 2)180$
$\Rightarrow n^2 - 25n + 144 = 0$
$\Rightarrow (n - 9)(n - 16) = 0$
$\Rightarrow n = 9, 16$
But n = 16 gives,
$T_{16} = a + 15d$
$= 120° + 15.5°$
= 195° which is impossible, as interior angle cannot be greater than 180°.
Hence, n = 9.

18. As given
$d = a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1}$
$\therefore$ $\sin d \{\text{cosec } a_1 \text{ cosec } a_2 + \ldots + \text{cosec } a_{n-1} \text{ cosec } a_n\}$
$= \frac{\sin(a_2 - a_1)}{\sin a_1 . \sin a_2} + \ldots + \frac{\sin(a_n - a_{n-1})}{\sin a_{n-1} \sin a_n}$
$= (\cot a_1 - \cot a_2) + (\cot a_2 - \cot a_3) + \ldots + (\cot a_{n-1} - \cot a_n)$
$= \cot a_1 - \cot a_n$

19. $a_1, a_2, a_3, \ldots, a_{n+1}$ are in A.P. and common difference = d
Let $S = \frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots\ldots + \frac{1}{a_na_{n+1}}$
$\Rightarrow S = \frac{1}{d}\left\{\frac{d}{a_1a_2} + \frac{d}{a_2a_3} + \ldots + \frac{d}{a_n\, a_{n+1}}\right\}$
$\Rightarrow S = \frac{1}{d}\left\{\frac{a_2 - a_1}{a_1a_2} + \frac{a_3 - a_2}{a_2a_3} + \ldots + \frac{a_{n+1} - a_n}{a_n\, a_{n+1}}\right\}$
$\Rightarrow S = \frac{1}{d}\left\{\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \ldots + \frac{1}{a_n} - \frac{1}{a_{n+1}}\right\}$
$\Rightarrow S = \frac{1}{d}\left\{\frac{1}{a_1} - \frac{1}{a_{n+1}}\right\} = \frac{1}{d}\left\{\frac{a_{n+1} - a_1}{a_1a_{n+1}}\right\}$
$\Rightarrow S = \frac{1}{d}\left(\frac{nd}{a_1a_{n+1}}\right) = \frac{n}{a_1a_{n+1}}$
Trick: Check for n = 2.

20. $164 = (3m^2 + 5m) - \{3(m - 1)^2 + 5(m - 1)\}$
$= (3m^2 + 5m) - 3m^2 + 6m - 3 - 5m + 5$
$\Rightarrow 164 = 6m + 2 \Rightarrow m = 27$

21. $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$
$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$
$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$
But $2y = x + z$ [$\because x, y, z$ are in A.P.]
$\therefore$ $1 - y^2 = 1 - xz$
$\Rightarrow y^2 = xz$
$\because$ x, y, z are both in G.P. and A.P.,
$\therefore$ $x = y = z$

22. Let a – d, a, a + d be the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$
Then, (a – d) + a + (a + d) = 12 and $(a - d)a(a + d) = 28$
$\Rightarrow 3a = 12$ and $a(a^2 - d^2) = 28$
$\Rightarrow a = 4$ and $a(a^2 - d^2) = 28$
$\Rightarrow 16 - d^2 = 7$
$\Rightarrow d = \pm 3$

23. Let the first term be a and common difference be d.
Given, $\frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + \ldots + a_q} = \frac{p^2}{q^2}$
$\Rightarrow \frac{pa + d[1 + 2 + \ldots + (p-1)]}{qa + d[1 + 2 + \ldots + (q-1)]} = \frac{p^2}{q^2}$

$$\Rightarrow \frac{pa+\frac{p(p-1)}{2}d}{qa+\frac{q(q-1)}{2}d}=\frac{p^2}{q^2} \Rightarrow \frac{a+\left(\frac{p-1}{2}\right)d}{a+\left(\frac{q-1}{2}\right)d}=\frac{p}{q}$$

We have to find, $\frac{a_6}{a_{21}}=\frac{a+5d}{a+20d}$

Put $\frac{p-1}{2}=5$ and $\frac{q-1}{2}=20$

$\Rightarrow p = 11$ and $\Rightarrow q = 41$

$\therefore \quad \frac{a+5d}{a+20d}=\frac{11}{41}$

24. Here, a = ₹ 200, d = ₹ 40
Saving in first two months = ₹ 400
Remained saving = 200 + 240 + 280 + ... upto n terms

$\Rightarrow \frac{n}{2}[400+(n-1)40] = 11040-400$

$\Rightarrow 200n + 20n^2 - 20n = 10640$

$\Rightarrow 20n^2 + 180n - 10640 = 0$

$\Rightarrow n^2 + 9n - 532 = 0$

$\Rightarrow (n + 28)(n - 19) = 0$

$\Rightarrow n = 19$

$\therefore$ Number of months = 19 + 2 = 21

25. $\frac{1}{S_1S_2}+\frac{1}{S_2S_3}+...+\frac{1}{S_{100}S_{101}}=\frac{1}{6}$

$\Rightarrow \frac{1}{d}\left[\frac{S_2-S_1}{S_1S_2}+\frac{S_3-S_2}{S_2S_3}+...+\frac{S_{101}-S_{100}}{S_{100}S_{101}}\right]=\frac{1}{6}$

....[$\because S_2 - S_1 = S_3 - S_2 = \ldots = d$]

$\Rightarrow \frac{1}{d}\left[\frac{1}{S_1}-\frac{1}{S_2}+\frac{1}{S_2}-\frac{1}{S_3}+...+\frac{1}{S_{100}}-\frac{1}{S_{101}}\right]=\frac{1}{6}$

$\Rightarrow \frac{1}{d}\left[\frac{1}{S_1}-\frac{1}{S_{101}}\right]=\frac{1}{6} \Rightarrow \frac{1}{d}\left[\frac{1}{S_1}-\frac{1}{S_1+100d}\right]=\frac{1}{6}$

$\Rightarrow \frac{1}{d}\left[\frac{100d}{S_1.(S_1+100d)}\right]=\frac{1}{6}$

$\Rightarrow S_1.(S_1 + 100d) = 600$ ....(i)

Given, $S_1 + S_{101} = 50$

$\Rightarrow S_1 + (S_1 + 100d) = 50 \Rightarrow 2S_1 + 100d = 50$

$\Rightarrow S_1 + 50d = 25$

$\Rightarrow S_1 = 25 - 50d$ ....(ii)

Putting (ii) in (i), we get

$(25 - 50d).(25 + 50d) = 600$

$\Rightarrow 625 - 2500 d^2 = 600$

$\Rightarrow d^2 = \frac{1}{100} \Rightarrow d = \pm\frac{1}{10}$

$\therefore \quad |S_1 - S_{101}| = |S_1 - (S_1 + 100d)|$

$= |-100d| = 100\,|d|$ ....[$\because |xy| = |x|.|y|$]

$\therefore \quad |S_1 - S_{101}| = 10$ ....[$\because d = \pm 1/10$]

26. Since, $a_1 = 0$

$\therefore \quad a_2 = d, a_3 = 2d, \ldots$

$\therefore \quad \left(\frac{a_3}{a_2}+\frac{a_4}{a_3}+...+\frac{a_n}{a_{n-1}}\right)-a_2\left(\frac{1}{a_2}+\frac{1}{a_3}+...+\frac{1}{a_{n-2}}\right)$

$= \left(\frac{2d}{d}+\frac{3d}{2d}+...+\frac{(n-1)d}{(n-2)d}\right)$

$-d\left(\frac{1}{d}+\frac{1}{2d}+...+\frac{1}{(n-3)d}\right)$

$= \left(\frac{2}{1}+\frac{3}{2}+...+\frac{n-1}{n-2}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-3}\right)$

$= \left[(1+1)+\left(1+\frac{1}{2}\right)+...+\left(\frac{n-1}{n-2}\right)\right]$

$-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n-3}\right)$

$= (n-3)+\frac{n-1}{n-2} = (n-3)+1+\frac{1}{n-2}$

$= (n-2)+\frac{1}{n-2}$

27. Since, a, 9, 3a – b and 3a + b are in A.P.

$\therefore \quad 9 - a = (3a + b) - (3a - b)$

$\Rightarrow 9 - a = 2b \Rightarrow a + 2b = 9$ ....(i)

Also, $9 - a = (3a - b) - 9$

$\Rightarrow 4a - b = 18$ ....(ii)

Eliminating b from (i) and (ii), we get

$4a - 18 = (9 - a)/2$

$\Rightarrow 8a - 36 = 9 - a \Rightarrow 9a = 45 \Rightarrow a = 5$

So, first 2 terms of the A.P. are 5 and 9

So, a = 5, d = 4

$\therefore \quad 2011^{th}$ term $= a + 2010d$

$= 5 + 2010 \times 4 = 8045$

28. The sequence can be written as log a, (2 log a – log b), (3 log a – 2 log b), ... which are in A.P. having common difference as log a – log b.

29. According to the given condition,

$4500 = 150 \times 10$

$+ \{148 + 146 + \ldots$ upto n terms$\}$

$= 1500 + \frac{n}{2}\{296 + (n - 1)(-2)\}$

$\Rightarrow n^2 - 149n + 3000 = 0 \Rightarrow (n - 24)(n - 125) = 0$

$\Rightarrow n = 24$ ....[$\because n \neq 125$]

So, total time taken = 10 + 24 = 34 min.

30. 12, 19, ..., 96 is the series of numbers which are of two digits and leave remainder 5 when divided by 7.
Here, a = 12, d = 7
Last term = 96
$\therefore$ $96 = 12 + (n-1)7 \Rightarrow n = 13$
$\therefore$ $S_{13} = \frac{13}{2}[2(12)+(13-1)7] = 702$

31. Let the first term be a and common difference be d.
The last 3 terms are T_{23}, T_{22} and T_{21}.
According to the given condition,
$T_{21} + T_{22} + T_{23} = 261$
$\Rightarrow (a + 20d) + (a + 21d) + (a + 22d) = 261$
$\Rightarrow 3a + 63d = 261$ ....(i)
Also, sum of 3 middle terms = 141
$\Rightarrow T_{11} + T_{12} + T_{13} = 141$
$\Rightarrow (a + 10d) + (a + 11d) + (a + 12d) = 141$
$\Rightarrow 3a + 33d = 141$ ....(ii)
Solving (i) and (ii), we get a = 3

32. $S_1 = a_2 + a_4 + a_6 + a_8 + ... + a_{100}$
$S_2 = a_1 + a_3 + a_5 + a_7 + ... + a_{99}$
$\therefore$ $S_1 - S_2 = (a_2 - a_1) + (a_4 - a_3) + ... + (a_{100} - a_{99})$
$= d + d + ... + d = 50d \Rightarrow d = \frac{S_1 - S_2}{50}$

33. According to the given condition,
$100(a + 99d) = 50(a + 49d)$
$\Rightarrow 2a + 198d = a + 49d$
$\Rightarrow a + 149d = 0$
$\therefore$ $T_{150} = a + 149d = 0$

34. Let first term of G.P.= A and common ratio = r
We know that n^{th} term of G.P. = Ar^{n-1}
Now $t_4 = a = Ar^3$, $t_7 = b = Ar^6$ and $t_{10} = c = Ar^9$
Relation $b^2 = ac$ is true because
$b^2 = (Ar^6)^2 = A^2r^{12}$ and $ac = (Ar^3)(Ar^9) = A^2r^{12}$
Alternate method : As we know, if p, q, r in A.P., then p^{th}, q^{th}, r^{th} terms of a G.P. are always in G.P., therefore, a, b, c will be in G.P. i.e. $b^2 = ac$.

35. Given that x, $2x + 2$, $3x + 3$ are in G.P.
Therefore,
$(2x + 2)^2 = x(3x + 3)$
$\Rightarrow x^2 + 5x + 4 = 0$
$\Rightarrow (x + 4)(x + 1) = 0$
$\Rightarrow x = -1, -4$
Now, first term: $a = x$
and second term: $ar = 2(x + 1)$
$\Rightarrow r = \frac{2(x+1)}{x}$
then 4^{th} term $= ar^3 = x\left[\frac{2(x+1)}{x}\right]^3$
$= \frac{8}{x^2}(x+1)^3$
Putting, $x = -4$
We get, $t_4 = \frac{8}{16}(-3)^3 = -\frac{27}{2} = -13.5$

36. x, y, z are in G.P., then $y^2 = x.z$
Now $a^x = b^y = c^z = m$
$\Rightarrow x \log_e a = y \log_e b = z \log_e c = \log_e m$
$\Rightarrow x = \log_a m, y = \log_b m, z = \log_c m$
Again as x, y, z are in G.P., so $\frac{y}{x} = \frac{z}{y}$
$\Rightarrow \frac{\log_b m}{\log_a m} = \frac{\log_c m}{\log_b m}$
$\Rightarrow \log_b a = \log_c b$

37. $t_5 = ar^4 = \frac{1}{3}$ .....(i)
and $t_9 = ar^8 = \frac{16}{243}$ .....(ii)
Solving (i) and (ii), we get $r = \frac{2}{3}$ and $a = \frac{27}{16}$
Now 4^{th} term $= ar^3 = \frac{3^3}{2^4}.\frac{2^3}{3^3} = \frac{1}{2}$

38. Let first term and common ratio of G.P. are respectively a and r, then under condition,
$t_n = t_{n-1} + t_{n-2}$
$\Rightarrow ar^{n-1} = ar^{n-2} + ar^{n-3}$
$\Rightarrow ar^{n-1} = ar^{n-1}r^{-1} + ar^{n-1}r^{-2}$
$\Rightarrow 1 = \frac{1}{r} + \frac{1}{r^2}$
$\Rightarrow r^2 - r - 1 = 0$
$\Rightarrow r = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$
Taking only (+) sign ($\because r > 1$)

39. $\therefore$ n^{th} term of series $= ar^{n-1} = a(3)^{n-1}$
$= 486$ .....(i)
and sum of n terms of series.
$S_n = \frac{a(3^n - 1)}{3-1} = 728$ ($\because r > 1$) ......(ii)

From (i), $a\left(\frac{3^n}{3}\right) = 486$ or $a.3^n = 3 \times 486$

$= 1458$

From (ii), $a.3^n - a = 728 \times 2$

or $a.3^n - a = 1456$

$\therefore$ $1458 - a = 1456$

$\Rightarrow a = 2$

40. Let G_1, G_2, G_3, G_4, G_5 be the G.M.'s are inserted between 486 and $\frac{2}{3}$. So total terms are 7.

$t_n = ar^{n-1}$

$\Rightarrow \frac{2}{3} = 486(r)^6 \Rightarrow r = \frac{1}{3}$

Hence, 4^{th} G.M. will be, $t_5 = ar^4$

$= 486\left(\frac{1}{3}\right)^4$

$= 6.$

41. Since $n^m + 1$ divides $1 + n + n^2 + \ldots\ldots + n^{127}$

Therefore, $\frac{1 + n + n^2 + \ldots\ldots + n^{127}}{n^m + 1}$ is an integer

$\Rightarrow \frac{1 - n^{128}}{1 - n} \times \frac{1}{n^m + 1}$ is an integer

$\Rightarrow \frac{(1 - n^{64})(1 + n^{64})}{(1 - n)(n^m + 1)}$

is an integer, when largest m = 64.

42. $0.2\dot{3}\dot{4} = \frac{234 - 2}{990} = \frac{232}{990}$

43. $3 + 3\alpha + 3\alpha^2 + 3\alpha^3 + \ldots\ldots \infty = \frac{45}{8}$

$\Rightarrow 3\left[\frac{1}{1 - \alpha}\right] = \frac{45}{8} \Rightarrow 8 = 15(1 - \alpha) \Rightarrow \alpha = \frac{7}{15}$

44. Since the series are in G.P., therefore

$x = \frac{1}{1 - a}$ and $y = \frac{1}{1 - b}$

$\therefore$ $a = \frac{x - 1}{x}, b = \frac{y - 1}{y}$

$\therefore$ $1 + ab + a^2b^2 + \ldots\ldots \infty$

$= \frac{1}{1 - ab} = \frac{1}{1 - \frac{x - 1}{x} \cdot \frac{y - 1}{y}} = \frac{xy}{x + y - 1}$

45. $0.4\dot{2}\dot{3} = \frac{423 - 4}{990} = \frac{419}{990}$

46. $y = x - x^2 + x^3 - x^4 + \ldots\ldots \infty$

Then $xy = x^2 - x^3 + x^4 - \ldots\ldots \infty$

Adding, $y + xy = x + 0 + 0 \ldots\ldots + 0$

$\Rightarrow x - xy = y$

$\Rightarrow x(1 - y) = y$

$\Rightarrow x = \frac{y}{1 - y}$

Alternate method:

$y = \frac{x}{1 - (-x)} \Rightarrow y = \frac{x}{1 + x}$

$\Rightarrow y + yx = x \Rightarrow x = \frac{y}{1 - y}$

47. We have $\frac{a}{1 - r} = x$

and $\frac{a^2}{1 - r^2} = \frac{a}{1 - r} \cdot \frac{a}{1 + r} = y$

$\Rightarrow y = x \cdot \frac{a}{1 + r} = x\frac{x(1 - r)}{1 + r}$

$\Rightarrow \frac{y}{x^2} = \frac{1 - r}{1 + r} \Rightarrow \frac{x^2}{y} = \frac{1 + r}{1 - r}$

$\Rightarrow \frac{x^2}{y}(1 - r) = 1 + r$

$\Rightarrow r\left[1 + \frac{x^2}{y}\right] = -1 + \frac{x^2}{y}$

$\Rightarrow r = \frac{x^2 - y}{x^2 + y}$

48. Let r be the common ratio of the G.P. Then

$S = \frac{a}{1 - r} \Rightarrow r = 1 - \frac{a}{S}$

Now S_n = Sum of n terms

$= a\left(\frac{1 - r^n}{1 - r}\right) = \frac{a}{1 - r}(1 - r^n)$

$= S\left[1 - \left(1 - \frac{a}{S}\right)^n\right]$

49. $0.14189189189\ldots.$

$= 0.14 + 0.00189 + 0.00000189 + \ldots\ldots$

$= \frac{14}{100} + 189\left[\frac{1}{10^5} + \frac{1}{10^8} + \ldots.\infty\right]$

$= \frac{7}{50} + 189\left[\frac{\frac{1}{10^5}}{1 - \left(\frac{1}{10^3}\right)}\right]$

$= \frac{7}{50} + 189\left[\frac{1}{10^5}\times\frac{10^3}{999}\right]$

$= \frac{7}{50} + \frac{189}{999\times100} = \frac{7}{50} + \frac{7}{3700}$

$= \frac{7}{50} + \frac{7}{25\times148} = \frac{21}{148}$

Alternate Method:

$0.14\dot{1}\dot{8}\dot{9}$

$= \frac{14189-14}{99900} = \frac{14175}{99900} = \frac{21}{148}$

50. Clearly it is a infinite G.P. whose common ratio is 0.24.

$\therefore$ $S_\infty = \frac{a}{1-r} = \frac{5.05}{1-0.24} = 6.64474$

51. Series 3 + 33 + 333 +.........+ n terms
Given series can be written as,

$= \frac{1}{3}[9 + 99 + 999 + \ldots + n \text{ terms}]$

$= \frac{1}{3}[(10-1) + (10^2-1) + (10^3-1) + \ldots + n \text{ terms}]$

$= \frac{1}{3}[10 + 10^2 + \ldots + 10^n] - \frac{1}{3}[1+1+1+\ldots+n \text{ terms}]$

$= \frac{1}{3}.\frac{10(10^n-1)}{10-1} - \frac{1}{3}.n$

$= \frac{1}{3}\left[\frac{10^{n+1}-10}{9} - n\right]$

$= \frac{1}{3}\left[\frac{10^{n+1}-9n-10}{9}\right]$

$= \frac{1}{27}[10^{n+1}-9n-10]$

52. $\frac{ar^n - a}{r-1} = 364$

$\Rightarrow \frac{ar^{n-1}.r - a}{r-1} = 364$ .....(i)

$\Rightarrow \frac{3\times243 - a}{2} = 364$

$\Rightarrow a = 1$

Now, putting this in (i), n = 6

53. Let α and β be the roots of equation $x^2 - 18x + 9 = 0$

$\therefore$ G.M. of α and $\beta = \sqrt{\alpha\beta} = \sqrt{9} = 3$ [$\because \alpha\beta = 9$]

54. Here, $\frac{a}{1-r} = 4$ and $ar = \frac{3}{4}$. Dividing these,

$r(1-r) = \frac{3}{16}$ or $16r^2 - 16r + 3 = 0$

or $(4r-3)(4r-1) = 0$

$r = \frac{1}{4}, \frac{3}{4}$ and $a = 3, 1$ so $(a, r) = \left(3, \frac{1}{4}\right), \left(1, \frac{3}{4}\right)$.

55. $\frac{\sqrt{2}+1}{\sqrt{2}-1}, \frac{1}{\sqrt{2}(\sqrt{2}-1)}, \frac{1}{2}, \ldots\ldots$

Common ratio of the series $= \frac{1}{\sqrt{2}(\sqrt{2}+1)}$

$\therefore$ sum $= \frac{a}{1-r} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \Big/ \left(1 - \frac{1}{\sqrt{2}(\sqrt{2}+1)}\right)$

$= \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)}.\frac{\sqrt{2}(\sqrt{2}+1)}{(1+\sqrt{2})}$

$= \sqrt{2}(\sqrt{2}+1)^2$

56. Common ratio (r) $= \frac{2}{x}$

For sum to be finite $r < 1 \Rightarrow \frac{2}{x} < 1$

$\Rightarrow 2 < x$

$\Rightarrow x > 2$

57. $1 - \cos\alpha = \frac{1}{2-\sqrt{2}} = 1 + \frac{1}{\sqrt{2}}$

$\left[\because \frac{a}{1-r} = 2-\sqrt{2}\right]$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$\Rightarrow \alpha = \frac{3\pi}{4}$

58. Let $a^{1/x} = b^{1/y} = c^{1/z}$

$\Rightarrow a = k^x, b = k^y, c = k^z$

Now, a, b, c are in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow k^{2y} = k^x.k^z = k^{x+z}$

$\Rightarrow 2y = x + z$

$\Rightarrow x, y, z$ are in A.P.

59. $r = \frac{1}{3}\sqrt{\frac{20}{3}.\frac{9}{10}} = \frac{\sqrt{60}}{10} = \sqrt{\frac{6}{10}} = \sqrt{\frac{3}{5}}$

$\therefore$ $t_5 = ar^4 = \left(\frac{10}{9}\right)\left(\frac{3}{5}\right)^2 = \frac{10}{9}.\frac{9}{25} = \frac{2}{5}$

60. The given series is a G.P. with a = i, r = –i

$\therefore$ $S_{100} = \dfrac{i(1-i^{100})}{1+i}$

$= \dfrac{i(1-(i^2)^{50})}{1+i}$

$= \dfrac{i(1-1)}{1+i} = 0$

61. Let the G.P. be a, ar, ar^2, ar^3, ..., ar^{48}, ar^{49} i.e., a_1 = a, a_2 = ar, a_3 = ar^2, ..., a_{49} = ar^{48} and $a_{50} = ar^{49}$

$\therefore$ $\dfrac{a_1 - a_3 + a_5 - ..... + a_{49}}{a_2 - a_4 + a_6 - ..... + a_{50}}$

$= \dfrac{a - ar^2 + ar^4 - ..... + ar^{48}}{ar - ar^3 + ar^5 - ..... + ar^{49}}$

$= \dfrac{\dfrac{a\left(1-(-r^2)^{25}\right)}{1-(-r^2)}}{\dfrac{ar\left(1-(-r^2)^{25}\right)}{1-(-r^2)}}$

$= \dfrac{1}{r} = \dfrac{a}{ar} = \dfrac{a_1}{a_2}$

62. $(32)(32)^{1/6}(32)^{1/36}....\infty = (32)^{1+\frac{1}{6}+\frac{1}{36}+....\infty}$

$= (32)^{\frac{1}{1-(1/6)}} = (32)^{\frac{1}{5/6}} = (32)^{\frac{6}{5}}$

$= 2^6 = 64$

63. $1 + \sin x + \sin^2 x + ...$ upto $\infty = 4 + 2\sqrt{3}$

$\Rightarrow \dfrac{1}{1-\sin x} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \sin x = \dfrac{1}{2(2+\sqrt{3})}$

$\Rightarrow \sin x = \dfrac{4+2\sqrt{3}-1}{2(2+\sqrt{3})}$

$\Rightarrow \sin x = \dfrac{\sqrt{3}}{2}$

$\Rightarrow x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

64. Let the first four terms be a, –ar, ar^2, $-ar^3$, where r > 0, a > 0

According to the given conditions,

$a - ar = 12$ and $ar^2 - ar^3 = 48$

By solving, we get r = 2 (r > 0)

So, a = –12

65. Let a – d, a, a + d be three numbers in A.P.

$\therefore$ a + d + a + a – d = 15

$\Rightarrow$ a = 5

a – d + 1, a + 4, a + d + 19 are in G.P.

$\Rightarrow$ 6 – d, 9, 24 + d are in G.P.

$\therefore$ 81 = (6 – d) (24 + d)

$\Rightarrow 81 = 144 + 6d - 24d - d^2$

$\Rightarrow d^2 + 18d - 63 = 0$

$\therefore$ d = 3, –21

$\therefore$ the numbers are 2, 5, 8 and 26, 5, –16

66. According to the given condition,

$\dfrac{a}{1-r} = \dfrac{4}{3}$

$\Rightarrow \dfrac{3}{4}\left(\dfrac{1}{1-r}\right) = \dfrac{4}{3} \Rightarrow r = 1 - \dfrac{9}{16} = \dfrac{7}{16}$

67. Let the G.P. be a, ar, ar^2, ar^3, ar^4, ...

$t_2 + t_5 = ar + ar^4 = 216$

$\dfrac{t_4}{t_6} = \dfrac{ar^3}{ar^5} = \dfrac{1}{4}$

$\Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

For r = 2,

$a(2 + 2^4) = 216$

$\Rightarrow a(18) = 216$

$\Rightarrow a = \dfrac{216}{18} = 12$

For r = –2,

$a(-2 + 2^4) = 216$

$\Rightarrow a(14) = 216$

$\Rightarrow a = \dfrac{216}{14} = \dfrac{108}{7}$

$\therefore$ a = 12

68. According to the given condition,

$4 = \dfrac{a}{1-r}$

$\Rightarrow 4 \Rightarrow a = 4 - 4r$

$\Rightarrow 4r = 4 - a$

Only option (D) satisfies this condition.

69. Since, a, b, c are in G.P.

$\therefore$ $b^2 = ac$

$\Rightarrow \log_e b^2 = \log_e ac$

$\Rightarrow \log_e a - 2\log_e b + \log_e c = 0$

Given, $(\log_e a)x^2 - (2\log_e b)x + \log_e c = 0$

Since, 1 satisfies this equation.

Therefore, 1 is one root and other root say β.

$\therefore$ $1.\beta = \dfrac{\log_e c}{\log_e a}$

$\therefore$ $\beta = \log_a c$

70. Series, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, …… are in H.P.

$\Rightarrow \frac{1}{2}, \frac{2}{5}, \frac{3}{10}$, …… will be in A.P.

Now, first term $a = \frac{1}{2}$ and

common difference $d = -\frac{1}{10}$

So, 5^{th} term of the A.P.

$= \frac{1}{2} + (5-1)\left(-\frac{1}{10}\right) = \frac{1}{10}$

Hence, 5^{th} term in H.P. is 10.

71. Since $a_1, a_2, a_3, \ldots\ldots a_n$ are in H.P

Therefore $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots \frac{1}{a_n}$ will be in A.P.

Which gives, $\frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \ldots\ldots$

$= \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$

$\Rightarrow \frac{a_1 - a_2}{a_1a_2} = \frac{a_2 - a_3}{a_2a_3} = \ldots\ldots = \frac{a_{n-1} - a_n}{a_{n-1}a_n} = d$

$\Rightarrow a_1 - a_2 = da_1a_2,\ a_2 - a_3 = da_2a_3$

and $a_{n-1} - a_n = da_{n-1}a_n$

Adding these, we get

$d(a_1a_2 + a_2a_3 + \ldots\ldots + a_{n-1}a_n)$

$= (a_1 + a_2 + \ldots\ldots + a_{n-1}) - (a_2 + a_3 + \ldots\ldots + a_n)$

$= a_1 - a_n$ …..(i)

Also n^{th} term of this A.P. is given by

$\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \Rightarrow d = \frac{a_1 - a_n}{a_1a_n(n-1)}$

Substituting this value of d in (i)

$(a_1 - a_n) = \frac{a_1 - a_n}{a_1a_n(n-1)}(a_1a_2 + a_2a_3 + \ldots\ldots + a_{n-1}a_n)$

$(a_1a_2 + a_2a_3 + \ldots\ldots + a_{n-1}a_n) = a_1a_n(n-1)$

72. Here, 5^{th} term of the corresponding

A.P. $= a + 4d = 45$ …..(i)

and 11^{th} term of the corresponding

A.P. $= a + 10d = 69$ …..(ii)

From (i) and (ii), we get $a = 29$, $d = 4$

Therefore, 16^{th} term of the corresponding A.P.

$= a + 15d = 29 + 15 \times 4 = 89$

Hence, 16^{th} term of the H.P. is $\frac{1}{89}$.

73. Let roots be α, β then

$\alpha + \beta = -\frac{b}{a} = 10$

$\alpha\beta = \frac{c}{a} = 11$

H.M. $= \frac{2\alpha\beta}{\alpha+\beta} = \frac{11 \times 2}{10} = \frac{11}{5}$

74. We know that, $x_n = \frac{(n+1)ab}{na+b}$

$\therefore$ Sixth H.M. i.e. $x_6 = \frac{7.3.\left(\frac{6}{13}\right)}{\left(6.3+\frac{6}{13}\right)}$

$= \frac{126}{240} = \frac{63}{120}$

75. We have $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{2ab}{a+b}$

$\Rightarrow a^{n+2} + ab^{n+1} + ba^{n+1} + b^{n+2} = 2a^{n+1}b + 2b^{n+1}a$

$\Rightarrow a^{n+1}(a-b) = b^{n+1}(a-b)$

or $\left(\frac{a}{b}\right)^{n+1} = (1) = \left(\frac{a}{b}\right)^0$

Hence, $n = -1$

76. Given that A.M. = 8 and G.M. = 5, if α, β are roots of quadratic equation, then quadratic equation is

$x^2 - x(\alpha + \beta) + \alpha\beta = 0$

A.M. $= \frac{\alpha+\beta}{2} = 8 \Rightarrow \alpha + \beta = 16$

and G.M. $= \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 25$

So the required quadratic equation will be

$x^2 - 16x + 25 = 0$.

77. a,b,c are in H.P. $\Rightarrow b = \frac{2ac}{a+c}$

By inspection, we get (A) False (B) False (C) False

78. Given $x_1.x_2.x_3 \ldots\ldots x_n = 1$

$\because$ A.M. $\geq$ G.M.

$\therefore \left(\frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}\right) \geq (x_1.x_2.x_3\ldots\ldots x_n)^{\frac{1}{n}}$

$= (1)^{\frac{1}{n}}$

$= 1$

$\therefore x_1 + x_2 + x_3 + \ldots\ldots + x_n \geq n$

$\therefore x_1 + x_2 + x_3 + \ldots\ldots + x_n$ can never be less than n.

79. As given $H = \frac{2pq}{p+q}$

$\therefore \quad \frac{H}{p} + \frac{H}{q} = \frac{2q}{p+q} + \frac{2p}{p+q} = \frac{2(p+q)}{p+q} = 2$

80. Let three numbers a, b and c in G.P., then $b^2 = ac$

$\Rightarrow 2 \log_e b = \log_e a + \log_e c$ or

$\log_e b = \frac{\log_e a + \log_e c}{2}$

Thus, their logarithms are in A.P.

81. x, l, z are in A.P., then $2 = x + z$ ......(i)

and $4 = xz$ ......(ii)

Divide (ii) by (i), we get

$\frac{x.z}{x+z} = \frac{4}{2}$ or $\frac{2xz}{x+z} = 4$

Hence, x, 4, z will be in H.P.

82. Given that a, A_1, A_2, b are in A.P.

Therefore, $A_1 = \frac{a + A_2}{2}$, $A_2 = \frac{A_1 + b}{2}$

$\Rightarrow A_1 + A_2 = \frac{1}{2}(a + b + A_1 + A_2)$

$\Rightarrow \frac{1}{2}(A_1 + A_2) = \frac{1}{2}(a+b)$ or

$A_1 + A_2 = a + b$

and a, G_1, G_2, b are in G.P.

Therefore, $G_1^2 = aG_2, G_2^2 = bG_1$

$\Rightarrow G_1^2 G_2^2 = abG_1G_2 \Rightarrow G_1G_2 = ab$

Hence, $\frac{A_1 + A_2}{G_1G_2} = \frac{a+b}{ab}$

Trick : Let a = 1, b = 2,

then $A_1 + A_2 = 1 + 2 = 3$

and $G_1 . G_2 = 2 \times 1 = 2$

$\therefore \quad \frac{A_1 + A_2}{G_1G_2} = \frac{3}{2}$, which is given by (A).

83. Given A.M. = 2(G.M.) or $\frac{1}{2}(a+b) = 2\sqrt{ab}$

or $\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{2+1}{2-1} = \frac{3}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{3}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{3}}{1} \Rightarrow \frac{a}{b} = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$

$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$ or $a : b = (2+\sqrt{3}) : (2-\sqrt{3})$.

84. $x + y + z = 15$, if 9, x, y, z, a are in A.P.

$\text{Sum} = 9 + 15 + a = \frac{5}{2}(9+a)$

$\Rightarrow 24 + a = \frac{5}{2}(9+a) \Rightarrow 48 + 2a = 45 + 5a$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$, if 9, x, y, z, a are in H.P.

$\text{Sum} = \frac{1}{9} + \frac{5}{3} + \frac{1}{a} = \frac{5}{2}\left[\frac{1}{9} + \frac{1}{a}\right] \Rightarrow a = 1$

85. If $\frac{x+y}{2}, y, \frac{y+z}{2}$ are in H.P., then

$$y = \frac{2\left(\frac{x+y}{2} . \frac{y+z}{2}\right)}{\frac{x+y}{2} + \frac{y+z}{2}} = \frac{\frac{2}{4}(x+y)(y+z)}{\frac{1}{2}(x+2y+z)}$$

$$y = \frac{xy + xz + y^2 + yz}{x + 2y + z}$$

$\Rightarrow xy + 2y^2 + yz = xy + xz + y^2 + yz$

$\Rightarrow y^2 = xz$

Thus, x, y, z will be in G.P.

86. We have H.M. $= \frac{2ab}{a+b}$ and G.M. $= \sqrt{ab}$

So $\frac{H.M.}{G.M.} = \frac{4}{5} \Rightarrow \frac{2ab/(a+b)}{\sqrt{ab}} = \frac{4}{5}$

$\Rightarrow \frac{2\sqrt{ab}}{(a+b)} = \frac{4}{5} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{9}{1} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3}{1}$

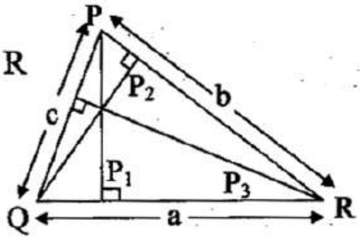
$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{3+1}{3-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{4}{2} \Rightarrow \left(\frac{a}{b}\right) = 2^2 = 4$

$\Rightarrow$ a : b = 4 : 1 or b : a = 1 : 4

87. Sum of the roots of $x^2 - 2ax + b^2 = 0$ is 2a,
Therefore, A = A.M. of the roots = a.
Product of the roots of $x^2 - 2bx + a^2 = 0$ is a^2
Therefore, G.M. of the roots is G = a
Thus, A = G

88. Let P_1, P_2, P_3 be altitudes from P, Q and R



$P_1 = c \sin Q = \lambda bc$,
$P_2 = a \sin R = \lambda ca$,
$P_3 = b \sin P = \lambda ab$

$$\ldots\left[\because \frac{\sin P}{a} = \frac{\sin Q}{b} = \frac{\sin R}{c} = \lambda\right]$$

$\Rightarrow P_1, P_2, P_3$ are in A.P.
$\Rightarrow \lambda bc, \lambda ca, \lambda ab$ are in A.P.
$\Rightarrow$ bc, ca, ab are in A.P.
$\Rightarrow \frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c}$ are in A.P
$\therefore$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.
$\therefore$ a, b, c are in H.P. i.e., sides of the triangle are in H.P.

89. Let, the distance of school from home = d and time taken are t_1 and t_2.

$\therefore$ $t_1 = \frac{d}{x}$ and $t_2 = \frac{d}{y}$

Avg. velocity $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{2d}{\left(\frac{d}{x} + \frac{d}{y}\right)} = \frac{2xy}{x+y}$, which is the H.M. of x and y.

90. x, y, z are in G.P.
Hence, $y^2 = xz$
$\therefore$ $2 \log y = \log x + \log z$
$\Rightarrow 2(\log y + 1) = (1 + \log x) + (1 + \log z)$
$\Rightarrow 1 + \log x, 1 + \log y, 1 + \log z$ are in A.P.
$\Rightarrow \frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are is H.P.

91. If x, y, z are in H.P., then $y = \frac{2xz}{x+z}$
$\therefore$ $\log_e(x+z) + \log_e(x - 2y + z)$
$= \log_e\{(x+z)(x - 2y + z)\}$
$= \log_e\left[(x+z)\left(x + z - \frac{4xz}{x+z}\right)\right]$
$= \log_e[(x+z)^2 - 4xz]$
$= \log_e(x - z)^2$
$= 2 \log_e(x - z)$

92. Since, b^2, a^2, c^2 are in A.P.
$\therefore$ $a^2 - b^2 = c^2 - a^2$
$\Rightarrow (a - b)(a + b) = (c - a)(c + a)$
$\Rightarrow \frac{1}{b+c} - \frac{1}{a+b} = \frac{1}{c+a} - \frac{1}{b+c}$
$\Rightarrow \frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ are in A.P.
$\therefore$ $(a + b), (b + c), (c + a)$ are in H.P.

93. Since, H_1, H_2 are two harmonic means between a and b.
$\therefore$ $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{b}$ are in A.P.
We know that $2A = a + b$ and $G^2 = ab$
$\therefore$ $2 \times \frac{1}{H_1} = \frac{1}{a} + \frac{1}{H_2}$
Similarly, $2 \times \frac{1}{H_2} = \frac{1}{b} + \frac{1}{H_1}$
On adding and solving we get,
$2\left(\frac{1}{H_1} + \frac{1}{H_2}\right) - \left(\frac{1}{H_1} + \frac{1}{H_2}\right) = \frac{1}{a} + \frac{1}{b}$
$\frac{1}{H_1} + \frac{1}{H_2} = \frac{a+b}{ab} = \frac{2A}{G^2}$

94. Given, a, b, c are in G.P.
$\therefore$ $b^2 = ac$
$x = \frac{a+b}{2}$, $y = \frac{b+c}{2}$
$\therefore$ $\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}$
$= \frac{2(ab + bc + 2ca)}{ab + ac + b^2 + bc}$
$= \frac{2(ab + bc + 2ca)}{(ab + ac + ac + bc)}$
$= 2$ ....$[\because b^2 = ac]$

95. According to the given condition,
$\frac{\frac{x+y}{2}}{\sqrt{xy}} = \frac{p}{q}$
$\Rightarrow \frac{x+y}{2(\sqrt{xy})} = \frac{p}{q}$ ....(i)

$\Rightarrow \frac{x^2+y^2+2xy}{4xy}=\frac{p^2}{q^2}$

$\Rightarrow \frac{x^2+y^2+2xy-4xy}{4xy}=\frac{p^2-q^2}{q^2}$

$\Rightarrow \frac{(x-y)^2}{4xy}=\frac{p^2-q^2}{q^2}$

$\Rightarrow \frac{x-y}{2\sqrt{xy}}=\frac{\sqrt{p^2-q^2}}{q}$ ....(ii)

Dividing (ii) by (i), we get

$\frac{x+y}{x-y}=\frac{p}{\sqrt{p^2-q^2}} \Rightarrow \frac{x}{y}=\frac{p+\sqrt{p^2-q^2}}{p-\sqrt{p^2-q^2}}$

96. Since, p, q, r are in G.P.

$\therefore$ $q^2 = pr$

Also, $\tan^{-1} p$, $\tan^{-1} q$, $\tan^{-1} r$ are in A.P.

$\Rightarrow \tan^{-1} p + \tan^{-1} r = 2 \tan^{-1} q$

$\Rightarrow p + r = 2q$

$\Rightarrow$ p, q, r are in A.P.

Here, p, q, r are both in A.P. and G.P., which is possible only, if p = q = r.

97. Given numbers a and 2.

A.M. = $\frac{a+2}{2}$ and G.M. = $\sqrt{2a}$

According to the given condition,

A.M. – G.M. = 1

$\Rightarrow \frac{a+2}{2}-\sqrt{2a}=1$

$\Rightarrow \frac{a}{2}+1-1=\sqrt{2a}$

$\Rightarrow a = 2\sqrt{2a} \Rightarrow a^2 = 8a$

$\Rightarrow a(a-8) = 0$

$\Rightarrow a = 0$ or 8

Since, $a \neq 0$

$\therefore$ a = 8

98. Let a and b be two numbers.

Sum of n A.M.'s = n × single A.M.

$\Rightarrow A_1 + A_2 = 2 \times \left(\frac{a+b}{2}\right) = a+b$

Product of n G.M.'s = (Single G.M.)^n

$\Rightarrow G_1.G_2 = \left(\sqrt{ab}\right)^2 = ab$

$\Rightarrow \frac{1}{a},\frac{1}{H_1},\frac{1}{H_2},\frac{1}{b}$ are in A.P.

$\Rightarrow \frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$

$\Rightarrow \frac{H_1H_2}{H_1+H_2}=\frac{G_1G_2}{A_1+A_2}$

$\Rightarrow \frac{G_1G_2}{H_1H_2}\times\frac{H_1+H_2}{A_1+A_2}=1$

99. Given, a, b, c are in G.P.

$\Rightarrow \log_x a, \log_x b \log_x c$ are in A.P.

$\Rightarrow \frac{\log a}{\log x},\frac{\log b}{\log x},\frac{\log c}{\log x}$ are in A.P.

$\Rightarrow \frac{\log x}{\log a},\frac{\log x}{\log b},\frac{\log x}{\log c}$ are in H.P.

i.e., $\log_a x$, $\log_b x$, $\log_c x$ are in H.P.

100. $(y-x)$, $2(y-a)$, $(y-z)$ are in H.P.

$\Rightarrow \frac{1}{y-x},\frac{1}{2(y-a)},\frac{1}{y-z}$ are in A.P.

$\Rightarrow \frac{1}{2(y-a)}-\frac{1}{(y-x)}=\frac{1}{(y-z)}-\frac{1}{2(y-a)}$

$\Rightarrow \frac{y-x-2y+2a}{y-x}=\frac{2y-2a-y+z}{y-z}$

$\Rightarrow \frac{-x-y+2a}{(y-x)}=\frac{y+z-2a}{(y-z)}$

$\Rightarrow \frac{(x-a)+(y-a)}{(x-a)-(y-a)}=\frac{(y-a)+(z-a)}{(y-a)-(z-a)}$

$\Rightarrow \frac{(x-a)}{(y-a)}=\frac{(y-a)}{(z-a)}$

$\Rightarrow (x-a), (y-a), (z-a)$ are in G. P.

101. Given, a, b, c are in A.P.

$\Rightarrow 2b = a + c \Rightarrow b - c = a - b$

Also, a^2, b^2, c^2 are in H.P.

$\frac{1}{b^2}-\frac{1}{a^2}=\frac{1}{c^2}-\frac{1}{b^2}$

$\Rightarrow \frac{a^2-b^2}{a^2b^2}=\frac{b^2-c^2}{b^2c^2}$

$\Rightarrow (a-b)\,[c^2(a+b)-a^2(b+c)]=0$ ....[$\because$ $(b-c)=(a-b)$]

$\Rightarrow a = b$ or $c^2a + c^2b - a^2b - a^2c = 0$

$\Rightarrow c^2a + c^2b - a^2b - a^2c = 0$

$\Rightarrow ac(c-a) = b(a^2-c^2)$

$\Rightarrow ac = -b(c+a)$

$\Rightarrow -ac = b.2b$

$\Rightarrow b^2 = -\left(\frac{a}{2}\right)c$

$\therefore$ $-\frac{a}{2}$, b, c are in G.P.

102. A.M. ≥ G.M.

$$\Rightarrow \frac{a_1 + a_2 + ... + a_{n-1} + 2a_n}{n} \geq (a_1.a_2,...a_{n-1}2a_n)^{\frac{1}{n}} \geq (2c)^{\frac{1}{n}}$$

∴ Minimum value of

$a_1 + a_2 + ... + a_{n-1} + 2a_n = n(2c)^{\frac{1}{n}}$

103. Let the positive numbers be a_1 and a_2.

a_1, A, a_2, ...are in A.P. then $A = \frac{a_1 + a_2}{2}$

Also, a_1, G, a_2, ... are in G.P.

∴ $G = \sqrt{a_1 a_2}$

$\frac{1}{a_1}, \frac{1}{H}, \frac{1}{a_2}$, ... are in H.P.

∴ $\frac{2}{H} = \frac{1}{a_1} + \frac{1}{a_2} \Rightarrow H = \frac{2a_1 a_2}{a_1 + a_2} \Rightarrow H = \frac{G^2}{A}$

104. Let the two numbers be x, y.

∴ $x - y = 48$ ....(i)

and $\frac{x+y}{2} - \sqrt{xy} = 18$

$\Rightarrow x + y - 2\sqrt{xy} = 36$

$\Rightarrow 48 + y + y - 2\sqrt{(48 + y)y} = 36$ ....[From (i)]

$\Rightarrow 12 + 2y = 2\sqrt{y(48 + y)}$

$\Rightarrow 144 + 4y^2 + 48y = 4(48y + y^2)$

$\Rightarrow 36 + y^2 + 12y = 48y + y^2$

$\Rightarrow 36y = 36 \Rightarrow y = 1$

∴ $x = 48 + 1 = 49$

105. Since, a, b, c are in H.P.

∴ $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow b = \frac{2ac}{a+c}$

Consider option (B),

$$\frac{1}{ca} = \frac{2\left(\frac{1}{bc}.\frac{1}{ab}\right)}{\frac{1}{bc} + \frac{1}{ab}} = \frac{\left(\frac{2}{ab^2c}\right)}{\frac{a+c}{abc}}$$

$$= \frac{2(abc)}{ab^2c(a+c)} = \frac{2}{b(a+c)}$$

$$\Rightarrow ca = \frac{b(a+c)}{2} \Rightarrow b = \frac{2ac}{a+c}$$

∴ $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in H.P.

106. G.M. of 1, 2, 2^2, 2^3, ..., 2^n

Here, no. of terms = (n + 1)

∴ $G.M. = (1.2.2^2.2^3 .....2^n)^{\frac{1}{(n+1)}}$

$$= (2^{0+1+2+....+n})^{\frac{1}{(n+1)}} = \left[2^{\frac{n(n+1)}{2}}\right]^{1/(n+1)}$$

∴ $G.M. = 2^{\frac{n}{2}}$

107. $\frac{x^{n+1} + y^{n+1}}{x^n + y^n} = \sqrt{xy} \Rightarrow x^{n+1} + y^{n+1} = \sqrt{xy}\,(x^n + y^n)$

$$\Rightarrow x^{n+\frac{1}{2}}\left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right) = y^{n+\frac{1}{2}}\left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right)$$

$$\Rightarrow \left(\frac{x}{y}\right)^{n+\frac{1}{2}} = 1 \Rightarrow n = -\frac{1}{2}$$

108. Given, $\sqrt{ab} = 10$

$\Rightarrow ab = 100$ and $\frac{2ab}{a+b} = 8$

$\Rightarrow a + b = 25$

∴ a = 5, b = 20

109. $1^2 - 2^2 + 3^2 - 4^2 + ... + 11^2$

$= (1^2 - 2^2) + (3^2 - 4^2) + ... + (9^2 - 10^2) + 11^2$

Now, $a^2 - b^2 = (a - b)(a + b)$

∴ $1^2 - 2^2 + 3^2 - 4^2 + ... + 11^2$

$= (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + ... + (9 - 10)(9 + 10) + 11^2$

$= (-1)[1 + 2 + 3 + ... + 9 + 10] + 11^2$

$= (-1).\frac{10 \times 11}{2} + 11^2 = 66$

110. $f(x) = x + \frac{1}{2} = \frac{2x+1}{2}$

$f(2x) = \frac{4x+1}{2}$

$f(4x) = \frac{8x+1}{2}$

f(x), f($2x$), f($4x$) are in H.P.

∴ $f(2x) = \frac{2f(x)f(4x)}{f(x) + f(4x)}$

$\Rightarrow x = 0, \frac{1}{4}$

At $x = 0$, terms are equal, so only solution is $x = \frac{1}{4}$

111. It is an arithmetico-geometric series

$\therefore \quad S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-\frac{1}{2}} + \frac{2\times\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}$

$= 2 + 4$

$= 6$

112. $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 1, r = \frac{1}{5}, d = 3$

$\therefore \quad S_\infty = \frac{1}{1-\frac{1}{5}} + \frac{3\times\frac{1}{5}}{\left(1-\frac{1}{5}\right)^2} = \frac{35}{16}$

113. Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ to ∞

$\Rightarrow (S-1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ to ∞ ....(i)

$\Rightarrow (S-1)\frac{1}{3} = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots$ to ∞ ....(ii)

Subtracting (ii) from (i), we get

$\frac{2}{3}(S-1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots$ to ∞

$\Rightarrow \frac{2}{3}(S-1) = \frac{2}{3} + \frac{\frac{4}{3^2}}{1-\frac{1}{3}}$

$\Rightarrow \frac{2}{3}(S-1) = \frac{2}{3} + \frac{2}{3}$

$\Rightarrow S = 3$

114. $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \ldots + \frac{1}{t_{2003}}$

$= 4\left[\frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \ldots + \frac{1}{(2005).(2006)}\right]$

$= 4\left[\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \ldots + \frac{1}{2005} - \frac{1}{2006}\right]$

$= 4\left[\frac{1}{3} - \frac{1}{2006}\right]$

$= 4.\frac{2003}{3(2006)}$

$= \frac{4006}{3009}$

115. $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots\infty = \frac{\pi^4}{90}$

$\therefore \quad \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\infty + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots\infty\right) = \frac{\pi^4}{90}$

$\therefore \quad \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots \infty + \frac{1}{16}\times\frac{\pi^4}{90} = \frac{\pi^4}{90}$

$\therefore \quad \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots + \infty$

$= \frac{\pi^4}{90} - \frac{1}{16}\left(\frac{\pi^4}{90}\right) = \frac{15}{16}\left(\frac{\pi^4}{90}\right) = \frac{\pi^4}{96}$

116. Here $t_n = \frac{n(n+1)}{2}$

$\therefore \quad S_n = \frac{1}{2}\left(\Sigma n^2 + \Sigma n\right) = \frac{n(n+1)(n+2)}{6}$

117. $S_n = 1(1!) + 2(2!) + 3(3!) + \ldots + n(n!)$

$= (2-1)(1!) + (3-1)(2!) + (4-1)(3!) + \ldots + [(n+1)-1](n!)$

$= (2.1! - 1!) + (3.2! - 2!) + (4.3! - 3!) + \ldots + [(n+1)! - (n!)]$

$= (2! - 1!) + (3.2! - 2!) + (4.3! - 3!) + \ldots + [(n+1)(n!) - (n!)]$

$= (n+1)! - 1!$

118. $S = 3.6 + 4.7 + \ldots$ upto $n-2$ terms

$= (1.4 + 2.5 + 3.6 + 4.7 + \ldots$ upto n terms$) - 14$

$= \Sigma n(n+3) - 14$

$= \frac{1}{6}(2n^3 + 12n^2 + 10n) - 14$

$= \left(\frac{2n^3 + 12n^2 + 10n - 84}{6}\right)$,

where $n = 3, 4, 5 \ldots$

Trick : $S_1 = 18, S_2 = 46$

Now put in options $(n-2) = 1,2$ i.e. $n = 3,4$

Option (B) gives the values.

119. $t_n = n(n+1)(n+2) = n(n^2 + 3n + 2)$

$= n^3 + 3n^2 + 2n$

$\therefore \quad S_n = \Sigma(n^3) + \Sigma(3n^2) + \Sigma(2n)$

$S_n = \left[\frac{n(n+1)}{2}\right]^2 + \frac{3.n(n+1)(2n+1)}{6} + \frac{2.n(n+1)}{2}$

$S_n = \frac{1}{4}n(n+1)(n+2)(n+3)$

120. Sum of cubes of 'n' natural number

$= \frac{n^2(n+1)^2}{4}$

$= \frac{15^2(16)^2}{4}$

$= 14,400$

121. Given series $\frac{1}{1} + \frac{1+2}{2} + \frac{1+2+3}{3} + \ldots\ldots$

So, n^{th} term of series is given by

$t_n = \frac{1+2+3+\ldots\ldots+n}{n}$

$= \frac{\frac{1}{2}n(n+1)}{n}$

$= \frac{n+1}{2}$

122. Here, t_n of the A.P. 1, 2, 3, ... = n

and t_n of the A.P. 3, 5, 7, ... = 2n + 1

∴ t_n of given series $= n(2n+1)^2 = 4n^3 + 4n^2 + n$

Hence,

$S = \sum_{1}^{20} t_n$

$= 4\sum_{1}^{20} n^3 + 4\sum_{1}^{20} n^2 + \sum_{1}^{20} n$

$= 4.\frac{1}{4}20^2.21^2 + 4.\frac{1}{6}20.21.41 + \frac{1}{2}20.21$

$= 188090$

123. $t_n = \frac{(2n+1)}{\frac{n(n+1)(2n+1)}{6}}$

$= \frac{6}{n(n+1)}$

$S_n = \Sigma(t_n)$

$= \Sigma 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$= 6\left[1 - \frac{1}{n+1}\right]$

$S_n = \frac{6n}{n+1}$

124. $1 + 3 + 7 + \ldots.. + t_n$

$= 2 - 1 + 2^2 - 1 + 2^3 - 1 + \ldots\ldots. + 2^n - 1$

$= (2 + 2^2 + \ldots\ldots + 2^n) - n$

$= 2^{n+1} - 2 - n$

125. We have $S = 2 + 4 + 7 + 11 + 16 + \ldots.. + t_n$

Again, $S = 2 + 4 + 7 + 11 + \ldots\ldots + t_{n-1} + t_n$

Subtracting, we get

$0 = 2 + \{2 + 3 + 4 + 5 + \ldots. + (t_n - t_{n-1})\} - t_n$

$t_n = 2 + \frac{1}{2}(n-1)\{(4 + (n-2)1\}$

$= \frac{1}{2}(n^2 + n + 2)$

Now,

$S = \Sigma t_n = \frac{1}{2}\Sigma(n^2 + n + 2)$

$= \frac{1}{2}(\Sigma n^2 + \Sigma n + 2\Sigma 1)$

$= \frac{1}{2}\{\frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1) + 2n\}$

$= \frac{n}{12}\{(n+1)(2n+1+3) + 12\}$

$= \frac{n}{6}\{(n+1)(n+2) + 6\}$

$= \frac{n}{6}(n^2 + 3n + 8)$

126. Let n^{th} term of series is t_n, then

$S_n = 12 + 16 + 24 + 40 + \ldots. + t_n$

Again $S_n = 12 + 16 + 24 + \ldots\ldots + t_n$

On subtraction

$0 = (12 + 4 + 8 + 16 + \ldots. + \text{upto n terms}) - t_n$

$\Rightarrow t_n = 12 + [4 + 8 + 16 + \ldots. + \text{upto } (n-1) \text{ terms}]$

$= 12 + \frac{4(2^{n-1} - 1)}{2-1}$

$= 2^{n+1} + 8$

On putting n = 1, 2, 3 .....

$t_1 = 2^2 + 8,\ t_2 = 2^3 + 8,\ t_3 = 2^4 + 8$ .... etc.

$S_n = t_1 + t_2 + t_3 + \ldots. + t_n$

$= (2^2 + 2^3 + 2^4 + \ldots.. \text{upto n terms}) + (8 + 8 + 8 + \ldots. \text{upto n terms})$

$= \frac{2^2(2^n - 1)}{2-1} + 8n$

$= 4(2^n - 1) + 8n$

127. Let, $S = 2 + 7 + 14 + 23 + 34 + ... + t_n +$ .....(i)

and $S = 2 + 7 + 14 + ... + t_{n-1} + t_n +$ ....(ii)

From (i) and (ii), we get

$0 = 2 + [5 + 7 + 9 + 11 ..... + t_n - t_{n-1}] - t_n$

$\Rightarrow t_n = 2 + \left[\frac{n-1}{2}\{2\times 5 + (n-2)2\}\right]$

$\Rightarrow t_n = 2 + (n-1)(n+3)$

Now,

put n = 99

$\Rightarrow t_{99} = 2 + 98 \times 102$

$= 9998$

128. $S_n = cn^2$

$S_{n-1} = c(n-1)^2 = cn^2 + c - 2cn$

$T_n = 2cn - c$

$T_n^2 = (2cn - c)^2$

$= 4c^2 n^2 + c^2 - 4c^2 n$

$\therefore$ Required sum $= \sum T_n^2$

$= \frac{4c^2.n(n+1)(2n+1)}{6} + nc^2 - 2c^2n(n+1)$

$= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2n(n+1)}{3}$

$= \frac{nc^2(4n^2 + 6n + 2 + 3 - 6n - 6)}{3}$

$= \frac{nc^2(4n^2 - 1)}{3}$

129. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n}\sum_{j=1}^{i} j$

$= \sum_{i=1}^{n}\left(\frac{i(i+1)}{2}\right)$

$= \frac{1}{2}\left[\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right]$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$= \frac{1}{4}n(n+1)\left[\frac{2n+1}{3} + 1\right]$

$= \frac{n(n+1)}{4}\left[\frac{2n+1+3}{3}\right]$

$= \frac{n(n+1)(n+2)}{6}$

130. $(1 + 2) + (1 + 2 + 2^2) + ...$ upto n terms

$\therefore T_n = 1 + 2 + 2^2 + ... + 2^n$

$\therefore T_n = \frac{1(2^{n+1} - 1)}{2-1} = 2^{n+1} - 1$

$\therefore S_n = \sum T_n = \sum(2^{n+1} - 1)$

$\therefore S_n = \sum 2^{n+1} - \sum 1$

$= 2^2 + 2^3 + 2^4 + ... + 2^n - (n)$

$= 2^{n+2} - 4 - n$

131. $\sum_{k=1}^{n} k(k+2) = \sum_{k=1}^{n}(k^2 + 2k)$

$= \sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$

$= \frac{n(n+1)(2n+1)}{6} + \frac{2.n(n+1)}{2}$

$= n(n+1)\left(\frac{2n+1}{6} + 1\right)$

$= \frac{n(n+1)(2n+7)}{6}$

132. General term $t_n = \frac{1^2 + 2^2 + ... + n^2}{1 + 2 + ... + n}$

$\Rightarrow t_n = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}$

$= \frac{1}{3}.(2n+1)$

$\therefore \sum t_n = \frac{2}{3}\sum n + \frac{1}{3}n$

$= \frac{2}{3}.\frac{n(n+1)}{2} + \frac{1}{3}n$

$= \frac{1}{3}n.(n+1) + \frac{1}{3}n$

$= \frac{n(n+2)}{3}$

133. $1^3 + 3^3 + 5^3 + 7^3 + ... = \sum(2n-1)^3$

$= \sum(8n^3 - 3.4n^2 + 3.2n - 1)$

$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$

$= 2n^4 + 4n^3 + 2n^2 - 2n(2n^2 + 3n + 1) + 3n^2 + 3n - n$

$= 2n^4 + 4n^3 + 2n^2 - 4n^3 - 6n^2 - 2n + 3n^2 + 3n - n$

$= 2n^4 - n^2 = n^2(2n^2 - 1)$