22 Probability

Formulae

1. i. Probability of an event A over a sample space S is

$$P(A) = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

i.e., $P(A) = \frac{n(A)}{n(S)}$

ii. Probability of an **impossible event** is zero.

iii. Probability of a **sure event** is one.

i.e., P(S) = 1, where S is the sure event

2. Exhaustive events:

Two events A and B of the sample space are said to be exhaustive if $A \cup B = S$ i.e., $A \cup B$ contains all sample points.

3. Mutually Exclusive events:

Two events A & B of the sample space S are said to be mutually exclusive if $A \cap B = \phi$

4. i. If two events A and B defined on the sample space S are mutually exclusive and exhaustive, then they are said to be complementary events.

ii. The complement of the event A is denoted by A' or $\overline{A}$ or A^c.

5. Addition theorem:

i. If A and B are any two events defined over a sample space S, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

or

P (A + B) – P(A) + P(B) – P(AB)

Where

P (A + B) or $P(A \cup B)$ = Probability of happening of events A or B and

P (AB) or $P(A \cap B)$ = Probability of happening of events A and B together.

ii. If A and B are two mutually exclusive events, then P(A n B) = 0

$\therefore$ $P(A \cup B) - P(A) + P(B)$

6. Elementary properties of probability:

i. P(A') = 1 – P(A) i.e., P(A') + P(A) = 1

ii. $0 \le P(A) \le 1$ for an event A

iii. $P(\phi) = 0$, where ϕ is a null set

iv. If $A \subseteq B$, then $P(A) \le P(B)$

v. $P(A \cap B') = P(A) - P(A \cap B)$

$P(A' \cap B) = P(B) - P(A \cap B)$

vi. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) + P(C \cap A) + P(A \cap B \cap C)$

where A, B, C are any events.

vii. $P(A \cup B \cup C) = P(A) + P(B) + P(C)$, if A, B, C are mutually exclusive events.

viii. $P(AB) \le P(A) \le P(A+B) \le P(A) + P(B)$

7. Conditional Probability:

The conditional probability of both the events A and B over the sample space S is

i. $P(A/B) = \frac{P(A \cap B)}{P(B)}$, where $B \ne \phi$

ii. $P(B/A) = \frac{P(A \cap B)}{P(A)}$, where $A \ne \phi$

8. Multiplication theorem:

If A and B are two events over the sample space S, then

i. $P(A \cap B) = P(B) . P(A/B)$

ii. $P(A \cap B) = P(A) . P(B/A)$

9. Independent events:

i. P(A/B) = P(A/B') = P(A)

ii. P(B/A) = P(B/A') = P(B)

iii. If A and B are independent events, then

a. $P(A \cap B) = P(A) . P$ b)

b. A and B' are also independent

c. A' and B' are also independent

10. Bayes theorem:

If B_1, B_2, ...B_n are mutually exclusive and exhaustive events and if A is an event consequent to these B_i's, then for each i = 1, 2, 3, ... n,

$$P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(A \cap B_i)}$$

11. Odds (Ratio of two complementary probabilities):

i. The odds in favour of an event A is P(A)/P(A')

ii. The odds against the happening of an event A is P(A') / P(A)

12. Some notations for events:

Event	Notation
Not A	$\overline{A}$
at least one of A, B occurs	$A \cup B$
both A and B occur	$A \cap B$
A occurs but not B	$A \cap B'$
B occurs but not A	$A' \cap B$
neither A nor B occur	$A' \cap B'$
at least one of A, B, C	$A \cup B \cup C$
Exactly one of A and B	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$
All three of A, B, C	$A \cap B \cap C$
Exactly two of A, B and C	$(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)$

Shortcuts

1. Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = 2^n

2. Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n , times) = 6^n

3. If odds in favour of an event are a : b, then the probability of the occurrence of that event is $\frac{a}{a+b}$ and the probability of non-occurrence of that event is $\frac{b}{a+b}$.

4. If odds against an event are a : b, then the probability of the occurrence of that event is $\frac{b}{a+b}$ and the probability of non-occurrence of that event is $\frac{a}{a+b}$

5. **Probability regarding n letters and their envelopes:**

If n letters corresponding to n envelopers are placed in the envelopes at random, then

i. Probability that all letters are in right envelopes $= \frac{1}{n!}$

ii. Probability of keeping at least one letter in wrong envelope $= 1 - \frac{1}{n!}$

iii. Probability of keeping all the n letters in wrong envelopes.

$$= \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + \frac{(-1)^n}{n!}$$

iv. Probability that exactly r letters are in right envelopes

$$= \frac{1}{r!}\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots\ldots + \frac{(-1)^{n-r}}{(n-r)!}\right]$$

v. Probability of keeping at least one letter in right envelope = 1 – p.

MULTIPLE CHOICE QUESTIONS

Classical Thinking

22.1 Types of events (Algebra of events, Concept of Probability)

1. 4 coins are tossed. The probability that they are all heads is

a) $\frac{1}{16}$ b) $\frac{2}{9}$

c) $\frac{3}{10}$ d) $\frac{4}{15}$

2. A card is drawn from a well shuffled pack of cards. The probability of getting a queen of club or king of heart is

a) $\frac{1}{52}$ b) $\frac{1}{26}$

c) $\frac{1}{13}$ d) $\frac{2}{13}$

3. A number is chosen at random from first ten natural numbers. The probability that number is odd and perfect square is

a) $\frac{2}{9}$ b) $\frac{2}{5}$

c) $\frac{3}{7}$ d) $\frac{1}{5}$

4. There are n letters and n addressed envelopes. The probability that all the letters are not kept in the right envelope, is

a) $\frac{1}{n!}$ b) $1-\frac{1}{n!}$

c) $1-\frac{1}{n}$ d) $\frac{1}{2!}-\frac{1}{3!}+.....+\frac{(-1)^n}{n!}$

5. Two dice are thrown. The probability that the sum of numbers appearing is more than 10, is

a) $\frac{1}{18}$ b) $\frac{1}{12}$

c) $\frac{1}{6}$ d) $\frac{1}{36}$

6. A card is drawn at random from a pack of 52 cards. The probability that the drawn card is a jack, a queen or a king, is

a) $\frac{3}{52}$ b) $\frac{3}{13}$

c) $\frac{4}{13}$ d) $\frac{3}{26}$

7. Three mangoes and three apples are in a box. If two fruits are chosen at random, then find the probability that one is a mango and other is an apple

a) $\frac{2}{3}$ b) $\frac{3}{5}$

c) $\frac{1}{3}$ d) $\frac{2}{15}$

8. From a group of 5 boys and 3 girls, three persons are chosen at random. Find the probability that there are more girls than boys

a) $\frac{3}{8}$ b) $\frac{4}{7}$

c) $\frac{5}{8}$ d) $\frac{2}{7}$

9. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is

a) $\frac{2}{11}$ b) $\frac{3}{11}$

c) $\frac{4}{11}$ d) 0

10. From 10,000 lottery tickets numbered from 1 to 10,000, one ticket is drawn at random. What is the probability that the number marked on the drawn ticket is divisible by 20

a) $\frac{1}{100}$ b) $\frac{1}{50}$

c) $\frac{1}{20}$ d) $\frac{1}{10}$

11. The chance of getting a same number on 2 dice is

a) $\frac{2}{3}$ b) $\frac{1}{6}$

c) $\frac{5}{6}$ d) $\frac{5}{36}$

12. If E is any event associated with an experiment, then

a) $P(E) \leq 0$ b) $P(E) \geq 1$

c) $P(E) \geq 0$ d) $0 \leq P(E) \leq 1$

13. The probability of an impossible event is

a) 1 b) 2

c) $\frac{1}{2}$ d) 0

14. If in a lottery there are 5 prizes and 20 blanks, then the probability of getting a prize is

a) $\frac{1}{5}$ b) $\frac{2}{5}$

c) $\frac{4}{5}$ d) None of these

15. If A is a sure event, then the $P(\bar{A})$ is

a) 0 b) – 1

c) 1 d) 2

16. Six dice are thrown simultaneously. The probability that all of them show the same face, is

a) $\frac{1}{6^6}$ b) $\frac{1}{6^5}$

c) $\frac{1}{6}$ d) 6^6

17. For any event A

a) $P(A) + P(\bar{A}) = 0$

b) $P(A) + P(\bar{A}) = 1$

c) $P(A) > 1$

d) $P(\bar{A}) < 1$

18. The probability of a sure event is

a) 1 b) 2

c) $\frac{1}{2}$ d) 0

19. If E_1, E_2 E_3, E_4 are mutually exclusive and exhaustive events with respective probabilities p_1, p_2, p_3 and p_4, then which of the following is possible ?

a) $p_1 = 0.1, p_2 = 0.2, p_3 = 0.3, p_4 = 0.4$

b) $p_1 = 0.25, p_2 = 0.35, p_3 = 0.10, p_4 = 0.05$

c) $p_1 = 0.4, p_2 = -0.2, p_3 = 0.5, p_4 = 0.3$

d) $P_1 = 0.6, p_2 = 0.3, p_3 = 0.1, p_4 = 0.1$

20. Three letters are written to different persons, along with their addresses on three envelopes. Without looking at the letters, the probability that letters go into right envelopes is

a) $\frac{1}{24}$ b) $\frac{1}{6}$

c) $\frac{23}{24}$ d) $\frac{9}{2}$

21. The probability that an ordinary or a non-leap year has 53 Sundays, is

a) $\frac{2}{7}$ b) $\frac{1}{7}$

c) $\frac{3}{7}$ d) None of these

22.2 Addition theorem Probability and Conditional

22. If A and B are two independent events, then A and B are

a) Not independent

b) Also independent

c) Mutually exclusive

d) None of these

23. If A and B are any two events associated with an experiment, then

a) $P(A \cup B) = P(A) + P(B)$ if A and B are independent

b) $P(A \cup B) = 1 - P(A') P(B')$ if A and B are independent

c) $P(A \cap B) = P(A) P(B)$ if A and B are exclusive

d) $P(A \cap B') = P(A) + P(A \cap B)$

24. Two events A and B have probabilities 0.25 and 0.5 respectively. The probabilities that A and B occur simultaneously is 0.15. Then the probability that A or B occurs is

a) 0.6 b) 0.7

c) 0.61 d) 0.72

25. A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is $\frac{1}{2}$ and that of the woman's selection is $\frac{1}{3}$. What is the probability that none of them will be selected

a) $\frac{1}{3}$ b) $\frac{1}{12}$

c) $\frac{1}{4}$ d) $\frac{2}{3}$

26. The probabilities of a student getting first class or second class or third class in an examination are $\frac{2}{7}, \frac{3}{5}, \frac{1}{10}$ respectively. The probability that the student fails is

a) $\frac{6}{70}$ b) $\frac{11}{70}$

c) $\frac{3}{35}$ d) $\frac{1}{70}$

27. The probability that a card drawn at random from a pack of 52 cards is a king or a heart is

a) $\frac{1}{13}$ b) $\frac{1}{52}$

c) $\frac{1}{4}$ d) $\frac{16}{52}$

28. The probability that at least one of A or B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then P (A') + P (B') is

a) 0.9 b) 1.15

c) 1.1 d) 1.2

29. Two events A and B have probability 0.28 and 0.55 respectively. The probability that A and B occur simultaneously is 0.14. Find the probability that neither A nor B occurs

a) 0.39 b) 0.41

c) 0.4 d) 0.31

30. A coin is tossed twice. If events A and B are defined as :

A = head on first toss, B = head on second toss. Then the probability of $A \cup B$ =

a) $\frac{1}{4}$ b) $\frac{1}{2}$

c) $\frac{1}{8}$ d) $\frac{3}{4}$

31. If $P(A \cap B) = 0.15$, $P(B') = 0.10$, then P(A/B) is

a) $\frac{1}{4}$ b) $\frac{1}{2}$

c) $\frac{1}{8}$ d) $\frac{3}{4}$

32. If A and B are two events such that

$P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\overline{B}) = \frac{1}{3}$,

then P(A) =

a) $\frac{1}{4}$ b) $\frac{1}{3}$

c) $\frac{1}{2}$ d) $\frac{2}{3}$

33. If A and B are two events such that $P(A) = \frac{3}{8}$,

$P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$ then $P\left(\frac{A}{B}\right) =$

a) $\frac{2}{5}$ b) $\frac{2}{3}$

c) $\frac{3}{5}$ d) $\frac{5}{2}$

34. If P(A) = 0.4, P(B) = x, $P(A \cup B) = 0.7$ and the events A and B are mutually exclusive, then x =

a) $\frac{3}{10}$ b) $\frac{1}{2}$

c) $\frac{2}{5}$ d) $\frac{1}{5}$

35. If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cup B) = \frac{5}{8}$, then $P(A \cap B)$ is equal to

a) $\frac{3}{8}$ b) $\frac{1}{8}$

c) $\frac{2}{8}$ d) $\frac{5}{8}$

36. If the events A and B are mutually exclusive, then P(A/B) =

a) 0 b) 1

c) $\frac{P(A \cap B)}{P(A)}$ d) $\frac{P(A \cap B)}{P(B)}$

37. A and B are two events such that

P(A) = 0.8, P(B) = 0.6 and $P(A \cap B)$= 0.5, then the value of P(A/B) is

a) $\frac{5}{6}$ b) $\frac{5}{8}$

c) $\frac{9}{10}$ d) $\frac{6}{5}$

38. Events A and B are independent if

a) $P(A \cap B) = P(A/B).P(B)$
b) $P(A \cap B) = P(B/A).P(A)$
c) $P(A \cap B) = P(A) + P(B)$
d) $P(A \cap B) = P(A).P(B)$

39. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$ then, $P(B/A) =$

a) 1
b) 0
c) $\frac{1}{2}$
d) $\frac{1}{3}$

40. If A and B are two events such that $P(A) \neq 0$ and $P(B) \neq 1$, then $P\left(\frac{\overline{A}}{\overline{B}}\right) =$

a) $1 - P\left(\frac{A}{B}\right)$
b) $1 - P\left(\frac{\overline{A}}{B}\right)$
c) $\frac{1 - P(A \cup B)}{P(B)}$
d) $\frac{P(\overline{A})}{P(\overline{B})}$

22.3 B ayes' theorem and Odds

41. A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then the probability for the ball chosen to be white is

a) $\frac{2}{15}$
b) $\frac{7}{15}$
c) $\frac{8}{15}$
d) $\frac{14}{15}$

42. In solving any problem, odds against A are 4 to 3 and in favour of B in solving the same is 7 to 5. The probability that problem will be solved is

a) $\frac{5}{21}$
b) $\frac{16}{21}$
c) $\frac{15}{84}$
d) $\frac{69}{84}$

43. If the odds against an event be 2 : 3, then the probability of its occurrence is

a) $\frac{1}{5}$
b) $\frac{2}{5}$
c) $\frac{3}{5}$
d) 1

44. For an event, odds against is 6 : 5. The probability that event does not occur, is

a) $\frac{5}{6}$
b) $\frac{6}{11}$
c) $\frac{5}{11}$
d) $\frac{1}{6}$

Critical Thinking

22.1 Types of events
(Algebra of events, Concept of Probability)

1. A cricket club has 16 members out of which 6 can bowl. If a team of 11 members is selected. Find the probability that the team will contain exactly four bowlers

a) $\frac{5}{146}$
b) $\frac{7}{1456}$
c) $\frac{5}{1456}$
d) $\frac{72}{182}$

2. A person draws two cards with replacement from a pack of 52 cards. What is the chance that he gets both cards of the same suit?

a) $\frac{1}{4}$
b) $\frac{3}{13}$
c) $\frac{1}{16}$
d) $\frac{2}{13}$

3. From 4 children, 2 women and 4 men, 4 are selected. Probability that there are exactly 2 children among the selected is

a) $\frac{2}{7}$
b) $\frac{3}{7}$
c) $\frac{10}{21}$
d) $\frac{2}{10}$

4. A drawer contains 5 black socks and 4 blue socks well mixed. A person pulls out 2 socks at random from drawer. The probability that they match is

a) $\frac{5}{8}$
b) $\frac{4}{9}$
c) $\frac{5}{9}$
d) $\frac{41}{81}$

5. An organization consists of 25 members including 4 doctors. A committee of 4 is to be formed at random. The probability that the committee contains at least 3 doctors is

a) $\frac{17}{2530}$ b) $\frac{4}{2300}$

c) $\frac{1}{12640}$ d) $\frac{1}{2300}$

6. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first. The probability of all five persons leaving at different floors, is

a) $\frac{{}^7C_5}{7^5}$ b) $\frac{{}^7C_5 \times 5!}{5^7}$

c) $\frac{{}^7C_5 \times 5!}{7^5}$ d) $\frac{5!}{7^5}$

7. A group of 4 boys and 3 girls are arranged at random, one after the other. Probability that girls and boys occupy, alternate seats is,

a) $\frac{1}{34}$ b) $\frac{1}{35}$

c) $\frac{31}{36}$ d) $\frac{25}{36}$

8. In a single throw of two dice, what is the probability of getting a total 13

a) 0 b) 1

c) $\frac{13}{36}$ d) $\frac{25}{36}$

9. Three persons work independently on a problem. If the respective probabilities that they will solve it are 1/3, 1/4 and 1/5, then the probability that none can solve it is

a) $\frac{2}{5}$ b) $\frac{3}{5}$

c) $\frac{1}{3}$ d) None of these

10. Two dice are thrown. The number of sample points in the sample space when six does not appear on either dice is

a) 11 b) 30

c) 18 d) 25

11. A fair coin is tossed three times. The probability that there is atleast one tail is

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{3}{8}$ d) $\frac{7}{8}$

12. A digit is selected at random from either of the two sets $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the chance that the sum of the digits selected is 10 ?

a) $\frac{1}{9}$ b) $\frac{10}{81}$

c) $\frac{10}{18}$ d) $\frac{1}{81}$

13. Two coins are tossed. What is the probability of getting 2 heads or 2 tails?

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) $\frac{3}{4}$

14. From a book containing 100 pages, one page is selected randomly. The probability that the sum of the digits of the page number of the selected page is 11, is

a) $\frac{2}{25}$ b) $\frac{9}{100}$

c) $\frac{11}{100}$ d) $\frac{1}{100}$

15. From a pack of 52 cards, the cards are drawn till an ace appears. Probability that an ace does not come in first 26 cards is,

a) $\frac{{}^4C_1}{{}^{52}C_{26}}$ b) $\frac{{}^4C_1}{{}^{48}C_{26}}$

c) $\frac{1}{{}^{52}C_{26}}$ d) $\frac{{}^{48}C_{26}}{{}^{52}C_{26}}$

16. Three numbers are chosen from 1 to 30. The probability that they are not consecutive, is

a) $\frac{142}{145}$ b) $\frac{144}{145}$

c) $\frac{143}{145}$ d) $\frac{1}{145}$

17. A coin is tossed once. If a head comes up, then it is tossed again and if a tail comes up, a dice is thrown. The number of points in the sample space of experiment is

a) 24 b) 12
c) 4 d) 8

18. In shuffling a pack of playing cards, fc cards are accidently dropped. The probabill: that the missing cards should be one from each suit is

a) $\frac{1}{256}$ b) $\frac{4}{20825}$
c) $\frac{2197}{20825}$ d) $\frac{4}{52}$

19. Probability of getting a number between and 100, which is divisible by 1 and itst only, is

a) $\frac{1}{4}$ b) $\frac{25}{99}$
c) $\frac{25}{98}$ d) None of these

20. There is an objective type question with 4 answer choices exactly one of which correct. A student has not studied the topic c which the question has been set. The probability that the student guesses the correct answer, is

a) $\frac{1}{2}$ b) $\frac{1}{4}$
c) $\frac{1}{8}$ d) 4

21. The probability that a leap year selected i random will contain 53 Sundays is

a) $\frac{1}{7}$ b) $\frac{2}{7}$
c) $\frac{2}{9}$ d) $\frac{3}{7}$

22. Six dice are thrown. The probability tha different numbers will turn up is equal to

a) $\frac{5}{36}$ b) $\frac{5}{324}$
c) $\frac{3}{324}$ d) $\frac{1}{324}$

23. A card is drawn at random from a pack of 52 cards. The probability of getting red queen is

a) $\frac{1}{3}$ b) $\frac{1}{26}$
c) $\frac{1}{2}$ d) $\frac{7}{23}$

24. A car is parked by an owner amongst 25 cars in a row, not at either end. On his return finds that exactly 15 places are still occupi The probability that both the neighbour! places are empty is

a) $\frac{15}{99}$ b) $\frac{15}{92}$
c) $\frac{15}{184}$ d) $\frac{15}{25}$

25. The letters of the word FATHER are writt on separate cards, two cards are drawn random. Probability that both are vowels is

a) $\frac{2}{15}$ b) $\frac{1}{25}$
c) $\frac{3}{15}$ d) $\frac{1}{15}$

26. A box contains 10 sample watches, 2 of which are defective. If 2 are selected at random, the probability that both selected are defective is

a) $\frac{2}{25}$ b) $\frac{9}{20}$
c) $\frac{1}{25}$ d) $\frac{1}{45}$

27. Mr. A gave his telephone number to Mr. B. Mr. B remembers that the first two digits were 40 and the remaining four digits were two 3 one 6 and one 8. He is not certain about tf order of the digits. Mr. B dials 403638. The probability that he will get A's house is

a) $\frac{1}{2}$ b) $\frac{1}{4}$
c) $\frac{1}{8}$ d) $\frac{1}{12}$

28. An urn contains 5 blue and an unknow number x of red balls. Two balls are drawn i random from this urn. If probability of both of them being blue is $\frac{5}{14}$, then x = ?

a) 1 b) 2
c) 3 d) 4

29. All the letters of the word HAMSANAND are placed at random in a row. The probability that the word ANAND occurs without getting split is

a) $\frac{1}{42}$ b) $\frac{1}{60}$

c) $\frac{1}{420}$ d) $\frac{1}{329}$

30. Three horses H_1, H_2 and H_3 are in a race which is won by one of them. If H_1 is twice as likely to win as H_2 and H_2 is twice as likely to win as H_3, then their respective probabilities of winning are

a) $\frac{4}{7},\frac{2}{7},\frac{1}{7}$ b) $\frac{2}{7},\frac{4}{7},\frac{1}{7}$

c) $\frac{1}{7},\frac{2}{7},\frac{4}{7}$ d) None of these

31. Three different numbers are selected at random from the set A = (1, 2, ..., 10}. The probability that the product of two of the numbers is equal to third is

a) $\frac{3}{4}$ b) $\frac{1}{40}$

c) $\frac{1}{8}$ d) $\frac{39}{40}$

32. Two cards are drawn at random from a pack of 52 cards. Find the probability that they are both Aces if the first card is not replaced?

a) $\frac{1}{169}$ b) $\frac{1}{221}$

c) $\frac{4}{13}$ d) $\frac{3}{13}$

33. Two dice are thrown simultaneously. The probability of getting the sum 2 or 8 or 12 is

a) $\frac{5}{18}$ b) $\frac{7}{36}$

c) $\frac{7}{18}$ d) $\frac{5}{36}$

34. Three identical dice are rolled. The probability that the same number will appear on each of them is

a) $\frac{1}{6}$ b) $\frac{1}{36}$

c) $\frac{1}{18}$ d) $\frac{3}{28}$

22.2 Addition theorem and Conditional Probability

35. If P(A) = 0.4, P(B) = x, P(A ∪ B) = 0.7 and the events A and B are independent, then x =

a) $\frac{1}{3}$ b) $\frac{1}{2}$

c) $\frac{2}{3}$ d) None of these

36. If two events A and B are such that

$P(A+B)=\frac{5}{6}$, $P(AB)=\frac{1}{3}$ and $P(\bar{A})=\frac{1}{2}$, then the events A and B are

a) Independent

b) Mutually exclusive

c) Mutually exclusive and independent

d) None of these

37. A speaks truth in 60% of the cases and B in 90%. Percentage of cases in which they are likely to contradict each other, while stating the same fact, is

a) 36 b) 48

c) 42 d) 30

38. If the probabilities that A and B will die within a year are p and q respectively, then probability that only one of them will be alive at the end of the year is,

a) p + q

b) p + q – 2pq

c) p + q – pq

d) p + q + pq

39. If A and B are two mutually exclusive events such that P(B) = 2P(A) and A ∪ B = S, then P(B) is

a) $\frac{3}{4}$ b) $\frac{1}{3}$

c) $\frac{2}{3}$ d) $\frac{1}{2}$

40. The probability that an event A happens in a trial is 0.4. Three independent trials are made. The probability that A happens at least once is

a) 0.216 b) 0.784

c) 0.64 d) 0.936

41. You are given a box with 20 cards in it. 10 of these cards have the letter I printed on them. The other ten have the letter T printed on them. If you pick up 3 cards at random and keep them in the same order, the probability of making the word IIT is

a) $\frac{9}{80}$ b) $\frac{1}{8}$

c) $\frac{4}{27}$ d) $\frac{5}{38}$

42. The event A is independent of itself if and only if P(A) =

a) 0 b) 1

c) 0, 1 d) 1, 1

43. If A and B are two events such that

$P(A \cup B) + P(A \cap B) = \frac{7}{8}$ and $P(A) = 2P(B)$,

then $P(A) =$

a) $\frac{7}{12}$ b) $\frac{7}{24}$

c) $\frac{5}{12}$ d) $\frac{17}{24}$

44. Three athletes A, B and C participate in a race competition. The probability of winning for A and B is twice of winning for C. Then the probability that the race is won by A or B, is

a) $\frac{2}{3}$ b) $\frac{1}{2}$

c) $\frac{4}{5}$ d) $\frac{1}{3}$

45. If A and B are two events such that $A \subseteq B$, then

$P\left(\frac{B}{A}\right) =$

a) 0 b) 1

c) 1/2 d) 1/3

46. If A and B are two independent events, then

$P\left(\frac{A}{B}\right) =$

a) 0 b) 1

c) P(A) d) P(B)

47. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then, the probability that neither A nor B occurs is

a) 0.39 b) 0.375

c) 0.49 d) 0.59

48. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is

a) 1 b) $\frac{2}{5}$

c) $\frac{3}{5}$ d) 0

49. If A and B are two events and $P(A) = \frac{3}{8}$,

$P(B) = \frac{1}{2}$, $P(A \cap B) = i$, then $P(A' \cup B') =$

a) $\frac{3}{8}$ b) $\frac{3}{4}$

c) $\frac{1}{4}$ d) $\frac{5}{8}$

50. If A and B are two events. The probability that exactly one of them occurs is equal to

a) $P(A) + P(B) - 2\,P(A \cap B)$

b) $P(A) + P(B) + P(A \cap B)$

c) $P(A) + P(B)$

d) $P(A) + P(B) - P(A \cap B)$

51. There are two boxes. One box contains 3 white balls and 2 black balls. The other box contains 7 yellow balls and 3 black balls. If a box is selected at random and from it, a ball is drawn, the probability that the ball is black is

a) $\frac{7}{20}$ b) $\frac{1}{5}$

c) $\frac{3}{20}$ d) $\frac{1}{3}$

52. Out of 80 students in a class, 30 passed in Mathematics, 20 in Electronics and 10 in both. If one student is selected at random. The probability that he has passed in none of the subject is

a) $\frac{3}{5}$ b) $\frac{1}{4}$

c) $\frac{3}{2}$ d) $\frac{1}{2}$

53. There are two childrens in a family. The probability that both of them are boys is

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) $\frac{2}{3}$

54. If $P(A \cap B) = \frac{1}{2}$ and $P(A' \cap B') = \frac{1}{3}$, $P(A) = p$ and $P(B) = 2p$, then the value of p is

a) $\frac{7}{18}$ b) $\frac{1}{3}$

c) $\frac{4}{9}$ d) $\frac{1}{9}$

55. The probability that a leap year will have 53 Fridays or 53 Saturdays, is

a) $\frac{2}{7}$ b) $\frac{3}{7}$

c) $\frac{4}{7}$ d) $\frac{1}{7}$

56. A letter is taken from the word MULTIPLE and another letter is taken from the word CHOICE, the probability that both letters chosen are vowels is

a) $5\frac{5}{8}$ b) $\frac{1}{2}$

c) $\frac{1}{6}$ d) $\frac{3}{16}$

57. If $P(E_1) = p_1$ and $P(E_2) = p_2$ and E_1 and E_2 are independent, then P(neither E_1 nor E_2) =

a) $1 - p_1)(1 - p_2)$ b) $1 - (p_1 + p_2)$

c) $p_1 + p_2 - p_1 p_2$ d) $p_1 - p_2$

58. If A and B are two events with

$P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{4}$ and $P(B/A) = \frac{1}{2}$ then

a) A and B are mutually exclusive

b) A and B are independent.

c) A is sub-event of B

d) B is sub-event of A

59. In a single throw of two dice, the probability of getting a total of 7 or 9 is

a) $\frac{4}{48}$ b) $\frac{1}{3}$

c) $\frac{5}{18}$ d) $\frac{1}{18}$

60. A class consists of 80 students 25 of them are girls and 55 boys. If 10 are rich and remaining poor and also 20 of them are intelligent, then the probability of selecting intelligent rich girls is

a) $\frac{5}{128}$ b) $\frac{25}{128}$

c) $\frac{5}{512}$ d) $\frac{5}{64}$

61. The probability that a man will live 10 more years is $\frac{1}{4}$ and the probability that his wife will live 10 more years is $\frac{1}{3}$. Then the probability that neither will be alive in 10 years is

a) $\frac{5}{12}$ b) $\frac{1}{2}$

c) $\frac{7}{12}$ d) $\frac{11}{12}$

62. Let E and F be two independent events. The probability that both E and F happens is $\frac{1}{12}$ and probability that neither happens is $\frac{1}{2}$. Then

a) $P(E) = \frac{1}{13}, P(F) = \frac{1}{4}$

b) $P(E) = \frac{1}{2}, P(F) = \frac{1}{6}$

c) $P(E) = \frac{1}{6}, P(F) = \frac{1}{12}$

d) $P(E) = \frac{1}{4}, P(F) = \frac{1}{3}$

63. The chances to fail in Physics are 20% and the chances to fail in Mathematics are 10%. What are the chances to fail in at least one subject

a) 28 % b) 38%

c) 72% d) 82%

64. The probability that in a throw of two dice we get, an even sum or sum less than 5 is

a) $\frac{1}{2}$ b) $\frac{1}{6}$

c) $\frac{2}{3}$ d) $\frac{5}{9}$

65. In a town 40% of the people have brown hair, 25% have brown eyes and 15% have both. If a person selected at random from the town has brown hair, the probability that he has brown eyes is

a) $\frac{1}{5}$ b) $\frac{3}{8}$

c) $\frac{1}{5}$ d) $\frac{2}{3}$

66. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ..., 98, 99. If X and Y denote respectively the sum and the product of the digits on the tickets, then $\left(\frac{X=9}{Y=0}\right)=$

a) $\frac{2}{17}$ b) $\frac{2}{19}$

c) $\frac{2}{21}$ d) $\frac{2}{11}$

67. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B)=\frac{1}{4}$ and $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{\overline{B}}{\overline{A}}\right)=$

a) $\frac{37}{40}$ b) $\frac{37}{45}$

c) $\frac{23}{40}$ d) $\frac{1}{3}$

22.3 Bayes' theorem and Odds

68. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is

a) $\frac{37}{40}$ b) $\frac{1}{37}$

c) $\frac{36}{37}$ d) $\frac{1}{9}$

69. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

a) $\frac{5}{17}$ b) $\frac{12}{17}$

c) $\frac{17}{30}$ d) $\frac{3}{5}$

70. A purse contains 4 copper coins and 3 silver coins, the second purse contains 6 copper coins and 2 silver coins. A coin is taken out from any purse, the probability that it is a copper coin is

a) $\frac{4}{7}$ b) $\frac{37}{56}$

c) $\frac{3}{7}$ d) $\frac{1}{3}$

71. Bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted it is green. The probability that it conies from bag B is

a) $\frac{2}{7}$ b) $\frac{2}{3}$

c) $\frac{3}{7}$ d) $\frac{1}{3}$

72. There are 3 bags which are known to contain 2 white and 3 black balls; 4 white and 1 black balls and 3 white and 7 black balls respectively. A ball is drawn at random from one of the bags and found to be a black ball. Then the probability that it was drawn from the bag containing the most black balls is

a) $\frac{7}{15}$ b) $\frac{5}{19}$

c) $\frac{3}{4}$ d) $\frac{7}{10}$

73. One and only one of the two events must 2 occur. If the chance of one is $\frac{2}{3}$ of the other, then odds in favour of the other are

a) 1 : 3 b) 3 : 1
c) 2 : 3 d) 3 : 2

74. If the odds in favour of an event be 3:: 5, then the probability of non-occurrence of fhe event is

a) $\frac{3}{5}$ b) $\frac{5}{3}$
c) $\frac{3}{8}$ d) $\frac{5}{8}$

75. The odds against a certain event is 5 : 2 and the odds in favour of another event is 6 : 5. If both the events are independent, then the probability that at least one of the events will happen is

a) $\frac{50}{77}$ b) $\frac{52}{77}$
c) $\frac{25}{88}$ d) $\frac{63}{88}$

76. In a horse race the odds in favour of three horses are 1 : 2, 1 : 3 and 1 : 4. The probability that one of the horse will win the race is

a) $\frac{37}{60}$ b) $\frac{47}{60}$
c) $\frac{1}{4}$ d) $\frac{3}{4}$

77. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is

a) $\frac{3}{8}$ b) $\frac{1}{5}$
c) $\frac{3}{4}$ d) $\frac{1}{4}$

78. A card is drawn from a pack of 52 cards. A gambler bets that it is a spade or an ace. What are the odds against his winning this bet

a) 17 : 52 b) 52 : 17
c) 9 : 4 d) 4 : 9

79. An event has odds in favour 4 : 5, then the probability that event occurs, is

a) $\frac{1}{5}$ b) $\frac{4}{5}$
c) $\frac{4}{9}$ d) $\frac{5}{9}$

80. Three ships A, B and C sail from England to India. If the ratio of their arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively, then the probability of all the ships for arriving safely is

a) $\frac{18}{595}$ b) $\frac{6}{17}$
c) $\frac{3}{10}$ d) $\frac{2}{7}$

Competitive Thinking

22.1 Types of events
(Algebra of events, Concept of Probability)

1. If two balanced dice are tossed once, the probability of the event, that the sum of the integers coming on the upper sides of the two dice is 9, is

a) $\frac{7}{18}$ b) $\frac{5}{36}$
c) $\frac{1}{9}$ d) $\frac{1}{6}$

2. The probability that an event will fail to happen is 0.05. The probability that the event will take place on 4 consecutive occasions is

a) 0.00000625 b) 0.18543125
c) 0.00001875 d) 0.81450625

3. Two integers are chosen at random and multiplied. The probability that the product is an even integer is

a) $\frac{1}{2}$ b) $\frac{2}{3}$
c) $\frac{3}{4}$ d) $\frac{4}{5}$

4. If a coin is tossed n times, then probability that the head comes odd times is

a) $\frac{1}{2}$ b) $\frac{1}{2^n}$
c) $\frac{1}{2^{n-1}}$ d) None of these

5. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

a) $\frac{1}{3}$ b) $\frac{1}{6}$

c) $\frac{1}{2}$ d) $\frac{1}{4}$

6. In four schools B_1, B_2, B_3, B_4 the percentage of girls students is 12, 20, 13, 17 respectively. From a school selected at random, one student is picked up at random and it is found that the student is a girl. The probability that the school selected is B_2, is

a) $\frac{6}{31}$ b) $\frac{10}{31}$

c) $\frac{13}{62}$ d) $\frac{17}{62}$

7. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

a) $\frac{1}{2}$ b) $\frac{7}{15}$

c) $\frac{2}{15}$ d) $\frac{1}{3}$

8. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or is a nail

a) $\frac{3}{16}$ b) $\frac{5}{16}$

c) $\frac{11}{16}$ d) $\frac{14}{16}$

9. The probability of getting a total of 5 or 6 in a single throw of 2 dice is

a) $\frac{1}{2}$ b) $\frac{1}{4}$

c) $\frac{1}{3}$ d) $\frac{1}{6}$

10. Three dice are thrown simultaneously. What is the probability of obtaining a total of 17 or 18

a) $\frac{1}{9}$ b) $\frac{1}{72}$

c) $\frac{1}{54}$ d) None of these

11. Two dice are thrown. The probability that the total score is a prime number, is

a) $\frac{1}{6}$ b) $\frac{5}{12}$

c) $\frac{1}{2}$ d) None of these

12. The chance of throwing at least 9 in a single throw with two dice, is

a) $\frac{1}{18}$ b) $\frac{5}{18}$

c) $\frac{7}{18}$ d) $\frac{11}{18}$

13. From the word 'POSSESSIVE', a letter is chosen at random. The probability of it to be S is

a) $\frac{3}{10}$ b) $\frac{4}{10}$

c) $\frac{3}{6}$ d) $\frac{4}{6}$

14. In a throw of three dice, the probability that at least one die shows up 1, is

a) $\frac{5}{6}$ b) $\frac{91}{216}$

c) $\frac{1}{36}$ d) $\frac{125}{216}$

15. The corners of regular tetrahedrons are numbered 1, 2, 3, 4. Three tetrahedrons are tossed. The probability that the sum of upward corners will be 5 is

a) $\frac{5}{24}$ b) $\frac{5}{64}$

c) $\frac{3}{32}$ d) $\frac{3}{16}$

16. A coin is tossed 4 times. The probability that at least one head turns up is

a) $\frac{1}{16}$ b) $\frac{2}{16}$

c) $\frac{14}{16}$ d) $\frac{15}{16}$

17. A coin is tossed three
Event A : two head conies
Event B : last should be head
Then A and B are
a) independent b) dependent
c) both d) none of these

18. The probability that a number selected at random from the set of number {1,2, 3, ...,100} is a cube is
a) $\frac{1}{25}$ b) $\frac{2}{25}$
c) $\frac{3}{25}$ d) $\frac{4}{25}$

19. Two dice are thrown simultaneously. The probability of obtaining a total score of 5 is
a) $\frac{1}{9}$ b) $\frac{1}{18}$
c) $\frac{1}{36}$ d) $\frac{1}{12}$

20. If $P(A) = \frac{4}{5}$, $P(B') = \frac{2}{5}$, $P(A \cap B) = \frac{1}{2}$, then $P(A \cap B')$ is equal to
a) $\frac{3}{10}$ b) $\frac{1}{5}$
c) $\frac{4}{5}$ d) $\frac{1}{2}$

21. Probability of solving of sum correctly by A, B and C is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{5}$ respectively. The probability that at least one of them solves it correctly is
a) $\frac{11}{15}$ b) $\frac{4}{15}$
c) $\frac{1}{20}$ d) $\frac{19}{20}$

22. If 3 coins were tossed, then the probability of getting 2 heads is
a) $\frac{3}{8}$ b) $\frac{2}{8}$
c) $\frac{1}{8}$ d) none of these

23. If P(A) = 0.7, P(B) = 0.4, $P(A \cap B) = 0.3$, then $P(A \cap B') =$
a) $\frac{2}{5}$ b) 1
c) 0.7 d) 0.42

24. If P(A) = 0.25, P(B) = 0.50, $P(A \cap B) = 0.14$, then $P(\overline{A} \cap \overline{B}) =$
a) 0.38 b) 0.39
c) 0.40 d) None of these

25. Two dice, one black and one white are rolled. The probability that sum of two no. is 7 and no. of black greater than the no. of white is
a) $\frac{1}{12}$ b) $\frac{1}{6}$
c) $\frac{1}{4}$ d) $\frac{1}{2}$

26. In a non-leap year, the probability of having 53 Friday or Saturday is
a) $\frac{3}{7}$ b) $\frac{4}{7}$
c) $\frac{2}{7}$ d) $\frac{1}{7}$

27. 26 cards numbered from 1 to 26. One card is chosen. Probability that it is not divisible by 4 is
a) $\frac{3}{13}$ b) $\frac{4}{13}$
c) $\frac{2}{13}$ d) $\frac{10}{13}$

28. There are 5 red balls and x black balls. If two balls are drawn at random, probability that the balls drawn are red is $\frac{5}{14}$, find the value of x ?
a) 9 b) 12
c) 3 d) 6

22.2 Addition theorem and Conditional Probability

29. A problem of mathematics is given to three students whose chances of solving the problem are 1/3, 1/4 and 1/5 respectively. The probability that the question will be solved is
a) $\frac{2}{3}$ b) $\frac{3}{4}$
c) $\frac{4}{5}$ d) $\frac{3}{5}$

30. The probability of India winning a test match against West Indies is $\frac{1}{2}$. Assuming independence from match to match, the probability that in a 5 match series India's second win occurs at the third test, is

a) $\frac{2}{3}$ b) $\frac{1}{2}$

c) $\frac{1}{4}$ d) $\frac{1}{8}$

31. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact

a) $\frac{4}{5}$ b) $\frac{1}{5}$

c) $\frac{7}{20}$ d) $\frac{3}{20}$

32. Two cards are drawn one by one at random from a pack of 52 cards. The probability that both of them are king, is

a) $\frac{2}{13}$ b) $\frac{1}{169}$

c) $\frac{1}{221}$ d) $\frac{30}{221}$

33. From a pack of 52 cards two are drawn with replacement. The probability, that the first is a diamond and the second is a king, is

a) $\frac{1}{26}$ b) $\frac{17}{2704}$

c) $\frac{1}{52}$ d) None of these

34. A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected

a) $\frac{1}{7}$ b) $\frac{2}{7}$

c) $\frac{3}{7}$ d) None of these

35. A bag contains 3 black and 4 white balls. Two balls are drawn one by one at random without replacement. The probability that the second drawn ball is white, is

a) $\frac{4}{49}$ b) $\frac{1}{7}$

c) $\frac{4}{7}$ d) $\frac{12}{49}$

36. The probabilities of three mutually exclusive events are $\frac{2}{3}, \frac{1}{4}$ and $\frac{1}{6}$. The statement is

a) True b) Wrong

c) Could be either d) Do not know

37. For any two independent events E_1 and E_2.
$P\{(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)\}$ is

a) $< \frac{1}{4}$ b) $> \frac{1}{4}$

c) $\geq \frac{1}{4}$ d) None of these

38. For two given events A and B, $P(A \cap B) =$

a) Not less than $P(A) + P(B) - 1$

b) Not greater than $P(A) + P(B)$

c) Equal to $P(A) + P(B) - P(A \cup B)$

d) All of the above

39. A, B, C are any three events. If P (S) denotes the probability of S happening, then
$P(A \cap (B \cup C)) =$

a) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$

b) $P(A) + P(B) + P(C) - P(B) P(C)$

c) $P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

d) None of these

40. The probability that at least one of the events A and B occurs is 3/5. If A and B occur simultaneously with probability 1/5, then $P(A') + P(B')$ is

a) $\frac{2}{5}$ b) $\frac{4}{5}$

c) $\frac{6}{5}$ d) $\frac{7}{5}$

41. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(AB^c) = 0.5$, then $P[B/(A \cup B^c)]$ is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{1}{4}$ d) None of these

42. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the true relation is

a) $P(A) + P(B) = 0$

b) $P(A)+P(B)=P(A)P\left(\frac{B}{A}\right)$

c) $P(A)+P(B)=2P(A)P\left(\frac{B}{A}\right)$

d) None of these

43. The probability of happening of an event A is 0,5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of happening of neither A nor B is

a) 0.6

b) 0.2

c) 0.21

d) None of these

44. For any two events A and B in a sample space

a) $P\left(\frac{A}{B}\right) \geq \frac{P(A)+P(B)-1}{P(B)}, P(B) \neq 0$ always true

b) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ does not hold

c) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A and B are disjoint

d) None of these

45. Consider two events A and B such that

$P(A)=\frac{1}{4}, P\left(\frac{B}{A}\right)=\frac{1}{2}, P\left(\frac{A}{B}\right)=\frac{1}{4}$. For each of the following statements, which is true

I. $P(A^c/B^c)=\frac{3}{4}$

II. The events A and B are mutually exclusive

III. $P(A / B) + P(A / B^c) = 1$

a) I only b) I and II

c) I and III d) II and III

46. If the probability of X to fail in the examination is 0.3 and that for Y is 0.2, then the probability that either X or Y fails in the examination is

a) 0.5 b) 0.44

c) 0.6 d) None of these

47. If A and B are two independent events such that $P(A) = 0.40$, $P(B) = 0.50$. Find P (neither A nor B)

a) 0.90 b) 0.10

c) 0.2 d) 0.3

48. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events, then $P(B) =$

a) $\frac{5}{6}$ b) $\frac{5}{7}$

c) $\frac{3}{5}$ d) $\frac{2}{5}$

49. $P(A \cup B) = P(A \cap B)$ if and only if the relation between P(A) and P(B) is

a) $P(A)=P(\bar{A})$

b) $P(A \cup B) = P(A' \cap B')$

c) $P(A) = P(B)$

d) None of these

50. Let A and B be events for which $P(A) = x$, $P(B) = y$, $P(A \cap B) = z$, then $P(\bar{A} \cap B)$ equals

a) $(1 - x)y$ b) $1 - x + y$

c) $y - z$ d) $1 - x + y - z$

51. The probability of solving a question by three students are $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ respectively. Probability of question being solved will be

a) $\frac{33}{48}$ b) $\frac{35}{48}$

c) $\frac{31}{48}$ d) $\frac{37}{48}$

52. If P(A) = 0.25_, P(B)= 0.50 and $P(A \cap B) = 0.14$, then $P(A \cap \overline{B})$ is equal to
a) 0.61 b) 0.39
c) 0.48 d) 0.11

53. If P(A) = P(B) = x and $P(A \cap B) = P(A' \cap B') = \frac{1}{3}$, then x
a) $\frac{1}{2}$ b) $\frac{1}{3}$
c) $\frac{1}{4}$ d) $\frac{1}{6}$

54. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then
a) E and F^c (the complement of the event F) are independent
b) E^c and F^c are independent
c) $P\left(\frac{E}{F}\right) + P\left(\frac{E^c}{F^c}\right) = 1$
d) All of the above

55. If $4\,P(A) = 6\,P(B) = 10\,P(A \cap B) = 1$, then $P\left(\frac{B}{A}\right) =$
a) $\frac{2}{5}$ b) $\frac{3}{5}$
c) $\frac{7}{10}$ d) $\frac{19}{60}$

56. A coin is tossed three times in succession. If E is the event that there are at least two heads and F is the event in which first toss is a head, then $P\left(\frac{E}{F}\right) =$
a) $\frac{3}{4}$ b) $\frac{3}{8}$
c) $\frac{1}{2}$ d) $\frac{1}{8}$

57. If A and B are two events such that P(A) = 0.4, P(A + B) = 0.7 and P(AB) = 0.2, then P(B) =
a) 0.1 b) 0.3
c) 0.5 d) None of these

58. If P(S) = 0.3, P(T) = 0.4, S and T are independent events, then P(S/T) =
a) 0.2 b) 0.3
c) 0.12 d) 0.4

59. The probability of happening at least one of the events A and B is 0.6. If the events A and B happens simultaneously with the probability 0.2, then P(A) + P(B) =
a) 0.4 b) 0.8
c) 1.2 d) 1.4

60. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is
a) 1 b) $\frac{2}{5}$
c) $\frac{3}{5}$ d) 0

61. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then, P(B) is
a) $\frac{2}{3}$ b) $\frac{1}{2}$
c) $\frac{1}{6}$ d) $\frac{1}{3}$

62. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals
a) $\frac{1}{14}$ b) $\frac{1}{7}$
c) $\frac{5}{14}$ d) $\frac{1}{50}$

63. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where A stands for complement of event A. Then events A and B are
a) mutually exclusive and independent
b) independent but not equally likely
c) equally likely but not independent
d) equally likely and mutually exclusive

64. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by second plane is

a) 0.2
b) 0.7
c) 0.06
d) 0.14

65. If A and B are independent events of a random experiment such that $P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, then P(A) is equal to

a) $\frac{1}{4}$ b) $\frac{1}{3}$
c) $\frac{1}{6}$ d) $\frac{2}{3}$

66. If A and B are independent events such that $P(B) = \frac{2}{7}$, $P(A \cup \bar{B}) = 0.8$, then P(A) =

a) 0.1 b) 0.2
c) 0.3 d) 0.4

67. 'X' speaks truth in 60 % and 'Y' in 50 % of the cases. The probability that they contradict each other while narrating the same incident, is

a) $\frac{1}{4}$ b) $\frac{1}{3}$
c) $\frac{1}{2}$ d) $\frac{2}{3}$

68. If $P(B) = \frac{3}{4}$, $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$, $P(A \cap B \cap \bar{C}) = \frac{1}{3}$, then $P(B \cap C) =$

a) $\frac{1}{12}$ b) $\frac{3}{4}$
c) $\frac{5}{12}$ d) $\frac{23}{60}$

69. If the events A and B are independent if $P(\bar{A}) = \frac{2}{3}$ and $P(\bar{B}) = \frac{2}{7}$, then $P(A \cap B)$ is equal to

a) $\frac{4}{21}$ b) $\frac{5}{21}$
c) $\frac{1}{21}$ d) $\frac{3}{21}$

70. Ram is visiting a friend. Ram knows that his friend has 2 children and 1 of them is a boy. Assuming that a child is equally likely to be a boy or a girl, then the probability that the other child is a girl, is

a) $\frac{1}{2}$ b) $\frac{1}{3}$
c) $\frac{2}{3}$ d) $\frac{7}{10}$

71. If P(A) = 0.4, P(B) = x, $P(A \cup B) = 0.7$ and the events A and B are mutually exclusive, then x =

a) $\frac{3}{10}$ b) $\frac{1}{2}$
c) $\frac{2}{5}$ d) $\frac{1}{5}$

22.3 Bayes' theorem and Odds

72. A party of 23 persons take their seats at a round table. The odds against two persons sitting together are

a) 10 : 1
b) 1 : 11
c) 9 : 10
d) None of these

73. Odds 8 to 5 against a person who is 40 years old living till he is 70 and 4 to 3 against another person now 50 till he will be living 80. Probability that one of them will be alive next 30 years

a) $\frac{59}{91}$ b) $\frac{44}{91}$
c) $\frac{51}{91}$ d) $\frac{32}{91}$

74. For a biased die the probabilities for different faces to turn up are given below

Face :	1	2	3	4	5	6
Probability	0.1	0.32	0.21	0.15	0.05	0.17

The die is thrown and you are told that either face 1 or 2 has turned up. Then the probability that it is face 1, is

a) $\frac{5}{21}$ b) $\frac{5}{22}$

c) $\frac{4}{21}$ d) None of these

75. A bag 'A' contains 2 white and 3 red balis and bag 'B' contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from bag 'B' was

a) $\frac{5}{14}$ b) $\frac{5}{16}$

c) $\frac{5}{18}$ d) $\frac{25}{52}$

76. If odds against solving a question by three students are 2 : 1, 5 : 2 and 5 : 3 respectively, then probability that the question is solved only by one student is

a) $\frac{31}{56}$ b) $\frac{24}{56}$

c) $\frac{25}{56}$ d) None of these

77. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is

a) $\frac{3}{8}$ b) $\frac{1}{5}$

c) $\frac{3}{4}$ d) None of these

78. A bag 'A' contains 2 white and 3 red balls and bag 'B' contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from bag 'B' was

a) $\frac{5}{14}$ b) $\frac{5}{16}$

c) $\frac{5}{18}$ d) $\frac{25}{52}$

79. A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then, the probability for the chosen ball to be white is

a) $\frac{2}{15}$ b) $\frac{7}{15}$

c) $\frac{8}{15}$ d) $\frac{14}{15}$

80. In an entrance examination there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to the question, then the probability that he was guessing is

a) $\frac{1}{9}$ b) $\frac{36}{37}$

c) $\frac{1}{37}$ d) $\frac{37}{40}$

81. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is

a) $\frac{3p}{4p+3}$ b) $\frac{5p}{3p+2}$

c) $\frac{5p}{4p+1}$ d) $\frac{4p}{3p+1}$

Evaluation Test

1. Three numbers are chosen from 1 to 30. The probability that they are not consecutive is

a) $\frac{142}{145}$ b) $\frac{144}{145}$

c) $\frac{143}{145}$ d) $\frac{1}{145}$

2. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then

a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$

b) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$

d) $P(E)=\frac{6}{5}, P(F)=\frac{1}{5}$

3. One Indian and four American men and their wives are to be seated randomly around a circular table. The conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) $\frac{2}{5}$ d) $\frac{1}{5}$

4. Four cards are drawn from a pack of 52 cards, The probability of drawing exactly one pair is

a) 0.4 b) 0.5

c) 0.8 d) none of these

5. Three numbers are chosen at random without replacement from $\{1, 2, 3, ... 10\}$. The probability that the minimum of the chosen number is 3 or their maximum is 7, is

a) $\frac{7}{40}$ b) $\frac{5}{40}$

c) $\frac{11}{40}$ d) none of these

6. The probability that in a year of 22nd century chosen at random, there will be 53 Sundays is

a) $\frac{3}{28}$ b) $\frac{2}{28}$

c) $\frac{7}{28}$ d) $\frac{5}{28}$

7. Let A, B and C be three events such that $P(A) = 0.3, P(B) = 0.4, P(C) = 0.8$, $P(A \cap B) = 0.08, P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then $P(B \cap C)$ satisfies

a) $P(B \cap C) \leq 0.23$

b) $P(B \cap C) \leq 0.48$

c) $0.23 \leq P(B \cap C) \leq 0.48$

d) $0.23 \leq P(B \cap C) \geq 0.48$

8. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received atstation B is green, then the probability that the original signal was green is

a) $\frac{3}{5}$ b) $\frac{6}{7}$

c) $\frac{20}{23}$ d) $\frac{9}{20}$

9. The probability that a leap year selected at random contains either 53 Sundays or 53 Mondays, is

a) $\frac{2}{7}$ b) $\frac{4}{7}$

c) $\frac{3}{7}$ d) $\frac{1}{7}$

❍❍❍❍

Classical Thinking

1. (A)	2. (B)	3. (D)	4. (B)	5. (B)	6. (B)	7. (B)	8. (D)	9. (C)	10. (C)
11. (B)	12. (D)	13. (D)	14. (A)	15. (A)	16. (B)	17. (B)	18. (A)	19. (A)	20. (B)
21. (B)	22. (B)	23. (B)	24. (A)	25. (A)	26. (D)	27. (D)	28. (C)	29. (D)	30. (D)
31. (D)	32. (C)	33. (A)	34. (A)	35. (B)	36. (A)	37. (A)	38. (D)	39. (C)	40. (C)
41. (C)	42. (B)	43. (C)	44. (B)						

Critical Thinking

1. (D)	2. (A)	3. (B)	4. (B)	5. (A)	6. (C)	7. (B)	8. (A)	9. (A)	10. (D)
11. (D)	12. (A)	13. (A)	14. (A)	15. (D)	16. (B)	17. (D)	18. (C)	19. (C)	20. (B)
21. (B)	22. (B)	23. (B)	24. (B)	25. (D)	26. (D)	27. (D)	28. (C)	29. (C)	30. (A)
31. (B)	32. (B)	33. (B)	34. (B)	35. (B)	36. (A)	37. (C)	38. (B)	39. (C)	40. (B)
41. (D)	42. (C)	43. (A)	44. (C)	45. (B)	46. (C)	47. (A)	48. (A)	49. (B)	50. (A)
51. (A)	52. (D)	53. (C)	54. (A)	55. (B)	56. (D)	57. (A)	58. (B)	59. (C)	60. (C)
61. (B)	62. (D)	63. (A)	64. (D)	65. (B)	66. (B)	67. (A)	68. (B)	69. (B)	70. (B)
71. (C)	72. (A)	73. (D)	74. (D)	75. (B)	76. (B)	77. (A)	78. (C)	79. (C)	80. (A)

Competitive Thinking

1. (C)	2. (D)	3. (B)	4. (A)	5. (B)	6. (B)	7. (B)	8. (C)	9. (B)	10. (C)
11. (B)	12. (B)	13. (B)	14. (B)	15. (C)	16. (D)	17. (B)	18. (A)	19. (A)	20. (A)
21. (A)	22. (A)	23. (A)	24. (B)	25. (A)	26. (C)	27. (D)	28. (C)	29. (D)	30. (C)
31. (C)	32. (C)	33. (C)	34. (B)	35. (C)	36. (B)	37. (A)	38. (D)	39. (C)	40. (C)
41. (C)	42. (C)	43. (B)	44. (A)	45. (A)	46. (B)	47. (D)	48. (B)	49. (C)	50. (C)
51. (A)	52. (D)	53. (A)	54. (D)	55. (A)	56. (A)	57. (C)	58. (B)	59. (C)	60. (A)
61. (D)	62. (A)	63. (B)	64. (D)	65. (B)	66. (C)	67. (C)	68. (A)	69. (B)	70. (A)
71. (A)	72. (A)	73. (B)	74. (A)	75. (D)	76. (C)	77. (A)	78. (D)	79. (C)	80. (C)
81. (C)									

Answers to Evaluation Test

1. (B) 2. (A) 3. (C) 4.(D) 5. (C) 6.(D) 7.(C) 8.(C) 9.(C)

Answer Key

Classical Thinking

1. (A)	2. (B)	3. (D)	4. (B)	5. (B)	6. (B)	7. (B)	8. (D)	9. (C)	10. (C)
11. (B)	12. (D)	13. (D)	14. (A)	15. (A)	16. (B)	17. (B)	18. (A)	19. (A)	20. (B)
21. (B)	22. (B)	23. (B)	24. (A)	25. (A)	26. (D)	27. (D)	28. (C)	29. (D)	30. (D)
31. (D)	32. (C)	33. (A)	34. (A)	35. (B)	36. (A)	37. (A)	38. (D)	39. (C)	40. (C)
41. (C)	42. (B)	43. (C)	44. (B)						

Critical Thinking

1. (D)	2. (A)	3. (B)	4. (B)	5. (A)	6. (C)	7. (B)	8. (A)	9. (A)	10. (D)
11. (D)	12. (A)	13. (A)	14. (A)	15. (D)	16. (B)	17. (D)	18. (C)	19. (C)	20. (B)
21. (B)	22. (B)	23. (B)	24. (B)	25. (D)	26. (D)	27. (D)	28. (C)	29. (C)	30. (A)
31. (B)	32. (B)	33. (B)	34. (B)	35. (B)	36. (A)	37. (C)	38. (B)	39. (C)	40. (B)
41. (D)	42. (C)	43. (A)	44. (C)	45. (B)	46. (C)	47. (A)	48. (A)	49. (B)	50. (A)
51. (A)	52. (D)	53. (C)	54. (A)	55. (B)	56. (D)	57. (A)	58. (B)	59. (C)	60. (C)
61. (B)	62. (D)	63. (A)	64. (D)	65. (B)	66. (B)	67. (A)	68. (B)	69. (B)	70. (B)
71. (C)	72. (A)	73. (D)	74. (D)	75. (B)	76. (B)	77. (A)	78. (C)	79. (C)	80. (A)

Competitive Thinking

1. (C)	2. (D)	3. (B)	4. (A)	5. (B)	6. (B)	7. (B)	8. (C)	9. (B)	10. (C)
11. (B)	12. (B)	13. (B)	14. (B)	15. (C)	16. (D)	17. (B)	18. (A)	19. (A)	20. (A)
21. (A)	22. (A)	23. (A)	24. (B)	25. (A)	26. (C)	27. (D)	28. (C)	29. (D)	30. (C)
31. (C)	32. (C)	33. (C)	34. (B)	35. (C)	36. (B)	37. (A)	38. (D)	39. (C)	40. (C)
41. (C)	42. (C)	43. (B)	44. (A)	45. (A)	46. (B)	47. (D)	48. (B)	49. (C)	50. (C)
51. (A)	52. (D)	53. (A)	54. (D)	55. (A)	56. (A)	57. (C)	58. (B)	59. (C)	60. (A)
61. (D)	62. (A)	63. (B)	64. (D)	65. (B)	66. (C)	67. (C)	68. (A)	69. (B)	70. (A)
71. (A)	72. (A)	73. (B)	74. (A)	75. (D)	76. (C)	77. (A)	78. (D)	79. (C)	80. (C)
81. (C)									

Hints

Classical Thinking

1. Here, $n(S) = 2 \times 2 \times 2 \times 2 = 16$
   A: Event of getting all heads
   $\Rightarrow A = \{(HHHH)\}$

$\therefore$ $n(A) = 1$

$\Rightarrow P(A) = \dfrac{1}{16}$

2. Here, $n(S) = 52$
   There is one queen of club and one king of heart

$\therefore$ Favourable ways $= 1 + 1 = 2$

$\therefore$ Required Probability $= \dfrac{2}{52} = \dfrac{1}{26}$

3. Odd and perfect square (< 10) are 1, 9.
   Hence, required probability $= \dfrac{2}{10} = \dfrac{1}{5}$

4. Probability of keeping at least one letter in wrong envelope $= 1 - \dfrac{1}{n!}$

$\therefore$ option (B) is the correct answer.

5. Required probability $= \dfrac{3}{36} = \dfrac{1}{12}$.

6. Hence, required probability $= \dfrac{12}{52} = \dfrac{3}{13}$.

7. Two fruits out of 6 can be chosen in ${}^6C_2 = 15$ ways.
One mango and one apple can be chosen in = ${}^3C_1 \times {}^3C_1 = 9$ ways

$\therefore$ Probability = $\frac{9}{15} = \frac{3}{5}$

8. Three persons can be chosen out of 8 in ${}^8C_3 = 56$ ways.
The number of girls is more than that of the boys if either 3 girls are chosen or two girls and one boy is chosen. This can be done in ${}^3C_3 + {}^3C_2 \times {}^5C_1$ ways
$= 1 + 3 \times 5 = 16$ ways.

$\therefore$ Required probability = $\frac{16}{56} = \frac{2}{7}$

9. Since there are one A, two I and one O, hence the required probability = $\frac{1+2+1}{11} = \frac{4}{11}$

10. Number of tickets, numbered such that it is divisible by 20 are $\frac{10000}{20} = 500$

Hence, required probability = $\frac{500}{10000} = \frac{1}{20}$.

11. Total number of outcomes = 36
Favourable number of outcomes = 6
i.e., {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$\therefore$ Required probability = $\frac{6}{36} = \frac{1}{6}$.

14. Required probability = $\frac{5}{25} = \frac{1}{5}$

15. Here, P(A) = 1
$\therefore$ $P(\bar{A}) = 1 - P(A) = 0$

16. Sample space when six dice are thrown = 6^6
All dice show the same face means we are getting same number on all six dice which can be any one of the six numbers 1, 2, …, 6.
$\therefore$ No. of ways of selecting a number is ${}^6C_1$.

$\therefore$ Required probability = $\frac{{}^6C_1}{6^6} = \frac{1}{6^5}$

19. $p_1 + p_2 + p_3 + p_4$ should be equal to 1 and none of p_1, p_2, p_3, p_4 should be negative.
$\therefore$ option (A) is correct.

20. Total no. of ways = 3! = 6
Favourable ways = 1
$\Rightarrow$ Probability = $\frac{1}{6}$

21. In a non-leap year, we have 365 days i.e., 52 weeks and one day. So, we may have any day of seven days.

23. If A and B are independent, A′ and B′ are also independent.

24. $P(A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= 0.25 + 0.5 - 0.15 = 0.6$

25. Let E_1 be the event that man will be selected and E_2 be the event that woman will be selected. Then

$P(E_1) = \frac{1}{2}$, So $P(\overline{E_1}) = 1 - \frac{1}{2} = \frac{1}{2}$ and

$P(E_2) = \frac{1}{3}$, So $P(\overline{E_2}) = 1 - \frac{1}{3} = \frac{2}{3}$

Clearly, E_1 and E_2 are independent events.

$\therefore$ $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2})$
$= \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

26. Probability of getting either first class or second class or third class = P(A)
$= \frac{2}{7} + \frac{3}{5} + \frac{1}{10}$
$= \frac{69}{70}$

Probability of failing = $P(A') = 1 - P(A) = \frac{1}{70}$

27. There are 4 kings, 13 hearts and a king of hearts is common to the two blocks.

$\therefore$ Required probability = $\frac{4+13-1}{52} = \frac{16}{52}$

28. Here, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$
$\therefore$ $P(A) + P(B) = P(A \cup B) + P(A \cap B) = 0.9$
$\therefore$ $P(A') + P(B') = 1 - P(A) + 1 - P(B)$
$= 2 - 0.9 = 1.1$

29. $P(A) = 0.28$, $P(B) = 0.55$, $P(A \cap B) = 0.14$
$P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B)$
$= 1 - [P(A) + P(B) - P(A \cap B)]$
$= 1 - (0.28 + 0.55 - 0.14) = 0.31$

30. Total number of ways = {HH, HT, TH, TT}

$\therefore$ P (head on first toss) = $\frac{2}{4} = \frac{1}{2} = P(A)$,

P (head on second toss) = $\frac{2}{4} = \frac{1}{2} = P(B)$

and P (head on both toss) = $\frac{1}{4} = P(A \cap B)$

Hence, required probability is,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

31. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{1 - P(B')} = \frac{0.15}{1 - 0.10}$

$= \frac{1}{6}$

32. $P(A) = P(A \cap B) + P(A \cup B) - P(B)$

$= \frac{1}{3} + \frac{5}{6} - \frac{2}{3} = \frac{3}{6} = \frac{1}{2}$

33. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{(3/8) + (5/8) - (3/4)}{(5/8)}$

$= \frac{2}{5}$

34. Since, events are mutually exclusive, therefore $P(A \cap B) = 0$ i.e., $P(A \cup B) = P(A) + P(B)$

$\Rightarrow 0.7 = 0.4 + x \Rightarrow x = \frac{3}{10}$

35. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ $\frac{5}{8} = \frac{1}{4} + \frac{1}{2} - P(A \cap B)$

$\therefore$ $P(A \cap B) = \frac{1}{8}$

36. $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Since, A and B are mutually exclusive.

So, $P(A \cap B) = 0$.

Hence, $P(A/B) = \frac{0}{P(B)} = 0$

37. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.6} = \frac{5}{6}$

39. $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

40. $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{P(\bar{B})} = \frac{1 - P(A \cup B)}{P(\bar{B})}$

41. Let A be the event of selecting bag X, B be the event of selecting bag Y and E be the event of drawing a white ball, then

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(E/A) = \frac{2}{5}$

and $P(E/B) = \frac{4}{6} = \frac{2}{3}$

$\therefore$ $P(E) = P(A)\,P(E/A) + P(B)\,P(E/B)$

$= \frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{2}{3} = \frac{8}{15}$

42. Here, $P(A) = \frac{3}{7}$, $P(B) = \frac{7}{12}$

$\therefore$ $P(A') = \frac{4}{7}$ and $P(B') = \frac{5}{12}$

$\therefore$ P(Problem will be considered solved even if one person solves it)

$= 1 - [P(A') \cdot P(B')] = 1 - \frac{5}{21} = \frac{16}{21}$

43. Required probability = $\frac{3}{5}$

....$\left[\because \text{The probability of the occurrence} = \frac{b}{a + b}\right]$

44. Required probability = $\frac{6}{6+5} = \frac{6}{11}$

....$\left[\because \text{The probability of the occurrence} = \frac{a}{a + b}\right]$

Critical Thinking

1. $n(S) = {}^{16}C_{11}$

A: Event that the team has exactly four bowlers.

$\therefore$ $n(A) = {}^{6}C_4 \cdot {}^{10}C_7$

$\Rightarrow P(A) = \frac{{}^{6}C_4 \cdot {}^{10}C_7}{{}^{16}C_{11}} = \frac{75}{182}$

2. Since, cards are drawn with replacement.

$\therefore$ Total no. of ways = 52×52.

Now, we can choose one suit out of four in ${}^{4}C_1$ ways and two cards in 13×13 ways.

$\therefore$ Required Probability = $\frac{{}^{4}C_1 \times 13 \times 13}{52 \times 52} = \frac{1}{4}$

3. We have to select exactly 2 children

$\therefore$ selection contain 2 children out of 4 children and remaining 2 person can be selected from 2 women and 4 men

i.e., $4C_2 \times 6C_2$ ways

$\therefore$ Total favourable ways = 6 × 15 = 90

$\therefore$ Required probability = $\frac{90}{210} = \frac{3}{7}$

4. Total no. of ways in which 2 socks can be drawn out of 9 is ${}^9C_2$. The two socks match if either they are both black or they are both blue. So, two matching socks can be drawn in ${}^5C_2 + {}^4C_2$ ways.

$\therefore$ Required probability = $\frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{10+6}{36}$

$= \frac{4}{9}$

5. A committee of 4 can be formed in ${}^{25}C_4$ ways

A: Event that the committee contains at least 3 doctors

$\therefore$ $n(A) = {}^4C_3.{}^{21}C_1 + {}^4C_4 = 85$

$\therefore$ $P(A) = \frac{85}{{}^{25}C_4} = \frac{85}{12650} = \frac{17}{2530}$

6. Besides ground floor, there are 7 floors. Since a person can leave the cabin at any of the seven floors, total no. of ways in which each of the five persons can leave the cabin at any of the 7 floors = 7^5

Five persons can leave the cabin at five different floors in ${}^7C_5 \times 5!$ ways

Hence, required probability = $\frac{{}^7C_5 \times 5!}{7^5}$

7. Total no. of ways = 7!

Arrangement of boys and girls in alternate seats is B G B G B G B

Boys can occupy seat in 4! ways and girls in 3! ways.

$\therefore$ Required Probability = $\frac{3! \times 4!}{7!} = \frac{1}{35}$

8. Since, the total '13' can't be found.

9. Required probability

$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right) = \frac{2}{3}.\frac{3}{4}.\frac{4}{5} = \frac{2}{5}$

10. It six does not appear on either dice then, there are only five possible outcomes associated with one dice, the number of sample points is 5 × 5.

11. Here, n(S) = 2 × 2 × 2 = 8

If A is the event that there is no tail, then

A = {(HHH)}

$\Rightarrow$ n(A) = 1

$\Rightarrow P(A) = \frac{1}{8}$

$\therefore$ $P(A') = 1 - P(A) = 1 - \frac{1}{8} = \frac{7}{8}$

12. Two digits, one from each set can be selected in 9 × 9 = 81 ways.

Favourable outcomes are (1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2) and (9, 1).

$\therefore$ n(S) = 81

and n(A) = 9

$\therefore$ $P(A) = \frac{9}{81} = \frac{1}{9}$

13. Here, n(S) = 2 × 2 = 4

A: Event of getting 2 heads or 2 tails

$\therefore$ A = {(H, H), (T, T)}

$\Rightarrow$ n(A) = 2

$\Rightarrow P(A) = \frac{2}{4} = \frac{1}{2}$

14. Favourable ways = {29, 92, 38, 83, 47, 74, 56, 65}

Hence, required probability = $\frac{8}{100} = \frac{2}{25}$

15. Ace is not drawn in 26 cards.

It means 26 cards are drawn from 48 cards.

$\therefore$ Required Probability = $\frac{{}^{48}C_{26}}{{}^{52}C_{26}}$

16. Out of 30 numbers from 1 to 30, three numbers can be chosen in ${}^{30}C_3$ ways.

Three consecutive numbers can be chosen in one of the following ways:

{(1, 2, 3), (2, 3, 4),…,(28, 29, 30)} = 28 ways

$\therefore$ Probability that numbers are consecutive

$= \frac{28}{{}^{30}C_3} = \frac{1}{145}$

Hence, required probability = $1 - \frac{1}{145} = \frac{144}{145}$

17. When a coin is tossed, there are two outcomes and when a dice is rolled, there are six possible outcomes.
Hence, there are 8 (2 corresponding to head and six corresponding to tail at first toss) sample points in the sample space.
Sample space is {HH, HT, T1, T2, T3, T4, T5, T6}.

18. 4 cards can drop out of 52 in $^{52}C_4$ ways. They can be one from each suit in
$^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = (13\times13\times13\times13)$ ways.

$\therefore$ Required probability $= \dfrac{13\times13\times13\times13}{^{52}C_4}$

$= \dfrac{13\times13\times13\times13\times4!}{52\times51\times50\times49}$

$= \dfrac{2197}{20825}$

19. Between 1 and 100, there are 25 prime numbers.
$\therefore$ n(S) = 98 and n(A) = 25
$\therefore$ $P(A) = \dfrac{25}{98}$

20. Total cases = 4
So, probability of correct answer $= \dfrac{1}{4}$

21. In a leap year, there are 366 days in which 52 weeks and two days. The combination of 2 days may be: Sun – Mon, Mon – Tue, Tue – Wed, Wed – Thu, Thu – Fri, Fri – Sat, Sat – Sun.
$\therefore$ P(53 Sun) $= \dfrac{2}{7}$

22. When six dice are thrown, the total number of outcomes is 6^6.They can show different number in $^6P_6 = 6!$ ways
$\therefore$ Required probability $= \dfrac{6!}{6^6} = \dfrac{5!}{6^5} = \dfrac{5}{324}$

23. One card can be selected from a pack in $^{52}C_1$ ways.
$\therefore$ $n(S) = {}^{52}C_1 = 52$
A: Event of getting a red queen
$\therefore$ P(A) = P(diamond queen or heart queen)
$= \dfrac{^2C_1}{^{52}C_1}$

24. 15 places are occupied. This includes the owner's car also. 14 cars are parked in 24 places of which 22 places are available (excluding the neighbouring places) and so the required probability $\dfrac{^{22}C_{14}}{^{24}C_{14}} = \dfrac{15}{92}$

25. Here, $n(S) = {}^6C_2 = 15$
If both are vowels, then they are selected in $^2C_2$ ways = 1.
$\therefore$ Required probability $= \dfrac{1}{15}$

26. Here, $n(S) = {}^{10}C_2$
A: Event that the watches selected are defective
$\therefore$ $n(A) = {}^2C_2 = 1$
$\therefore$ $P(A) = \dfrac{1}{^{10}C_2} = \dfrac{1}{45}$

27. Two 3s, one 6 and one 8 can be dialled in $\dfrac{4!}{2!} = 12$ ways of which only one is the correct way of dialling.
$\therefore$ Required probability $= \dfrac{1}{12}$

28. Required probability $= \dfrac{^5C_2}{^{5+x}C_2}$

$\therefore$ $\dfrac{5}{14} = \dfrac{^5C_2}{^{(5+x)}C_2}$

$\Rightarrow \dfrac{5}{14} = \dfrac{5(4)}{(5+x)(4+x)}$

$\Rightarrow (x-3)(x+12) = 0$

$\Rightarrow x = 3$

29. Since there are 3 A's and 2 N's.
Total no. of arrangements $= \dfrac{10!}{3!2!}$
Hence, the number of arrangements in which ANAND occurs without any split = 6!
$\therefore$ Required probability $= \dfrac{6!3!2!}{10!} = \dfrac{1}{420}$

30. Probabilities of H_1, H_2 and H_3 winning a race must be in the ratio 4 : 2 : 1 (due to given condition) and should also add up to 1.

31. Three numbers can be chosen out of 10 numbers in $^{10}C_3$ ways.
The product of two numbers, out of the three chosen numbers, will be equal to the third number, if the numbers are chosen in one of the following ways:
$\{(2, 3, 6), (2, 4, 8), (2, 5, 10)\} = 3$ ways
Hence, required probability $= \frac{3}{^{10}C_3} = \frac{1}{40}$

32. Here, $n(S) = {}^{52}C_1 \times {}^{51}C_1 = 52 \times 51$
A: Event that both cards chosen are Ace.
$\therefore$ $n(A) = {}^4C_1 \times {}^3C_1 = 12$
$\therefore$ $P(A) = \frac{12}{52 \times 51} = \frac{1}{221}$

33. The sum 2 can be found in one way i.e., $\{(1, 1)\}$
The sum 8 can be found in five ways i.e., $\{(6, 2), (5, 3), (4, 4), (3, 5), (2, 6)\}$.
Similarly, the sum twelve can be found in one way i.e., $\{(6, 6)\}$.
Hence, required probability $= \frac{7}{36}$.

34. As $\{(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)\}$ are only favourable outcomes
$\Rightarrow$ Required probability $= \frac{6}{216}$

35. $0.7 = 0.4 + x - 0.4x$
$\Rightarrow x = \frac{1}{2}$

36. We have $P(A + B) = P(A) + P(B) - P(AB)$
$\Rightarrow \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3} \Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$
Thus, $P(A).P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} = P(AB)$
Hence, events A and B are independent.

37. Here, $P(A) = 0.6$; $P(B) = 0.9$
$\therefore$ Required pobability =
$P(A)\cdot P(\overline{B}) + P(B)\cdot P(\overline{A}) = (0.6)(0.1) + (0.9)(0.4)$
$= 0.06 + 0.36 = 42$

38. Here, $P(A) = p$
$\Rightarrow P(\overline{A}) = 1 - p$
and $P(B) = q \Rightarrow P(\overline{B}) = 1 - q$
Probability that one person is alive is the sum of two cases A dies B lives and A lives B dies
$= p(1 - q) + q(1 - p) = p + q - 2pq$

39. Since, $A \cup B = S$.
$\therefore$ $P(A \cup B) = P(S) = 1$
$\therefore$ $1 = P(A) + 2P(A)$ $[\because P(A \cup B) = P(A) + P(B)]$
$\Rightarrow 3(P(A)) = 1$
$\Rightarrow P(A) = \frac{1}{3}$
$\therefore$ $P(B) = \frac{2}{3}$

40. P(not happening) $= 1 - 0.4 = 0.6$
$\therefore$ Required Probability $= 1 - (0.6)^3 = 0.784$

41. We have to consider order for IIT
$\therefore$ Required probability $= \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18}$
$= \frac{5}{38}$

42. A is independent of itself, if
$P(A \cap A) = P(A).P(A)$
$\Rightarrow P(A) = P(A)^2$
$\Rightarrow P(A) = 0, 1$

43. Since, we have
$P(A \cup B) + P(A \cap B) = P(A) + P(B)$
$= P(A) + \frac{P(A)}{2}$
$\Rightarrow \frac{7}{8} = \frac{3P(A)}{2}$
$\Rightarrow P(A) = \frac{7}{12}$

44. Here, $P(A) = P(B) = 2\,P(C)$,
and $P(A) + P(B) + P(C) = 1$
$\Rightarrow P(C) = \frac{1}{5}$ and $P(A) = P(B) = \frac{2}{5}$
Hence, $P(A \cup B) = P(A) + P(B)$
$= \frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

45. Since, $A \subseteq B \Rightarrow A \cap B = B \cap A = A$
Hence, $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$

46. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).P(B)}{P(B)} = P(A)$.

47. P(neither A nor B)
$= 1 - P(\text{either A or B}) = 1 - P(A \cup B)$
$= 1 - [P(A) + P(B) - P(A \cap B)]$
$= 1 - 0.25 - 0.50 + 0.14 = 0.39$

48. Here, A = {4, 5, 6}
$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$
and B = {4, 3, 2, 1}
$\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$
$\therefore$ $A \cap B = \{4\}$
$\Rightarrow P(A \cap B) = \frac{1}{6}$
$\therefore$ $P(A \cup B) = \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = 1$

49. $P(A' \cup B') = P[(A \cap B)']$
$= 1 - P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

50. Required Probability
$= P[(A \cap B') \cup (A' \cap B)]$
$= P(A \cap B') + P(A' \cap B)$
$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= P(A) + P(B) - 2P(A \cap B)$

51. $\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{10} = \frac{7}{20}$

52. M: Event that student passed in Mathematics.
E: Event that student passed in Electronics
$\therefore$ $n(M) = 30, n(E) = 20, n(M \cap E) = 10$,
$n(S) = 80$.
$\therefore$ $P(M) = \frac{30}{80}, P(E) = \frac{20}{80}, P(M \cap E) = \frac{10}{80}$
$\therefore$ $P(M \cup E) = P(M) + P(E) - P(M \cap E)$
$= \frac{30}{80} + \frac{20}{80} - \frac{10}{80} = \frac{1}{2}$
$\therefore$ P(Student has passed in none of the subject)
$= P[(M \cup E)'] = 1 - P(M \cup E) = 1 - \frac{1}{2} = \frac{1}{2}$

53. For both to be boys, the probability
$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

54. $P(A' \cap B') = \frac{1}{3}$
$\Rightarrow P[(A \cup B)'] = \frac{1}{3}$
$\therefore$ $1 - P(A \cup B) = \frac{1}{3}$
$\Rightarrow P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3}$
$\therefore$ $P(A) + P(B) - P(A \cap B) = \frac{2}{3}$
$\therefore$ $p + 2p - \frac{1}{2} = \frac{2}{3}$
$\Rightarrow 3p = \frac{2}{3} + \frac{1}{2} = \frac{7}{6} \Rightarrow p = \frac{7}{18}$

55. In a leap year, there are 366 days in which 52 weeks and two days. The combination of 2 days may be: Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun.
$\therefore$ $P(53\text{ fri}) = \frac{2}{7}$; $P(53\text{ Sat}) = \frac{2}{7}$
There is one combination in common i.e., (Fri-Sat)
$\therefore$ $P(53\text{ Fri and }53\text{ Sat}) = \frac{1}{7}$
$\therefore$ $P(53\text{ Fri or }53\text{ Sat}) = P(53\text{ Fri}) + P(53\text{ Sat}) - P(53\text{ Fri and Sat})$
$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

56. In the word 'MULTIPLE' there are 3 vowels, out of total of 8, 1 vowel can be chosen in $^3C_1$ ways. In the word 'CHOICE' there are 3 vowels, out of the total of 6, 1 vowel can be chosen in $^3C_1$ ways.
$\therefore$ Required probability $= \frac{^3C_1}{8} \times \frac{^3C_1}{6} = \frac{3}{16}$

57. P(neither E_1 nor E_2 occurs) $= P\left(E_1' \cap E_2'\right)$
$= P\left(E_1'\right)P\left(E_2'\right)$
$= (1 - p_1)(1 - p_2)$

58. $P(A \cap B) = P(A) \cdot P(B/A) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Since, $P(A \cap B) = P(B)\, P(A/B)$

$\therefore \quad \frac{1}{8} = P(B) \times \frac{1}{4}$

$\Rightarrow P(B) = \frac{1}{2}$

$\therefore \quad P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = P(A \cap B)$

$\therefore$ A and B are independent

59. A total of 7 and a total of 9 cannot occur simultaneously.

$\therefore$ P(total of 7 or 9)

$=$ P(total of 7) + P(total of 9) $= \frac{6}{36} + \frac{4}{36} = \frac{5}{18}$

(A total of 7 and a total of 9 cannot occur simultaneously)

60. $P(G) = \frac{25}{80}, P(R) = \frac{10}{80}, P(I) = \frac{20}{80}$

Since events are independent,

$\therefore$ P(selecting rich and intelligent girls)

$= P(G) \cdot P(R) \cdot P(I) = \frac{5}{512}$

61. $P(M) = \frac{1}{4} \Rightarrow P(M') = \frac{3}{4}$

and $P(W) = \frac{1}{3} \Rightarrow P(W') = \frac{2}{3}$

Both events are independent so that probability that no one will be alive is

$P(W' \cap M') = P(W')\, P(M') = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

62. Since, E and F are independent

$\therefore \quad P(E \cap F) = P(E)\, P(F)$

$\Rightarrow P(E)\, P(F) = \frac{1}{12}$

Now, E and F are independent

$\therefore$ E′ and F′ are also independent

$\therefore \quad P(E' \cap F') = P(E') \cdot P(F') = \frac{1}{2}$

$\therefore \quad [1 - P(E)] \cdot [1 - P(F)] = \frac{1}{2}$

$\therefore \quad 1 - P(E) - P(F) + P(E) \cdot P(F) = \frac{1}{2}$

$\therefore \quad 1 - P(E) - P(F) + \frac{1}{12} = \frac{1}{2}$

$\Rightarrow P(E) + P(F) = \frac{7}{12}$

Solving, $P(E) = \frac{1}{4}, P(F) = \frac{1}{3}$

63. Let $P(A) = \frac{20}{100} = \frac{1}{5}$, $P(B) = \frac{10}{100} = \frac{1}{10}$

Since, events are independent and we have to find $P(A \cup B) = P(A) + P(B) - P(A).P(B)$

$= \frac{1}{5} + \frac{1}{10} - \frac{1}{5} \times \frac{1}{10}$

$= \frac{3}{10} - \frac{1}{50} = \frac{14}{50} \times 100$

$= 28\%$

64. A: Event of obtaining an even sum and B: Event of obtaining a sum less than five. Since A, B are not mutually exclusive,

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{6}{36} - \frac{4}{36} = \frac{5}{9}$

[$\because$ there are 18 ways to get an even sum i.e {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)} and there are 6 ways to get a sum < 5 i.e., {(1, 3), (3, 1), (2, 2), (1, 2), (2, 1), (1, 1)} and 4 ways to get an even sum < 5 i.e., {(1, 3), (3, 1), (2, 2), (1, 1)}]

65. A: Brown hair

$\Rightarrow P(A) = \frac{40}{100}$

B: Brown eyes

$\Rightarrow P(B) = \frac{25}{100} \quad \therefore \quad P(A \cap B) = \frac{15}{100}$

$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{15}{100}}{\frac{40}{100}} = \frac{3}{8}$

66. $(Y = 0)$ is $\{00, 01, \ldots, 09, 10, 20, \ldots, 90\}$.

Also, $(X = 9) \cap (Y = 0) = \{09, 90\}$.

We have, $P(Y = 0) = \frac{19}{100}$ and

$P[(X = 9) \cap (Y = 0)] = \frac{2}{100}$.

$\therefore \quad P\left(\frac{X = 9}{Y = 0}\right) = \frac{P[(X = 9) \cap (Y = 0)]}{P(Y = 0)} = \frac{2}{19}$

67. $P\left(\frac{\overline{B}}{\overline{A}}\right) = \frac{1-P(A\cup B)}{P(\overline{A})}$

$$= \frac{1-\frac{23}{60}}{1-\frac{1}{3}} = \frac{37}{60} \times \frac{3}{2} = \frac{37}{40}$$

68. We define the following events :

A_1: He knows the answer.

A_2 : He does not know the answer.

E : He gets the correct answer.

Then $P(A_1) = \frac{9}{10}$, $P(A_2) = 1 - \frac{9}{10} = \frac{1}{10}$,

$P\left(\frac{E}{A_1}\right) = 1$ and $P\left(\frac{E}{A_2}\right) = \frac{1}{4}$

$\therefore$ Required probability is

$$P\left(\frac{A_2}{E}\right) = \frac{P(A_2)P\left(\frac{E}{A_2}\right)}{P(A_1)P\left(\frac{E}{A_1}\right)+P(A_2)P\left(\frac{E}{A_2}\right)} = \frac{1}{37}$$

69. We define the following events :

A_1 : Selecting a pair of consecutive letter from the word LONDON.

A_2 : Selecting a pair of consecutive letters from the word CLIFTON.

E : Selecting a pair of letters 'ON'.

Then $P(A_1 \cap E) = \frac{2}{5}$; as there are 5 pairs of consecutive letters out of which 2 are ON.

$P(A_2 \cap E) = \frac{1}{6}$; as there are 6 pairs of consecutive letters of which one is ON.

$\therefore$ The required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E)+P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{6}}$$

$$= \frac{12}{17}$$

70. Required probability $= \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{6}{8}$

$$= \frac{37}{56}$$

71. It is based on Baye's theorem.

Probability of picked bag A, i.e., $P(A) = \frac{1}{2}$

Probability of picked bag B, i.e., $P(B) = \frac{1}{2}$

Probability of green ball picked from bag A

$= P(A).P\left(\frac{G}{A}\right) = \frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$

Probability of green ball picked from bag B

$= P(B).P\left(\frac{G}{B}\right) = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

$\therefore$ Total probability of green ball $= \frac{2}{7} + \frac{3}{14} = \frac{1}{2}$

$\therefore$ Probability of fact that green ball is drawn from bag B

$$= \frac{P(B)P\left(\frac{G}{B}\right)}{P(A)P\left(\frac{G}{A}\right)+P(B)P\left(\frac{G}{B}\right)} = \frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{7}} = \frac{3}{7}$$

72. Consider the following events :

$A \rightarrow$ Ball drawn is black;

$E_1 \rightarrow$ Bag I is chosen;

$E_2 \rightarrow$ Bag II is chosen and

$E_3 \rightarrow$ Bag III is chosen.

Then $P(E_1) = (E_2) = P(E_3) = \frac{1}{3}$, $P\left(\frac{A}{E_1}\right) = \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = \frac{1}{5}$, $P\left(\frac{A}{E_3}\right) = \frac{7}{10}$

$\therefore$ Required probability $= P\left(\frac{E_3}{A}\right)$

$$= \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{7}{15}$$

73. Let p be the probability of the other event. Then the probability of the first event is $\frac{2}{3}$p.

$\therefore$ $\frac{p}{p+\frac{2}{3}p} = \frac{3}{3+2}$

$\therefore$ odds in favour of the other are 3 : 2

74. Required probability $= \frac{5}{5+3} = \frac{5}{8}$

$$\dots\left[\because \text{If odds in favours of an event are a : b, then the probability of non}-\text{occurrence of that event is } \frac{b}{a+b}\right]$$

75. Let A and B be two given events. The odds against A are 5:2, therefore $P(A) = \frac{2}{7}$.

And the odds in favour of B are 6:5, therefore $P(B) = \frac{6}{11}$

$\therefore$ The required probability $= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \left(1-\frac{2}{7}\right)\left(1-\frac{6}{11}\right) = \frac{52}{77}$

76. Probabilities of winning the race by three horses are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$.

Hence, required probability $= \frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{47}{60}$

77. Let E denote the event that a six occurs and A is the event that the man reports that it is a '6', we have

$P(E) = \frac{1}{6}, P(E') = \frac{5}{6}, P(A/E) = \frac{3}{4}$ and

$P(A/E') = \frac{1}{4}$

$\therefore$ From Baye's theorem,

$$P(E/A) = \frac{P(E).P\left(\frac{A}{E}\right)}{P(E).P\left(\frac{A}{E}\right) + P(E').P\left(\frac{A}{E'}\right)}$$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}} = \frac{3}{8}$$

78. Probability of the card being a spade or an ace $= \frac{16}{52} = \frac{4}{13}$. Hence, odds in favour is 4 : 9.

So, the odds against his winning is 9: 4

79. Required probability $= \frac{4}{4+5} = \frac{4}{9}$

80. We have ratio of the ships A, B and C for arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively.

$\therefore$ The probability of ship A for arriving safely

$= \frac{2}{2+5} = \frac{2}{7}$

Similarly, for $B = \frac{3}{3+7} = \frac{3}{10}$ and for

$C = \frac{6}{6+11} = \frac{6}{17}$

$\therefore$ Probability of all the ships for arriving safely

$= \frac{2}{7}\times\frac{3}{10}\times\frac{6}{17} = \frac{18}{595}$.

Competitive Thinking

1. Required probability $= \frac{4}{36} = \frac{1}{9}$

2. We have $P(\bar{A}) = 0.05 \Rightarrow P(A) = 0.95$

Hence, the probability that the event will take place in 4 consecutive occasions

$= \{P(A)\}^4 = (0.95)^4 = 0.81450625$

3. If both integers are even, then product is even.
If both integers are odd, then product is odd.
If one integer is odd and other is even, then product is even.

$\therefore$ Required probability $= \frac{2}{3}$.

4. Total number of ways $= 2^n$

If head comes odd times, then favourable ways $= 2^{n-1}$.

$\therefore$ Required probability $= \frac{2^{n-1}}{2^n} = \frac{1}{2}$.

5. Number of ways in which two faulty machines may be detected (depending upon the test done to identify the faulty machines) $= {}^4C_2 = 6$

and Number of favourable cases = 1

[When faulty machines are identified in the first and the second test]

Hence, required probability $= \frac{1}{6}$.

6. Favorable number of cases = ${}^{20}C_1 = 20$

Sample space = ${}^{62}C_1 = 62$

$\therefore$ Required probability = $\frac{20}{62} = \frac{10}{31}$

7. The number of ways to arrange 7 white and 3 black balls in a row $= \frac{10!}{7!.3!} = \frac{10.9.8}{1.2.3} = 120$

Numbers of blank places between 7 balls are 6. There is 1 place before first ball and 1 place after last ball. Hence, total number of places are 8.

Hence, 3 black balls are arranged on these 8 places so that no two black balls are together in number of ways

$= {}^{8}C_3 = \frac{8\times7\times6}{1\times2\times3} = 56$

So required probability = $\frac{56}{120} = \frac{7}{15}$.

8. Total rusted items = 3 + 5 = 8;
unrusted nails = 3.

$\therefore$ Required probability = $\frac{3+8}{6+10} = \frac{11}{16}$.

9. Total number of ways = 36
and Favourable number of cases are
{(1, 4), (2, 3), (3, 2), (4, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)} = 9

Hence, the required probability = $\frac{9}{36} = \frac{1}{4}$.

10. Three dice can be thrown in 6 × 6 × 6 = 216 ways. A total 17 can be obtained as {(5, 6, 6), (6, 5, 6), (6, 6, 5)}. A total 18 can be obtained as (6, 6, 6).

Hence, the required probability = $\frac{4}{216} = \frac{1}{54}$

11. Prime numbers are {2, 3, 5, 7, 11}.
Hence, required probability

$= \frac{1+2+4+6+2}{36} = \frac{15}{36} = \frac{5}{12}$

12.

$$\text{Ways}\quad \begin{array}{cccc} 9 & 10 & 11 & 12 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 \end{array}$$

Hence, required probability = $\frac{10}{36} = \frac{5}{18}$

14. Required probability is 1 – P (no die show up 1)

$= 1 - \left(\frac{5}{6}\right)^3 = \frac{216-125}{216} = \frac{91}{216}$

15. Required combinations are {(2, 2, 1), (1, 2, 2), (2, 1, 2), (1, 3, 1,), (3, 1, 1), (1, 1, 3)}

$\therefore$ Required probability = $\frac{6}{4^3} = \frac{6}{64} = \frac{3}{32}$.

16. P (at least 1H) = 1 – P (No head)

$= 1 - P\text{ (four tail)} = 1 - \frac{1}{16} = \frac{15}{16}$

17. $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{2}$

$\therefore$ $P(A)\,P(B) = \frac{3}{8}\cdot\frac{1}{2} = \frac{3}{16}$

and $P(A \cap B) = \frac{2}{8} = \frac{1}{4} \neq P(A).P(B)$

$\therefore$ A and B are dependent.

18. Number which are cubes
$1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64$

$\therefore$ Required probability = $\frac{4}{100} = \frac{1}{25}$

19. n(S) = 36
E = {(1, 4), (4, 1), (2, 3), (3, 2)}

$\therefore$ $P(E) = \frac{4}{36} = \frac{1}{9}$

20. $P(A \cap B') = P(A) - P(A \cap B)$

$= \frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

21. Probability that A does not solve the problem

$= 1 - \frac{1}{2} = \frac{1}{2}$

Probability that B does not solve the problem

$= 1 - \frac{1}{3} = \frac{2}{3}$

Probability that C does not solve the problem

$= 1 - \frac{1}{5} = \frac{4}{5}$

Probability that at least one of them solve problem = 1 – no one solves the problem

$= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)$

$= 1 - \frac{4}{15} = \frac{11}{15}$

22. 3 coins are tossed

$\therefore$ S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

A: Event of getting 2 heads

$\Rightarrow$ A = {HHT, HTH, THH}

$\therefore$ n (A) = 3

$\Rightarrow P(A) = \frac{3}{8}$

23. $P(A \cap B') = P(A) - P(A \cap B)$

$= 0.7 - 0.3 = 0.4 = \frac{2}{5}$

24. $P(\overline{A} \cap \overline{B}) = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - 0.25 - 0.50 + 0.14 = 0.39$

25. Here, n(S) = 36

Also, n(F), where F is the set of favourable cases.

F = {(6, 1), (5, 2), (4, 3)}

where 1st number in ordered pair gives the number of black die and 2nd number gives the number on white die.

$\therefore$ required probability $= \frac{3}{36} = \frac{1}{12}$

26. In a non-leap year there are 365 days, 52 complete weeks and 1 day. That can be Monday, Tuesday, Wednesday, Thursday, Friday and Saturday.

$\therefore$ P (Friday) $= \frac{1}{7}$; P (Saturday) $= \frac{1}{7}$

$\therefore$ Required probability $= \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

27. Let E be the event that the numbers are divisible by 4.

$\therefore$ E = {4, 8, 12, 16, 20, 24}

$\therefore$ n(E) = 6

$\therefore$ $n(\overline{E}) = 20$

$\therefore$ Required probability $= P(\overline{E}) = \frac{20}{26} = \frac{10}{13}$

28. Total balls = $5 + x$

Two balls are drawn.

$\therefore$ $n(S) = {}^{5+x}C_2$

Given, probability of red balls drawn $= \frac{5}{14}$

$\therefore$ $\frac{5}{14} = \frac{{}^5C_2}{{}^{5+x}C_2}$

$\Rightarrow \frac{5}{14} = \frac{5!}{3!2!} \times \frac{(3+x)!\,2!}{(5+x)!}$

$\Rightarrow \frac{5}{14} = \frac{20}{1} \times \frac{1}{(5+x)(4+x)}$

$\Rightarrow (5+x)(4+x) = \frac{20 \times 14}{5}$

$\Rightarrow (5+x)(4+x) = 56 \Rightarrow x = 3$

29. The probability of students not solving the problem are $1 - \frac{1}{3} = \frac{2}{3}$, $1 - \frac{1}{4} = \frac{3}{4}$ and $1 - \frac{1}{5} = \frac{4}{5}$

Therefore, the probability that the problem is not solved by any one of them $= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

Hence, the probability that problem is solved $= 1 - \frac{2}{5} = \frac{3}{5}$.

30. The sample space is [LWW, WLW]

$\therefore$ P(LWW) + P(WLW)

= Probability that in 5 match series, it is India's second win

= P(L)P(W)P(W) + P(W)P(L)P(W)

$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

31. Here, $P(A) = \frac{3}{4}$, $P(B) = \frac{4}{5}$

$\therefore$ $P(\overline{A}) = \frac{1}{4}$ and $P(\overline{B}) = \frac{1}{5}$

$\therefore$ Required probability

$= P(A).P(\overline{B}) + P(\overline{A}).P(B) = \frac{7}{20}$.

32. Probability of first card to be a king $= \frac{4}{52}$

and probability of also second to be a king $= \frac{3}{51}$

Hence, required probability $= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$.

33. Required probability = P(Diamond).P(king)

$= \frac{13}{52} \cdot \frac{4}{52} = \frac{1}{52}$

34. The probability of husband is not selected

$= 1 - \frac{1}{7} = \frac{6}{7}$

The probability that wife is not selected

$= 1 - \frac{1}{5} = \frac{4}{5}$

The probability that only husband selected

$= \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

The probability that only wife selected

$= \frac{1}{5} \times \frac{6}{7} = \frac{6}{35}$

Hence, required probability $= \frac{6}{35} + \frac{4}{35} = \frac{10}{35}$

$= \frac{2}{7}$

35. Second white ball can draw in two ways.

i. First is white and second is white

Probability $= \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

ii. First is black and second is white

Probability $= \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$

Hence, required probability $= \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$.

36. Since, $P(A + B + C)$

$= P(A) + P(B) + P(C)$

$= \frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{13}{12}$, which is greater than 1.

Hence, the statement is wrong.

37. Since $\overline{E_1} \cap \overline{E_2} = \overline{E_1 \cup E_2}$

and $(E_1 \cup E_2) \cap (\overline{E_1 \cup E_2}) = \phi$

$\therefore \quad P\{(E_1 \cup E_2) \cap (\overline{E_1} \cap \overline{E_2})\} = P(\phi) = 0 < \frac{1}{4}$

39. $P[(A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

40. Given $P(A \cup B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{5}$

We know $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad \frac{3}{5} = 1 - P(\overline{A}) + 1 - P(\overline{B}) - \frac{1}{5}$

$\therefore \quad 2 - \frac{4}{5} = P(\overline{A}) + P(\overline{B})$

$\Rightarrow P(\overline{A}) + P(\overline{B}) = \frac{6}{5}$.

41. $P(B / (A \cup B^c)] = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$

$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$

$= \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)}$

$= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$

42. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cap B)$

$\{\because P(A \cap B) = P(A \cup B)\}$

$\Rightarrow 2\,P(A \cap B) = P(A) + P(B)$

$\Rightarrow 2\,P(A). \frac{P(A \cap B)}{P(A)} = P(A) + P(B)$

$\Rightarrow 2\,P(A). P\left(\frac{B}{A}\right) = P(A) + P(B)$

43. $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Since A and B are mutually exclusive, so

$P(A \cup B) = P(A) + P(B)$

Hence, required probability $= 1 - (0.5 + 0.3)$

$= 0.2$.

44. We know that $P(A / B) = \frac{P(A \cap B)}{P(B)}$

Also we know that $P(A \cup B) \le 1$

$\Rightarrow P(A) + P(B) - P(A \cap B) \le 1$

$\Rightarrow P(A \cap B) \ge P(A) + P(B) - 1$

$\Rightarrow \frac{P(A \cap B)}{P(B)} \ge \frac{P(A) + P(B) - 1}{P(B)}$

$\Rightarrow P(A / B) \ge \frac{P(A) + P(B) - 1}{P(B)}$

45. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \Rightarrow \frac{1}{2} = \frac{P(A \cap B)}{1/4}$

$\Rightarrow P(A \cap B) = \frac{1}{8}$

Hence, events A and B are not mutually exclusive.

$\therefore$ Statement II is incorrect.

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)} \Rightarrow P(B)=\frac{1}{2}$

$\ldots\left[\because\ P(A\cap B)=\frac{1}{8}=P(A).P(B)\right]$

$\therefore$ events A and B are independent events.

$\therefore\quad P\left(\frac{A^c}{B^c}\right)=\frac{P(A^c\cap B^c)}{P(B^c)}=\frac{P(A^c)P(B^c)}{P(B^c)}$

$=\frac{3}{4}.\frac{1}{2}.\frac{2}{1}=\frac{3}{4}$

Hence, statement I is correct.

Again $P\left(\frac{A}{B}\right)+P\left(\frac{A}{B^c}\right)=\frac{1}{4}+\frac{P(A\cap B^c)}{P(B^c)}$

$=\frac{1}{4}+\frac{P(A)-P(A\cap B)}{P(B^c)}$

$=\frac{1}{4}+\frac{\frac{1}{4}-\frac{1}{8}}{\frac{1}{2}}$

$=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Hence, statement III is incorrect.

46. Here, $P(X)=0.3;\ P(Y)=0.2$

Now $P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$

Since, these are independent events, so

$\therefore$ $P(X\cap Y)=P(X).P(Y)$

Thus, required probability

$=0.3+0.2-0.06=0.44$

47. P(neither A nor B) $=P\left(\overline{A}\cap\overline{B}\right)$

$=P(\overline{A}).P(\overline{B})=0.6\times 0.5$

$=0.3$

48. $0.8=0.3+x-0.3x\Rightarrow x=\frac{5}{7}$.

49. If $P(A)=P(B)$

As this gives,

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

or

$P(A)=2P(A)-P(A)$

$\Rightarrow P(A\cup B)=P(A\cap B)$

50. $P(\overline{A}\cap B)=P(B)-P(A\cap B)=y-z.$

51. i. This question can also be solved by one student

ii. This question can be solved by two students simultaneously

ii. This question can be solved by three students all together.

We have, $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{4}$, $P(C)=\frac{1}{6}$

$\therefore$ $P(A\cup B\cup C)=P(A)+P(B)+P(C)$
$-[P(A).P(B)+P(B).P(C)+P(C).P(A)]+$
$[P(A).P(B).P(C)]$

$=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}-\left[\frac{1}{2}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{2}\right]$
$+\left[\frac{1}{2}\times\frac{1}{4}\times\frac{1}{6}\right]$

$=\frac{33}{48}$

52. $P(A\cap\overline{B})=P(A)-P(A\cap B)$
$=0.25-0.14=0.11$

53. $P(A'\cap B')=1-P(A\cup B)$

$\Rightarrow P(A\cup B)=\frac{2}{3}$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\frac{2}{3}=x+x-\frac{1}{3}$

$\Rightarrow x=\frac{1}{2}$

54. $P(E\cap F)=P(E).P(F)$

Now, $P(E\cap F^c)=P(E)-P(E\cap F)$
$=P(E)[1-P(F)]$
$=P(E).P(F^c)$

and $P(E^c\cap F^c)=1-P(E\cup F)$
$=1-[P(E)+P(F)-P(E\cap F)$
$=[1-P(E)][1-P(F)]=P(E^c)P(F^c)$

Also $P\left(\frac{E}{F}\right)=P(E)$ and $P\left(\frac{E^c}{F^c}\right)=P(E^c)$

$\Rightarrow P\left(\frac{E}{F}\right)+P\left(\frac{E^c}{F^c}\right)=1.$

55. $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{\left(\frac{1}{10}\right)}{\left(\frac{1}{4}\right)}=\frac{2}{5}$

56. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore$ $n(E) = 4$, $n(F) = 4$ and $n(E \cap F) = 3$

$\therefore$ $P\left(\dfrac{E}{F}\right) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{3}{8}}{\frac{4}{8}} = \dfrac{3}{4}$

57. Since, we have

$P(A + B) = P(A) + P(B) - P(AB)$

$\Rightarrow 0.7 = 0.4 + P(B) - 0.2$

$\Rightarrow P(B) = 0.5.$

58. For S and T as independent events,

$P(S/T) = P(S)$. Thus, $P(S/T) = 0.3$.

59. Given, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

We know that, if A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.6 = 1 - P(\bar{A}) + 1 - P(\bar{B}) - 0.2$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - 0.8 = 1.2$

60. Here, $P(A) = \dfrac{3}{6} = \dfrac{1}{2}$, $P(B) = \dfrac{4}{6} = \dfrac{2}{3}$

and $P(A \cap B)$ = Probability of getting a number greater than 3 and less than 5

= Probability of getting 4 = $\dfrac{1}{6}$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{1}{2} + \dfrac{2}{3} - \dfrac{1}{6} = 1$

61. $P(A \cap B) = P(A)\, P(B/A)$

$\therefore$ $P(A \cap B) = \dfrac{1}{4} \times \dfrac{2}{3} = \dfrac{1}{6}$

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow \dfrac{1}{2} = \dfrac{1}{6} \times \dfrac{1}{P(B)}$

$\Rightarrow P(B) = \dfrac{1}{3}$

62. Consider the following events:

A = Sum of the digits on the selected tickets is 8.

B = Product of the digits on the selected ticket is zero.

There are 14 tickets having product of digits appearing on them as zero. The numbers on such tickets are 00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40.

$\therefore$ $P(B) = \dfrac{14}{50}$ and $P(A \cap B) = \dfrac{1}{50}$

$\therefore$ **Required probability** $= P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{1}{14}$

63. $P(\overline{A \cup B}) = \dfrac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \dfrac{1}{6}$

$\Rightarrow P(A \cup B) = \dfrac{5}{6}$

$\Rightarrow P(A) + P(B) - P(A \cap B) = \dfrac{5}{6}$

$\Rightarrow \dfrac{3}{4} + P(B) - \dfrac{1}{4} = \dfrac{5}{6} \Rightarrow P(B) = \dfrac{1}{3}$

Clearly, $P(A \cap B) = \dfrac{1}{4} = \dfrac{3}{4} \times \dfrac{1}{3} = P(A)\, P(B)$

So, A and B are independent.

Also, $P(A) \neq P(B)$. So, A and B are not equally likely.

64. Let $A_i (i = 1, 2)$ denote the event that i^{th} plane hits the target.

Clearly, A_1 and A_2 are independent events.

Required probability $= P(\bar{A}_1 \cap A_2)$

$= P(\bar{A}_1)\, P(A_2)$

$= (1 - 0.3)(0.2) = 0.14$

65. $P(A \cap B) = \dfrac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \dfrac{1}{3}$

$\Rightarrow P(A)\, P(B) = \dfrac{1}{6}$ and $P(\bar{A}) P(\bar{B}) = \dfrac{1}{3}$

$\Rightarrow xy = \dfrac{1}{6}$ and $(1 - x)(1 - y) = \dfrac{1}{3}$,

where $P(A) = x$, $P(B) = y$

$\Rightarrow xy = \dfrac{1}{6}$ and $1 - x - y + \dfrac{1}{6} = \dfrac{1}{3}$

$\Rightarrow xy = \dfrac{1}{6}$ and $x + y = \dfrac{5}{6}$

$\Rightarrow x = \dfrac{1}{2}$ and $y = \dfrac{1}{3}$ or $x = \dfrac{1}{3}$ and $y = \dfrac{1}{2}$

66. $P(A\cup\overline{B})=0.8$ and $P(B)=\frac{2}{7}\Rightarrow P(\overline{B})=\frac{5}{7}$

$\Rightarrow P(A)+P(\overline{B})-P(A\cap\overline{B})=0.8$

$\Rightarrow P(A)+\frac{5}{7}-\frac{5}{7}P(A)=0.8$

$\Rightarrow \frac{2}{7}P(A)=\frac{3}{35}\Rightarrow P(A)=0.3$

67. Consider the following events:
A = 'X' speaks truth, B = 'Y' speaks truth.

Then, $P(A)=\frac{60}{100}=\frac{3}{5}$ and $P(B)=\frac{50}{100}=\frac{1}{2}$

Required probability $=P\left((A\cap\overline{B})\cup(\overline{A}\cap B)\right)$

$=P(A\cap\overline{B})+P(\overline{A}\cap B)$

$=\frac{3}{5}\times\frac{1}{2}+\frac{2}{5}\times\frac{1}{2}=\frac{1}{2}$

68. $P(B\cap C)$

$=P(B)-\left[P(A\cap B\cap\overline{C})+P(\overline{A}\cap B\cap\overline{C})\right]$

$=\frac{3}{4}-\frac{2}{3}=\frac{1}{12}$

69. Since, A and B are independent events

$\therefore$ $P(A\cap B)=P(A).P(B)$

$=[1-P(\overline{A})]\,[1-P(\overline{B})]$

$=[1-2/3]\,[1-2/7]$

$=\frac{1}{3}\cdot\frac{5}{7}=\frac{5}{21}$

70. Event that at least one of them is a boy $\to$ A,
Event that other is girl $\to$ B,
So, required probability

$P(B/A)=\frac{P(B\cap A)}{P(A)}$

Now, total cases are 3 (BG, BB, GG)

$\therefore$ $\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$

....[$\because$ $B\cap A=\{BG\}$ and $A=\{BG,BB\}$]

71. Since events are mutually exclusive, therefore

$P(A\cap B)=0$ i.e., $P(A\cup B)=P(A)+P(B)$

$\Rightarrow 0.7=0.4+x\ \Rightarrow x=\frac{3}{10}$

72. Required probability $=\frac{(21)!2!}{(22)!}=\frac{1}{11}=\frac{1}{1+10}$

$\therefore$ Odds against = 10 : 1.

73. Probability [Person A will die in 30 years]

$=\frac{8}{8+5}$

$\therefore$ $P(A)=\frac{8}{13}\Rightarrow P(\overline{A})=\frac{5}{13}$

Similarly, $P(B)=\frac{4}{7}\Rightarrow P(\overline{B})=\frac{3}{7}$

There are two ways in which one person is alive after 30 years. $\overline{A}B$ and $A\overline{B}$ are independent events.

So, required probability

$=P(\overline{A}).P(B)+P(A).P(\overline{B})$

$=\frac{5}{13}\times\frac{4}{7}+\frac{8}{13}\times\frac{3}{7}=\frac{44}{91}$

74. Required probability $=\frac{0.1}{0.1+0.32}$

$=\frac{0.1}{0.42}$

$=\frac{5}{21}$

75. Let E_1 be the event that the ball is drawn from bag A, E_2 the event that it is drawn from bag B and E that the ball is red. We have to find $P\left(\frac{E_2}{E}\right)$.

Since, both the bags are equally likely to be selected, we have $P(E_1)=P(E_2)=\frac{1}{2}$

Also, $P\left(\frac{E}{E_1}\right)=\frac{3}{5}$ and $P\left(\frac{E}{E_2}\right)=\frac{5}{9}$.

Hence, by Bayes' theorem, we have

$$P\left(\frac{E_2}{E}\right)=\frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$=\frac{\frac{1}{2}\cdot\frac{5}{9}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{5}{9}}$$

$$=\frac{25}{52}$$

76. The probability of solving the question by these three students are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively.

$\therefore$ $P(A) = \frac{1}{3}; P(B) = \frac{2}{7}; P(C) = \frac{3}{8}$

Then, probability of question solved by only one student $= P(A\bar{B}\bar{C}$ or $\bar{A}B\bar{C}$ or $\bar{A}\bar{B}C)$

$$= P(A)\,P(\bar{B})P(\bar{C}) + P(\bar{A})P(B)\,P(\bar{C}) + P(\bar{A})\,P(\bar{B})P(C)$$

$$= \frac{1}{3}\cdot\frac{5}{7}\cdot\frac{5}{8} + \frac{2}{3}\cdot\frac{2}{7}\cdot\frac{5}{8} + \frac{2}{3}\cdot\frac{5}{7}\cdot\frac{3}{8}$$

$$= \frac{25+20+30}{168} = \frac{25}{56}$$

77. Let E denote the event that a six occurs and A be the event that the man reports, that it is a '6'. Then,

$P(E) = \frac{1}{6}$, $P(E') = \frac{5}{6}$, $P(A/E) = \frac{3}{4}$ and $P(A/E') = \frac{1}{4}$

From Baye's theorem,

$$P(E/A) = \frac{P(E).P(A/E)}{P(E).P(A/E)+P(E').P(A/E')}$$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}} = \frac{3}{8}$$

78. Let E_1 be the event that the ball is drawn from bag A, E_2 the event that it is drawn from bag B and E, that the ball is red. We have to find $P(E_2/E)$.

Since both the bags are equally likely to be selected, we have $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(E/E_1) = \frac{3}{5}$, $P(E/E_2) = \frac{5}{9}$

Hence by Baye's theorem, we have

$$P(E_2/E) = \frac{P(E_2)P(E/E_2)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{5}{9}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{5}{9}} = \frac{25}{52}$$

79. Let A be the event of selecting bag X, B be the event of selecting bag Y and E be the event of drawing a white ball, the $P(A) = 1/2$, $P(B) = 1/2$, $P(E/A) = 2/5$, $P(E/B) = 4/6 = 2/3$

$\therefore$ $P(E) = P(A)\,P(E/A) + P(B)P(E/B)$

$$= \frac{1}{2}\cdot\frac{2}{5}+\frac{1}{2}\cdot\frac{2}{3}$$

$$= \frac{8}{15}$$

80. Consider the following events:

$E_1 \to$ He knows the answer, $E_2 \to$ He guesses the answer

$A \to$ He gets the correct answer.

We have,

$P(E_1) = \frac{90}{100} = \frac{9}{10}$, $P(E_2) = \frac{1}{10}$,

$P(A/E_1) = 1$, $P(A/E_2) = \frac{1}{4}$

$\therefore$ Required probability $= P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{10}\times\frac{1}{4}}{\frac{9}{10}\times 1+\frac{1}{10}\times\frac{1}{4}}$$

$$= \frac{1}{37}$$

81. K = He knows the answers, NK = He randomly ticks the answers, C = He is correct

$$P\left(\frac{K}{C}\right) = \frac{P(K).P\left(\frac{C}{K}\right)}{P(K).P\left(\frac{C}{K}\right)+P(NK).P\left(\frac{C}{NK}\right)}$$

$$= \frac{p\times 1}{p\times 1+(1-p)\times\frac{1}{5}}$$

$$= \frac{5p}{4p+1}$$